# Convection

## Terms

$D$ = diameter (m)

$\bar{h}$ = average convection heat-transfer coefficient of the fluid [W/(m$^2$•K)]

$L$ = length (m)

$\overline{Nu}$ = average Nusselt number

Pr = Prandtl number = $\dfrac{c_P \mu}{k}$

$u_m$ = mean velocity of fluid (m/s)

$u_\infty$ = free stream velocity of fluid (m/s)

$\mu$ = dynamic viscosity of fluid [kg/(m•s)]

$\rho$ = density of fluid (kg/m$^3$)

## External Flow

In all cases, evaluate fluid properties at average temperature between that of the body and that of the flowing fluid.

Flat Plate of Length $L$ in Parallel Flow

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu}$$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.6640\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} \qquad \left(\mathrm{Re}_L < 10^5\right)$$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.0366\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \qquad \left(\mathrm{Re}_L > 10^5\right)$$

Cylinder of Diameter $D$ in Cross Flow

$$\mathrm{Re}_D = \frac{\rho u_\infty D}{\mu}$$

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C\,\mathrm{Re}_D^n\,\mathrm{Pr}^{1/3}$$

where

| $\mathrm{Re}_D$ | $C$ | $n$ |
|---|---|---|
| 1 – 4 | 0.989 | 0.330 |
| 4 – 40 | 0.911 | 0.385 |
| 40 – 4,000 | 0.683 | 0.466 |
| 4,000 – 40,000 | 0.193 | 0.618 |
| 40,000 – 250,000 | 0.0266 | 0.805 |

Flow Over a Sphere of Diameter, D

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 2.0 + 0.60\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\left(1 < \mathrm{Re}_D < 70{,}000; 0.6 < \mathrm{Pr} < 400\right)$$

## Internal Flow

$$\mathrm{Re}_D = \frac{\rho u_m D}{\mu}$$

Laminar Flow in Circular Tubes

For laminar flow ($\mathrm{Re}_D < 2300$), fully developed conditions

$\mathrm{Nu}_\mathrm{D} = 4.36$ (uniform heat flux)

$\mathrm{Nu}_\mathrm{D} = 3.66$ (constant surface temperature)

For laminar flow ($\mathrm{Re}_D < 2300$), combined entry length with constant surface temperature

$$Nu_D = 1.86\left(\frac{\mathrm{Re}_D \mathrm{Pr}}{\frac{L}{D}}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

where

$L$ = length of tube (m)

$D$ = tube diameter (m)

$\mu_b$ = dynamic viscosity of fluid [kg/(m•s)] at bulk temperature of fluid $T_b$

$\mu_s$ = dynamic viscosity of fluid [kg/(m•s)] at inside surface temperature of the tube $T_s$

Turbulent Flow in Circular Tubes

**Dittus-Boelter Equation**

$$Nu_D = 0.023\,\mathrm{Re}_D{}^{4/5}\,\mathrm{Pr}^n \qquad \text{where} \begin{bmatrix} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re}_D \gtrsim 10{,}000 \\ \frac{L}{D} \gtrsim 10 \end{bmatrix}$$

where

$n = 0.4$ for heating

$n = 0.3$ for cooling

Noncircular Ducts

In place of the diameter, $D$, use the equivalent (hydraulic) diameter ($D_H$) defined as

$$D_H = \frac{4 \times \text{cross-sectional area}}{\text{wetted perimeter}}$$

Circular Annulus ($D_o > D_i$)

In place of the diameter, $D$, use the equivalent (hydraulic) diameter ($D_H$) defined as

$$D_H = D_o - D_i$$

Liquid Metals ($0.003 < \mathrm{Pr} < 0.05$)

$$Nu_D = 6.3 + 0.0167\,\mathrm{Re}_D^{0.85}\mathrm{Pr}^{0.93} \text{ (uniform heat flux)}$$

$$Nu_D = 7.0 + 0.025\,\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.8} \text{ (constant wall temperature)}$$

## Heat Exchangers

The rate of heat transfer associated with either stream in a heat exchanger in which incompressible fluid or ideal gas with constant specific heats flows is

$$\dot{Q} = \dot{m}c_p\left(T_{\text{exit}} - T_{\text{inlet}}\right)$$

where

$c_p$ = specific heat (at constant pressure)

$\dot{m}$ = mass flow rate

The rate of heat transfer in a heat exchanger is

$$\dot{Q} = UAF\Delta T_{lm}$$

where

$A$ = any convenient reference area ($m^2$)

$F$ = correction factor for log mean temperature difference for more complex heat exchangers (shell and tube arrangements with several tube or shell passes or cross-flow exchangers with mixed and unmixed flow); otherwise $F = 1$.

$U$ = overall heat-transfer coefficient based on area A and the log mean temperature difference [$W/(m^2 \bullet K)$]

$\Delta T_{lm}$ = log mean temperature difference (K)

Log Mean Temperature Difference (LMTD)

For *counterflow* in tubular heat exchangers

$$\Delta T_{lm} = \frac{(T_{Ho} - T_{Ci}) - (T_{Hi} - T_{Co})}{\ln\left(\dfrac{T_{Ho} - T_{Ci}}{T_{Hi} - T_{Co}}\right)}$$

For *parallel flow* in tubular heat exchangers

$$\Delta T_{lm} = \frac{\left(T_{Ho} - T_{Co}\right) - \left(T_{Hi} - T_{Ci}\right)}{\ln\left(\dfrac{T_{Ho} - T_{Co}}{T_{Hi} - T_{Ci}}\right)}$$

where

$\Delta T_{lm}$ = log mean temperature difference (K)

$T_{Hi}$ = inlet temperature of the hot fluid (K)

$T_{Ho}$ = outlet temperature of the hot fluid (K)

$T_{Ci}$ = inlet temperature of the cold fluid (K)

$T_{Co}$ = outlet temperature of the cold fluid (K)

Heat Exchanger Effectiveness, ε

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{actual heat transfer rate}}{\text{maximum possible heat transfer rate}}$$

$$\varepsilon = \frac{C_H\left(T_{Hi} - T_{Ho}\right)}{C_{\min}\left(T_{Hi} - T_{Ci}\right)} \quad \text{or} \quad \varepsilon = \frac{C_C\left(T_{Co} - T_{Ci}\right)}{C_{\min}\left(T_{Hi} - T_{Ci}\right)}$$

where

$C = \dot{m}c_P =$ heat capacity rate (W/K)

$C_{\min}$ = smaller of $C_C$ or $C_H$

Number of Transfer Units ($NTU$)

$$NTU = \frac{UA}{C_{\min}}$$

Effectiveness-NTU Relations

$$C_r = \frac{C_{\min}}{C_{\max}} = \text{heat capacity ratio}$$

For *parallel flow concentric tube* heat exchanger

$$\varepsilon = \frac{1 - \exp\left[-NTU(1 + C_r)\right]}{1 + C_r}$$

$$NTU = -\frac{\ln\left[1 - \varepsilon(1 + C_r)\right]}{1 + C_r}$$

For *counterflow concentric tube* heat exchanger

$$\varepsilon = \frac{1 - \exp\left[-NTU(1 - C_r)\right]}{1 - C_r\exp\left[-NTU(1 - C_r)\right]} \qquad (C_r < 1)$$

$$\varepsilon = \frac{NTU}{1 + NTU} \qquad (C_r = 1)$$

$$NTU = \frac{1}{C_r - 1}\ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \qquad (C_r < 1)$$

$$NTU = \frac{\varepsilon}{1 - \varepsilon} \qquad (C_r = 1)$$

Overall Heat-Transfer Coefficient for Concentric Tube and Shell-and-Tube Heat Exchangers

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi kL} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o}$$

where

$A_i$ = inside area of tubes ($\text{m}^2$)

$A_o$ = outside area of tubes ($\text{m}^2$)

$D_i$ = inside diameter of tubes (m)

$D_o$ = outside diameter of tubes (m)

$h_i$ = convection heat-transfer coefficient for inside of tubes [W/($\text{m}^2\bullet$K)]

$h_o$ = convection heat-transfer coefficient for outside of tubes [W/($\text{m}^2\bullet$K)]

$k$ = thermal conductivity of tube material [W/(m•K)]

$R_{fi}$ = fouling factor for inside of tube [($\text{m}^2\bullet$K)/W]

$R_{fo}$ = fouling factor for outside of tube [($\text{m}^2\bullet$K)/W]