Heat Transfer

There are three modes of heat transfer: conduction, convection, and radiation.

Basic Heat-Transfer Rate Equations

Conduction

Fourier's Law of Conduction

$$\dot{Q} = -kA\frac{dT}{dx}$$

where

$\dot{Q}$ = rate of heat transfer (W)

k = thermal conductivity [W/(m•K)]

A = surface area perpendicular to direction of heat transfer (m^2)

Convection

Newton's Law of Cooling

$$\dot{Q} = hA\left(T_w - T_\infty\right)$$

where

h = convection heat-transfer coefficient of the fluid [W/(m^2•K)]

A = convection surface area (m^2)

T_w = wall surface temperature (K)

T_∞ = bulk fluid temperature (K)

Radiation

The radiation emitted by a body is given by

$$\dot{Q} = \varepsilon\sigma AT^4$$

where

ε = emissivity of the body

σ = Stefan-Boltzmann constant

= 5.67×10^{-8} W/(m^2•K^4)

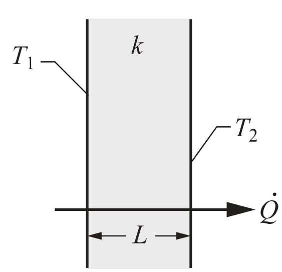
A = body surface area (m^2)

T = absolute temperature (K)

Conduction

Conduction Through a Plane Wall

$$\dot{Q} = \frac{-kA\left(T_2 - T_1\right)}{L}$$



where

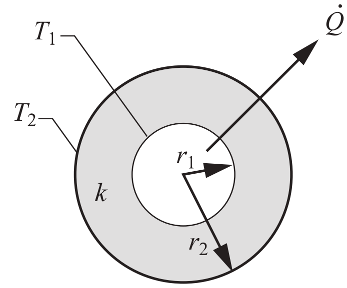
A = wall surface area normal to heat flow (m^2)

L = wall thickness (m)

T_1 = temperature of one surface of the wall (K)

T_2 = temperature of the other surface of the wall (K)

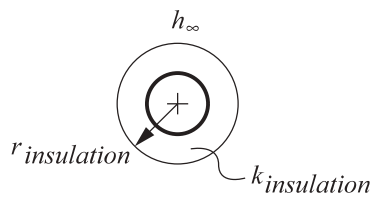
Conduction Through a Cylindrical Wall



Cylinder (Length = L)

$$\dot{Q} = \frac{2\pi k L (T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

Critical Insulation Radius



$$r_{cr} = \frac{k_{insulation}}{h_\infty}$$

Thermal Resistance (R)

$$\dot{Q} = \frac{\Delta T}{R_{total}}$$

Resistances in series are added:

$$R_{total} = \Sigma R$$

where

Plane Wall Conduction Resistance (K/W):

$$R = \frac{L}{kA}$$

where L = wall thickness

Cylindrical Wall Conduction Resistance (K/W):

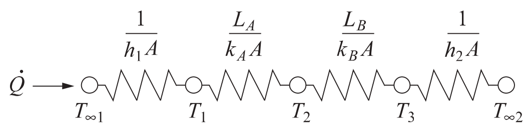
$$R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L}$$

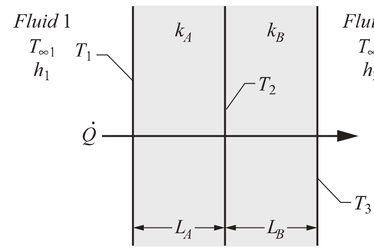
where L = cylinder length

Convection Resistance (K/W) :

$$R = \frac{1}{hA}$$

Composite Plane Wall

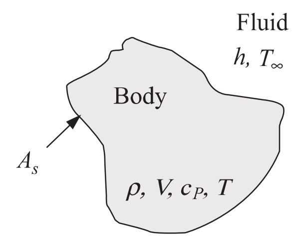




To evaluate surface or intermediate temperatures:

$$\dot{Q} = \frac{T_1 - T_2}{R_A} = \frac{T_2 - T_3}{R_B}$$

Transient Conduction Using the Lumped Capacitance Model



The lumped capacitance model is valid if

$$\text{Biot number, Bi} = \frac{hV}{kA_s} < 0.1$$

where

h = convection heat-transfer coefficient of the fluid [W/(m^2•K)]
V = volume of the body (m^3)
k = thermal conductivity of the body [W/(m•K)]
A_s = surface area of the body (m^2)

Constant Fluid Temperature

If the temperature may be considered uniform within the body at any time, the heat-transfer rate at the body surface is given by

$$\dot{Q} = hA_s\left(T - T_\infty\right) = -\rho V\left(c_P\right)\left(\frac{dT}{dt}\right)$$

where

T = body temperature (K)
T_∞ = fluid temperature (K)
ρ = density of the body (kg/m^3)
c_P = heat capacity of the body [J/(kg•K)]
t = time (s)

The temperature variation of the body with time is

$$T - T_\infty = \left(T_i - T_\infty\right)e^{-\beta t}$$

$$\beta = \frac{hA_s}{\rho V c_P}$$

where

$$\beta = \frac{1}{\tau}$$

τ = time constant (s)

The total heat transferred (Q_{total}) up to time t is

$$Q_{\text{total}} = \rho V c_P\left(T_i - T\right)$$

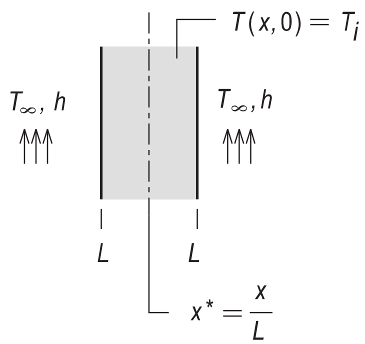
where T_i = initial body temperature (K)

Approximate Solution for Solid with Sudden Convection

The time dependence of the temperature at any location within the solid is the same as that of the midplane/centerline/centerpoint temperature T_o.

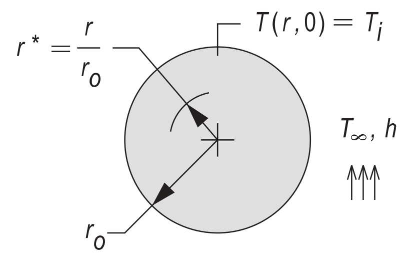
PLANE WALL

For $Fo = \dfrac{\alpha t}{L^2} > 0.2$



INFINITE CYLINDER AND SPHERE

For $Fo = \dfrac{\alpha t}{r_o^2} > 0.2$



where

T_∞ = bulk fluid temperature
T_i = initial uniform temperature of solid
T_o = temperature at midplane of wall, centerline of cylinder, centerpoint of sphere at time t
L = half-thickness of plane wall
x = distance from midplane of wall
r_o = radius of cylinder/sphere
r = radial distance from centerline of cylinder/centerpoint of sphere
h = convective heat transfer coefficient
t = time
α = thermal diffusivity = $\dfrac{k}{\rho c}$
k = thermal conductivity of solid
ρ = density of solid
c = specific heat of solid

$$\left(T_o - T_\infty\right)/\left(T_i - T_\infty\right) = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

where C_1 and ζ are obtained from the following table

Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction						
	Plane Wall		Infinite Cylinder		Sphere	
*Bi**	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1732	1.0049	0.2439	1.0075	0.2989	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2217	1.0082	0.3142	1.0124	0.3852	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3708	1.0173	0.4550	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0160	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6608	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8448	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.40	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.50	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.60	0.7051	1.0814	1.0185	1.1346	1.2644	1.1713
0.70	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.80	0.7910	1.1016	1.1490	1.1725	1.4320	1.2236
0.90	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1795	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7201
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8674
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8921
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5707	1.2733	2.4050	1.6018	3.1415	2.0000

*$Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere.

Incropera, Frank P. and David P. DeWitt, *Introduction to Heat Transfer*, 4th ed., John Wiley and Sons, 2002, pp. 256–261.

Fins

For a straight fin with uniform cross section (assuming negligible heat transfer from tip),

$$\dot{Q} = \sqrt{hPkA_c}\left(T_b - T_\infty\right)\tanh\left(mL_c\right)$$

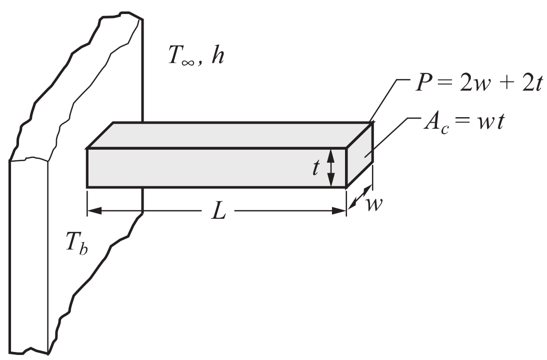
where

h = convection heat-transfer coefficient of the fluid [W/(m^2•K)]
P = perimeter of exposed fin cross section (m)
k = fin thermal conductivity [W/(m•K)]
A_c = fin cross-sectional area (m^2)
T_b = temperature at base of fin (K)
T_∞ = fluid temperature (K)

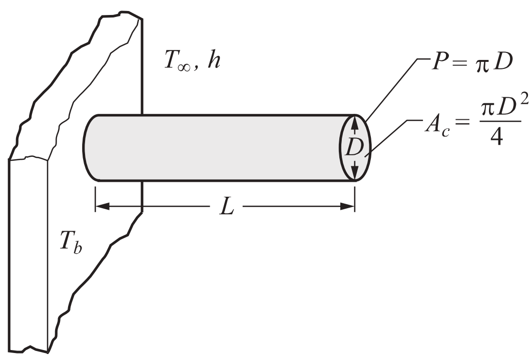
$$m = \sqrt{\frac{hP}{kA_c}}$$

$L_c = L + \frac{A_c}{P}$, corrected length of fin (m)

Rectangular Fin



Pin Fin



Convection

Terms

D = diameter (m)
$\bar{h}$ = average convection heat-transfer coefficient of the fluid [W/(m^2•K)]
L = length (m)
$\overline{Nu}$ = average Nusselt number
Pr = Prandtl number = $\frac{c_P \mu}{k}$
u_m = mean velocity of fluid (m/s)

u_∞ = free stream velocity of fluid (m/s)
μ = dynamic viscosity of fluid [kg/(m•s)]
ρ = density of fluid (kg/m^3)

External Flow

In all cases, evaluate fluid properties at average temperature between that of the body and that of the flowing fluid.

Flat Plate of Length L in Parallel Flow

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu}$$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.6640\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} \qquad \left(\mathrm{Re}_L < 10^5\right)$$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.0366\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \qquad \left(\mathrm{Re}_L > 10^5\right)$$

Cylinder of Diameter D in Cross Flow

$$\mathrm{Re}_D = \frac{\rho u_\infty D}{\mu}$$

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C\,\mathrm{Re}_D^n\,\mathrm{Pr}^{1/3}$$

where

Re_D	C	n
1 – 4	0.989	0.330
4 – 40	0.911	0.385
40 – 4,000	0.683	0.466
4,000 – 40,000	0.193	0.618
40,000 – 250,000	0.0266	0.805

Flow Over a Sphere of Diameter, D

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 2.0 + 0.60\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\left(1 < \mathrm{Re}_D < 70{,}000; 0.6 < \mathrm{Pr} < 400\right)$$

Internal Flow

$$\mathrm{Re}_D = \frac{\rho u_m D}{\mu}$$

Laminar Flow in Circular Tubes

For laminar flow ($\mathrm{Re}_D < 2300$), fully developed conditions

$\mathrm{Nu}_D = 4.36$ (uniform heat flux)
$\mathrm{Nu}_D = 3.66$ (constant surface temperature)

For laminar flow ($\mathrm{Re}_D < 2300$), combined entry length with constant surface temperature

$$Nu_D = 1.86\left(\frac{\mathrm{Re}_D\,\mathrm{Pr}}{\frac{L}{D}}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

where

L = length of tube (m)
D = tube diameter (m)
μ_b = dynamic viscosity of fluid [kg/(m•s)] at bulk temperature of fluid T_b
μ_s = dynamic viscosity of fluid [kg/(m•s)] at inside surface temperature of the tube T_s

<u>Turbulent Flow in Circular Tubes</u>

Dittus-Boelter Equation

$$Nu_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n \qquad \text{where}\begin{bmatrix} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re}_D \gtrsim 10{,}000 \\ \dfrac{L}{D} \gtrsim 10 \end{bmatrix}$$

where

$n = 0.4$ for heating

$n = 0.3$ for cooling

should be used for small to moderate temperature differences

Sieder-Tate Equation

$$Nu_D = 0.027\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad \text{where}\begin{bmatrix} 0.7 \le \mathrm{Pr} \le 16{,}700 \\ \mathrm{Re}_D \gtrsim 10{,}000 \\ \dfrac{L}{D} \gtrsim 10 \end{bmatrix}$$

should be used for flows characterized by large property variations.

Incropera, Frank P. and David P. DeWitt, *Fundamentals of Heat and Mass Transfer*, 3rd ed., Wiley, 1990, p. 496.

<u>Noncircular Ducts</u>

In place of the diameter, D, use the equivalent (hydraulic) diameter (D_H) defined as

$$D_H = \frac{4 \times \text{cross-sectional area}}{\text{wetted perimeter}}$$

<u>Circular Annulus ($D_o > D_i$)</u>

In place of the diameter, D, use the equivalent (hydraulic) diameter (D_H) defined as

$$D_H = D_o - D_i$$

<u>Liquid Metals (0.003 < Pr < 0.05)</u>

$$Nu_D = 6.3 + 0.0167\,\mathrm{Re}_D^{0.85}\mathrm{Pr}^{0.93} \text{ (uniform heat flux)}$$

$$Nu_D = 7.0 + 0.025\,\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.8} \text{ (constant wall temperature)}$$

Boiling

Evaporation occurring at a solid-liquid interface when

$T_{\text{solid}} > T_{\text{sat, liquid}}$

$q'' = h(T_s - T_{\text{sat}}) = h\Delta T_e$

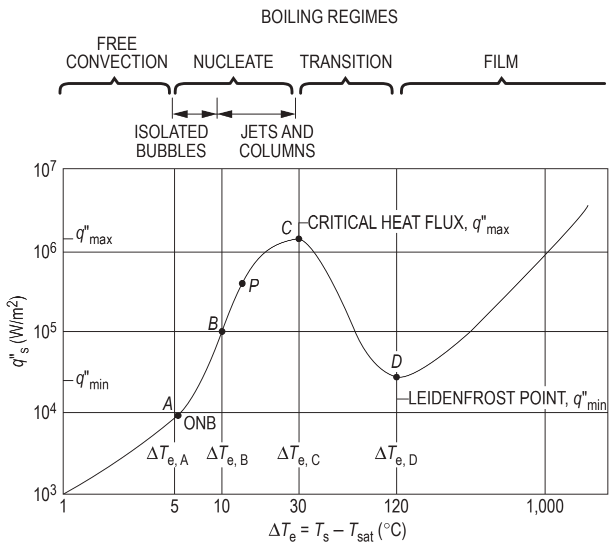
where ΔT_e = excess temperature

Pool Boiling – Liquid is quiescent; motion near solid surface is due to free convection and mixing induced by bubble growth and detachment.

Forced Convection Boiling – Fluid motion is induced by external means in addition to free convection and bubble-induced mixing.

Sub-Cooled Boiling – Temperature of liquid is below saturation temperature; bubbles forming at surface may condense in the liquid.

Saturated Boiling – Liquid temperature slightly exceeds the saturation temperature; bubbles forming at the surface are propelled through liquid by buoyancy forces.



Incropera, Frank P. and David P. DeWitt, *Fundamentals of Heat and Mass Transfer*, 3rd ed., Wiley, 1990. Reproduced with permission of John Wiley & Sons, Inc.

Typical boiling curve for water at one atmosphere: surface heat flux q''_s as a function of excess temperature, $\Delta T_e = T_s - T_{sat}$

Free Convection Boiling – Insufficient vapor is in contact with the liquid phase to cause boiling at the saturation temperature.

Nucleate Boiling – Isolated bubbles form at nucleation sites and separate from surface; vapor escapes as jets or columns.

For nucleate boiling a widely used correlation was proposed in 1952 by Rohsenow:

$$\dot{q}_{\text{nucleate}} = \mu_l \, h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf}\, h_{fg}\, \text{Pr}_l^n}\right]^3$$

where

$\dot{q}_{\text{nucleate}}$ = nucleate boiling heat flux (W/m^2)
μ_1 = viscosity of the liquid [kg/(m•s)]
h_{fg} = enthalpy of vaporization (J/kg)
g = gravitational acceleration (m/s^2)
ρ_l = density of the liquid (kg/m^3)
ρ_v = density of the vapor (kg/m^3)
σ = surface tension of liquid–vapor interface (N/m)
c_{pl} = specific heat of the liquid [J/(kg•°C)]
T_s = surface temperature of the heater (°C)
T_{sat} = saturation temperature of the fluid (°C)
C_{sf} = experimental constant that depends on surface–fluid combination
Pr_1 = Prandtl number of the liquid
n = experimental constant that depends on the fluid

Çengel, Yunus A., *Heat and Mass Transfer: A Practical Approach*, 3rd ed., New York: McGraw-Hill, 2007.

Peak Heat Flux

The maximum (or critical) heat flux (CHF) in nucleate pool boiling:

$$\dot{q}_{\max} = C_{cr}\, h_{fg}\left[\sigma g \rho^2{}_v (\rho_l - \rho_v)\right]^{1/4}$$

C_{cr} is a constant whose value depends on the heater geometry, but generally is about 0.15.

The CHF is independent of the fluid–heating surface combination, as well as the viscosity, thermal conductivity, and specific heat of the liquid.

The CHF increases with pressure up to about one-third of the critical pressure, and then starts to decrease and becomes zero at the critical pressure.

The CHF is proportional to h_{fg}, and large maximum heat fluxes can be obtained using fluids with a large enthalpy of vaporization, such as water.

Values of the coefficient C_{cr} for maximum heat flux (dimensionless parameter $L^* = L[g(\rho_l - \rho_v)/\sigma]^{1/2}$

Heater Geometry	C_{cr}	Charac. Dimension of Heater, L	Range of L^*
Large horizontal flat heater	0.149	Width or diameter	$L^* > 27$
Small horizontal flat heater[1]	$18.9\ K_1$	Width or diameter	$9 < L^* < 20$
Large horizontal cyclinder	0.12	Radius	$L^* > 1.2$
Small horizontal cyclinder	$0.12\ L^{*-0.25}$	Radius	$0.15 < L^* < 1.2$
Large sphere	0.11	Radius	$L^* > 4.26$
Small sphere	$0.227\ L^{*-0.5}$	Radius	$0.15 < L^* < 4.26$

[1] $K_1 = \sigma/[g(\rho_l - \rho_v)A_{\text{heater}}]$

Çengel, Yunus A., *Heat and Mass Transfer: A Practical Approach*, 3rd ed., New York: McGraw-Hill, 2007.

Minimum Heat Flux

Minimum heat flux, which occurs at the Leidenfrost point, it represents the lower limit for the heat flux in the film boiling regime.

Zuber derived the following expression for the minimum heat flux for a large horizontal plate

$$\dot{q}_{\min} = 0.09\ \rho_v\ h_{fg}\left[\frac{\sigma g(\rho_l - \rho_v)}{(\rho_l + \rho_v)^2}\right]^{1/4}$$

The relation above can be in error by 50% or more.

Transition Boiling – Rapid bubble formation results in vapor film on surface and oscillation between film and nucleate boiling.

Film Boiling – Surface completely covered by vapor blanket; includes significant radiation through vapor film.

Çengel, Yunus A., *Heat and Mass Transfer: A Practical Approach*, 3rd ed., New York: McGraw-Hill, 2007.

Film Boiling

The heat flux for film boiling on a horizontal cylinder or sphere of diameter D is given by

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{g k_v^3\ \rho_v(\rho_l - \rho_v)\left[h_{fg} + 0.4 c_{pv}(T_s - T_{\text{sat}})\right]}{\mu_v D(T_s - T_{\text{sat}})}\right]^{1/4}(T_s - T_{\text{sat}})$$

$$C_{\text{film}} = \begin{Bmatrix} 0.62 \text{ for horizontal cylinders} \\ 0.67 \text{ for spheres} \end{Bmatrix}$$

Çengel, Yunus A., *Heat and Mass Transfer: A Practical Approach*, 3rd ed., New York: McGraw-Hill, 2007.

Film Condensation of a Pure Vapor

On a Vertical Surface

$$\overline{Nu}_L = \frac{\bar{h}L}{k_l} = 0.943\left[\frac{\rho_l^2 g h_{fg} L^3}{\mu_l k_l \left(T_{\text{sat}} - T_s\right)}\right]^{0.25}$$

where

ρ_1 = density of liquid phase of fluid (kg/m^3)
g = gravitational acceleration (9.81 m/s^2)
h_{fg} = latent heat of vaporization (J/kg)
L = length of surface (m)
μ_1 = dynamic viscosity of liquid phase of fluid [kg/(s•m)]
k_1 = thermal conductivity of liquid phase of fluid [W/(m•K)]
T_{sat} = saturation temperature of fluid (K)
T_s = temperature of vertical surface (K)

Note: Evaluate all liquid properties at the average temperature between the saturated temperature T_{sat} and the surface temperature T_s.

Outside Horizontal Tubes

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.729\left[\frac{\rho_l^2 g h_{fg} D^3}{\mu_l k_l \left(T_{\text{sat}} - T_s\right)}\right]^{0.25}$$

where D = tube outside diameter (m)

Note: Evaluate all liquid properties at the average temperature between the saturated temperature T_{sat} and the surface temperature T_s.

Natural (Free) Convection

Vertical Flat Plate in Large Body of Stationary Fluid

Equation also can apply to vertical cylinder of sufficiently large diameter in large body of stationary fluid.

$$\bar{h} = C\left(\frac{k}{L}\right)\text{Ra}_L^n$$

where

L = length of the plate (cylinder) in the vertical direction

Ra_L = Rayleigh Number = $\dfrac{g\beta\left(T_s - T_\infty\right)L^3}{\upsilon^2}Pr$

T_s = surface temperature (K)
T_∞ = fluid temperature (K)
β = coefficient of thermal expansion (1/K)

(For an ideal gas: $\beta = \dfrac{2}{T_s + T_\infty}$ with T in absolute temperature)

υ = kinematic viscosity ($\text{m}^2\text{/s}$)

Range of Ra_L	C	n
$10^4 - 10^9$	0.59	1/4
$10^9 - 10^{13}$	0.10	1/3

Long Horizontal Cylinder in Large Body of Stationary Fluid

$$\overline{h} = C\left(\frac{k}{D}\right)\mathrm{Ra}_D^n$$

$$\mathrm{Ra}_D = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\upsilon^2}Pr$$

Ra_D	C	n
10^{-3} – 10^2	1.02	0.148
10^2 – 10^4	0.850	0.188
10^4 – 10^7	0.480	0.250
10^7 – 10^{12}	0.125	0.333

Heat Exchangers

The rate of heat transfer associated with either stream in a heat exchanger in which incompressible fluid or ideal gas with constant specific heats flows is

$$\dot{Q} = \dot{m}c_p\left(T_{\text{exit}} - T_{\text{inlet}}\right)$$

where

c_p = specific heat (at constant pressure)

$\dot{m}$ = mass flow rate

The rate of heat transfer in a heat exchanger is

$$\dot{Q} = UAF\Delta T_{lm}$$

where

A = any convenient reference area (m^2)

F = correction factor for log mean temperature difference for more complex heat exchangers (shell and tube arrangements with several tube or shell passes or cross-flow exchangers with mixed and unmixed flow); otherwise $F = 1$.

U = overall heat-transfer coefficient based on area A and the log mean temperature difference [$W/(m^2 \bullet K)$]

ΔT_{lm} = log mean temperature difference (K)

Log Mean Temperature Difference (LMTD)

For *counterflow* in tubular heat exchangers

$$\Delta T_{lm} = \frac{\left(T_{Ho} - T_{Ci}\right) - \left(T_{Hi} - T_{Co}\right)}{\ln\left(\dfrac{T_{Ho} - T_{Ci}}{T_{Hi} - T_{Co}}\right)}$$

For *parallel flow* in tubular heat exchangers

$$\Delta T_{lm} = \frac{\left(T_{Ho} - T_{Co}\right) - \left(T_{Hi} - T_{Ci}\right)}{\ln\left(\dfrac{T_{Ho} - T_{Co}}{T_{Hi} - T_{Ci}}\right)}$$

where

ΔT_{lm} = log mean temperature difference (K)

T_{Hi} = inlet temperature of the hot fluid (K)

T_{Ho} = outlet temperature of the hot fluid (K)

T_{Ci} = inlet temperature of the cold fluid (K)

T_{Co} = outlet temperature of the cold fluid (K)

Heat Exchanger Effectiveness, ε

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{actual heat transfer rate}}{\text{maximum possible heat transfer rate}}$$

$$\varepsilon = \frac{C_H\left(T_{Hi} - T_{Ho}\right)}{C_{\min}\left(T_{Hi} - T_{Ci}\right)} \quad \text{or} \quad \varepsilon = \frac{C_C\left(T_{Co} - T_{Ci}\right)}{C_{\min}\left(T_{Hi} - T_{Ci}\right)}$$

where

$C = \dot{m}c_P =$ heat capacity rate (W/K)

$C_{\min}$ = smaller of C_C or C_H

Number of Transfer Units (*NTU*)

$$NTU = \frac{UA}{C_{\min}}$$

Effectiveness-NTU Relations

$$C_r = \frac{C_{\min}}{C_{\max}} = \text{heat capacity ratio}$$

For *parallel flow concentric tube* heat exchanger

$$\varepsilon = \frac{1 - \exp\left[-NTU\left(1 + C_r\right)\right]}{1 + C_r}$$

$$NTU = -\frac{\ln\left[1 - \varepsilon\left(1 + C_r\right)\right]}{1 + C_r}$$

For *counterflow concentric tube* heat exchanger

$$\varepsilon = \frac{1 - \exp\left[-NTU\left(1 - C_r\right)\right]}{1 - C_r\exp\left[-NTU\left(1 - C_r\right)\right]} \qquad \left(C_r < 1\right)$$

$$\varepsilon = \frac{NTU}{1 + NTU} \qquad \left(C_r = 1\right)$$

$$NTU = \frac{1}{C_r - 1}\ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \qquad \left(C_r < 1\right)$$

$$NTU = \frac{\varepsilon}{1 - \varepsilon} \qquad \left(C_r = 1\right)$$

Overall Heat-Transfer Coefficient for Concentric Tube and Shell-and-Tube Heat Exchangers

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi kL} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o}$$

where

A_i = inside area of tubes (m^2)

A_o = outside area of tubes (m^2)

D_i = inside diameter of tubes (m)

D_o = outside diameter of tubes (m)

h_i = convection heat-transfer coefficient for inside of tubes [W/(m^2•K)]

h_o = convection heat-transfer coefficient for outside of tubes [W/(m^2•K)]

k = thermal conductivity of tube material [W/(m•K)]

R_{fi} = fouling factor for inside of tube [(m^2•K)/W]

R_{fo} = fouling factor for outside of tube [(m^2•K)/W]

Radiation

Types of Bodies

Any Body

For any body

$$\alpha + \rho + \tau = 1$$

where

α = absorptivity (ratio of energy absorbed to incident energy)

ρ = reflectivity (ratio of energy reflected to incident energy)

τ = transmissivity (ratio of energy transmitted to incident energy)

Opaque Body

For an opaque body

$$\alpha + \rho = 1$$

Gray Body

A gray body is one for which

$$\alpha = \varepsilon, (0 < \alpha < 1; 0 < \varepsilon < 1)$$

where

ε = the emissivity of the body

For a gray body

$$\varepsilon + \rho = 1$$

Real bodies are frequently approximated as gray bodies.

Black body

A black body is defined as one that absorbs all energy incident upon it. It also emits radiation at the maximum rate for a body of a particular size at a particular temperature. For such a body

$$\alpha = \varepsilon = 1$$

Shape Factor (View Factor, Configuration Factor) Relations

Reciprocity Relations

$$A_i F_{ij} = A_j F_{ji}$$

where

A_i = surface area (m^2) of surface i

F_{ij} = shape factor (view factor, configuration factor); fraction of the radiation leaving surface *i* that is intercepted by surface *j*; $0 \le F_{ij} \le 1$

Summation Rule for *N* Surfaces

$$\sum_{j=1}^{N} F_{ij} = 1$$

Net Energy Exchange by Radiation between Two Bodies

Body Small Compared to its Surroundings

$$\dot{Q}_{12} = \varepsilon\sigma A\left(T_1^4 - T_2^4\right)$$

where

$\dot{Q}_{12}$ = net heat-transfer rate from the body (W)

ε = emissivity of the body

σ = Stefan-Boltzmann constant [$\sigma = 5.67 \times 10^{-8}$ W/(m^2•K^4)]

A = body surface area (m^2)

T_1 = absolute temperature (K) of the body surface

T_2 = absolute temperature (K) of the surroundings

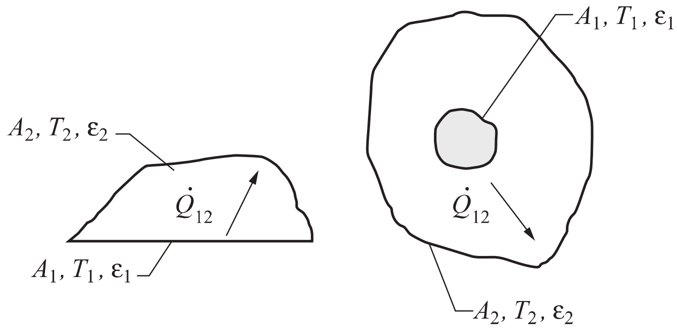
Net Energy Exchange by Radiation between Two Black Bodies

The net energy exchange by radiation between two black bodies that see each other is given by

$$\dot{Q}_{12} = A_1 F_{12}\,\sigma\left(T_1^4 - T_2^4\right)$$

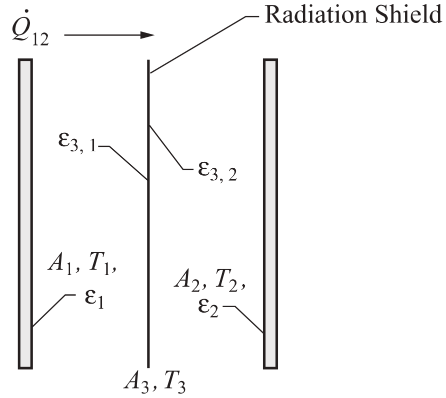
Net Energy Exchange by Radiation between Two Diffuse-Gray Surfaces that Form an Enclosure

Generalized Cases



$$\dot{Q}_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

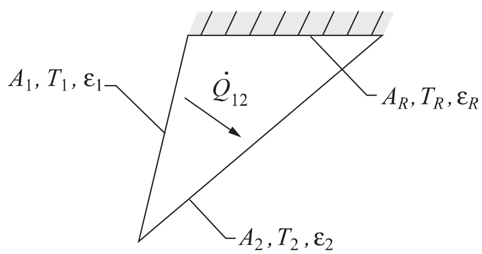
One-Dimensional Geometry with Thin Low-Emissivity Shield Inserted between Two Parallel Plates



$$\dot{Q}_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{13}} + \frac{1-\varepsilon_{3,1}}{\varepsilon_{3,1} A_3} + \frac{1-\varepsilon_{3,2}}{\varepsilon_{3,2} A_3} + \frac{1}{A_3 F_{32}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Reradiating Surface

Reradiating Surfaces are considered to be insulated or adiabatic $\left(\dot{Q}_R = 0\right)$.



$$\dot{Q}_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12} + \left[\left(\frac{1}{A_1 F_{1R}}\right) + \left(\frac{1}{A_2 F_{2R}}\right)\right]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$