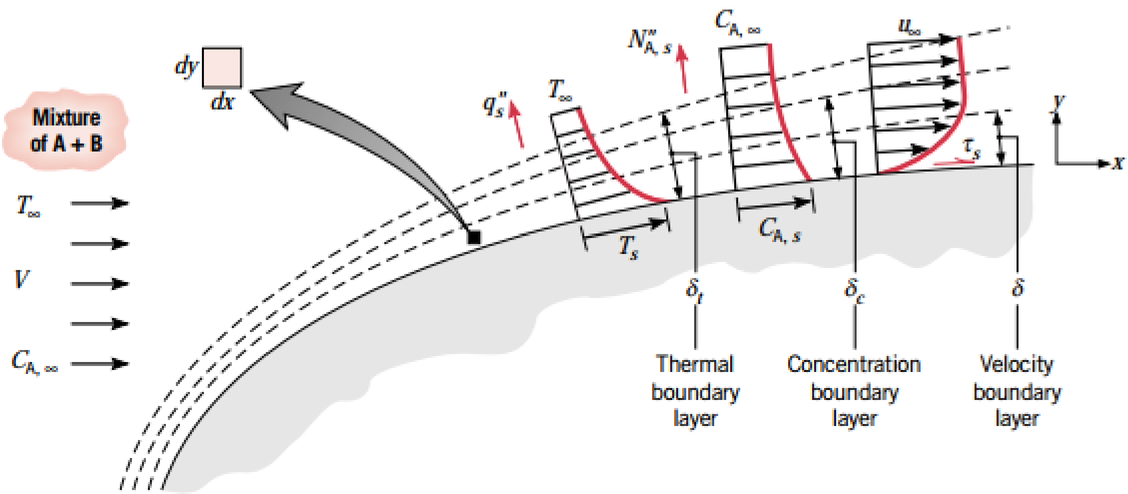
Boundary Layer Equations



Appendix D contains boundary layer equations in Cartesian coordinates for the following conditions:

- Laminar flow
- 2D flow
- Steady-state
- Incompressible fluid
- Constant properties

Appendix E contains boundary layer equations in Cartesian coordinates for the following conditions:

- Turbulent flow
- 2D flow
- Steady-state
- Incompressible fluid
- Constant properties

Boundary Layer Similarity Parameters for Heat Transfer

Dimensionless Length

$x^* = \frac{x}{L}$ $y^* = \frac{y}{L}$

Where L is the characteristic length (length of flat plate)

Dimensionless Velocity

$u^* = \frac{u}{V}$ $v^* = \frac{v}{V}$

Where V is the velocity upstream of the surface (flat plate)

Dimensionless Temperature and Concentration

$T^* = \frac{(T - T_S)}{(T_\infty - T_S)}$ $C^* = \frac{(C_A - C_{A,s})}{(C_{A.\infty} - C_{A,s})}$

Important Dimensionless Parmeters from Table 6.1

Reynolds Number:

$$RE_L = \frac{V * L}{v} = \frac{\rho * V * L}{\mu} = \frac{\rho * u * L}{\mu}$$

Prandtl Number:

$$RE_L, Pr = \frac{v}{\alpha}$$

Schmidt Number:

$$RE_L, Sc = \frac{v}{D_{AB}}$$

So long as the similarity parameters and dimensionless boundary conditions are the same for two sets of conditions → the solution of the differential equations for nondimensional velocity, temperature, and species concentration will also be the same!

Nondimensional Equations for Laminar Boundary Layers

TABLE 6.1 The boundary layer equations and their y-direction boundary conditions in nondimensional form

Boundary Layer	Conservation Equation	Boundary Conditions: Wall	Boundary Conditions: Free Stream	Similarity Parameter(s)
Velocity	$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$ (6.35)	$u^*(x^*,0) = 0$ $v^*(x^*,0) = 0$	$u^*(x^*,\infty) = \frac{u_\infty(x^*)}{V}$ (6.38)	$Re_L = \frac{VL}{\nu}$ (6.41)
Thermal	$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr}\frac{\partial^2 T^*}{\partial y^{*2}}$ (6.36)	$T^*(x^*,0) = 0$	$T^*(x^*,\infty) = 1$ (6.39)	$Re_L,\ Pr = \frac{\nu}{\alpha}$ (6.42)
Concentration	$u^*\frac{\partial C_A^*}{\partial x^*} + v^*\frac{\partial C_A^*}{\partial y^*} = \frac{1}{Re_L Sc}\frac{\partial^2 C_A^*}{\partial y^{*2}}$ (6.37)	$C_A^*(x^*,0) = 0$	$C_A^*(x^*,\infty) = 1$ (6.40)	$Re_L,\ Sc = \frac{\nu}{D_{AB}}$ (6.43)

Relationships Between Boundary Layers

TABLE 6.3 Functional relations pertinent to the boundary layer analogies

Fluid Flow		Heat Transfer		Mass Transfer	
$u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right)$	(6.44)	$T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right)$	(6.47)	$C_A^* = f\left(x^*, y^*, Re_L, Sc, \frac{dp^*}{dx^*}\right)$	(6.51)
$C_f = \frac{2}{Re_L}\frac{\partial u^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.45)	$Nu = \frac{hL}{k} = +\frac{\partial T^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.48)	$Sh = \frac{h_m L}{D_{AB}} = +\frac{\partial C_A^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.52)
$C_f = \frac{2}{Re_L} f(x^*, Re_L)$	(6.46)	$Nu = f(x^*, Re_L, Pr)$	(6.49)	$Sh = f(x^*, Re_L, Sc)$	(6.53)
		$\overline{Nu} = f(Re_L, Pr)$	(6.50)	$\overline{Sh} = f(Re_L, Sc)$	(6.54)

Additional Important Dimensionless Parameters

Nusselt Number

Dimensionless temperature gradient at the surface → used to find convection heat transfer coefficient

$$Nu \equiv \frac{hL}{k_f} = +\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

$$Nu = f(x^*, Re_L, Pr)$$

Average Nusselt Number

$$\overline{Nu} = \frac{\bar{h}L}{k_f} = f(Re_L, Pr)$$

Used to find average convection coefficient (and no longer dependent on x*)

Sherwood Number

Dimensionless concentration gradient at the surface → used to find convection mass transfer coefficient

$$Sh \equiv \frac{h_m L}{D_{AB}} = +\frac{\partial C_A^*}{\partial y^*}\bigg|_{y^*=0}$$

$$Sh = f(x^*, Re_L, Sc)$$

Average Sherwood Number

$$\overline{Sh} = \frac{\bar{h}_m L}{D_{AB}} = f(Re_L, Sc)$$

Used to find average convection mass transfer coefficient (also no longer dependent on x*)

TABLE 6.2 Selected dimensionless groups of heat and mass transfer

Group	Definition	Interpretation
Biot number (Bi)	$\frac{hL}{k_s}$	Ratio of the internal thermal resistance of a solid to the boundary layer thermal resistance.
Mass transfer Biot number (Bi_m)	$\frac{h_m L}{D_{AB}}$	Ratio of the internal species transfer resistance to the boundary layer species transfer resistance.
Bond number (Bo)	$\frac{g(\rho_l - \rho_v)L^2}{\sigma}$	Ratio of gravitational and surface tension forces.
Coefficient of friction (C_f)	$\frac{\tau_s}{\rho V^2/2}$	Dimensionless surface shear stress.
Eckert number (Ec)	$\frac{V^2}{c_p(T_s - T_\infty)}$	Kinetic energy of the flow relative to the boundary layer enthalpy difference.
Fourier number (Fo)	$\frac{\alpha t}{L^2}$	Ratio of the heat conduction rate to the rate of thermal energy storage in a solid. Dimensionless time.
Mass transfer Fourier number (Fo_m)	$\frac{D_{AB} t}{L^2}$	Ratio of the species diffusion rate to the rate of species storage. Dimensionless time.
Friction factor (f)	$\frac{\Delta p}{(L/D)(\rho u_m^2/2)}$	Dimensionless pressure drop for internal flow.
Grashof number (Gr_L)	$\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$	Measure of the ratio of buoyancy forces to viscous forces.
Colburn j factor (j_H)	$St\,Pr^{2/3}$	Dimensionless heat transfer coefficient.
Colburn j factor (j_m)	$St_m\,Sc^{2/3}$	Dimensionless mass transfer coefficient.
Jakob number (Ja)	$\frac{c_p(T_s - T_{sat})}{h_{fg}}$	Ratio of sensible to latent energy absorbed during liquid–vapor phase change.
Lewis number (Le)	$\frac{\alpha}{D_{AB}}$	Ratio of the thermal and mass diffusivities.
Nusselt number (Nu_L)	$\frac{hL}{k_f}$	Ratio of convection to pure conduction heat transfer.
Peclet number (Pe_L)	$\frac{VL}{\alpha} = Re_L\,Pr$	Ratio of advection to conduction heat transfer rates.
Prandtl number (Pr)	$\frac{c_p\mu}{k} = \frac{\nu}{\alpha}$	Ratio of the momentum and thermal diffusivities.

TABLE 6.2 *Continued*

Group	Definition	Interpretation
Reynolds number (Re_L)	$\frac{VL}{\nu}$	Ratio of the inertia and viscous forces.
Schmidt number (Sc)	$\frac{\nu}{D_{AB}}$	Ratio of the momentum and mass diffusivities.
Sherwood number (Sh_L)	$\frac{h_m L}{D_{AB}}$	Dimensionless concentration gradient at the surface.
Stanton number (St)	$\frac{h}{\rho V c_p} = \frac{Nu_L}{Re_L\, Pr}$	Modified Nusselt number.
Mass transfer Stanton number (St_m)	$\frac{h_m}{V} = \frac{Sh_L}{Re_L Sc}$	Modified Sherwood number.
Weber number (We)	$\frac{\rho V^2 L}{\sigma}$	Ratio of inertia to surface tension forces.