Finite Heat Release Model

- In the Otto cycle it is assumed that the heat is release instantaneously. A finite heat release model specifies heat release as a function of crank angle.

- This model can be used determine the effect of spark timing or heat transfer on engine work and efficiency.

- The cumulative heat release or “burn fraction” for SI engines is given by:

$$x_b(\theta) = 1 - \exp\left[-a\left(\frac{\theta - \theta_s}{\theta_d}\right)^n\right]$$

where θ = crank angle

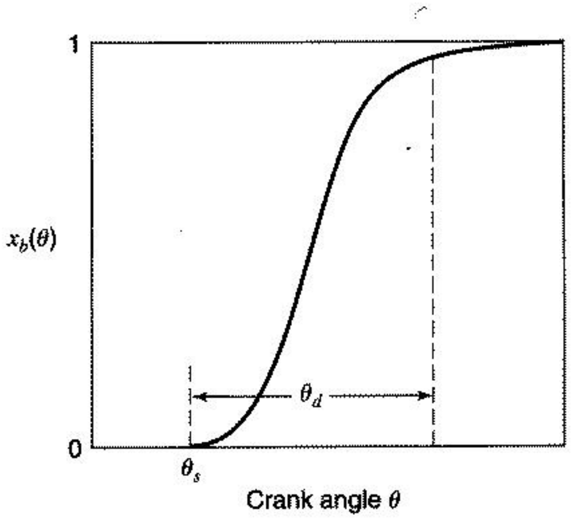
θ_s = start of heat release

θ_d = duration of heat release

$$\left.\begin{array}{l}\text{n = form factor}\\ \text{a = efficiency factor}\end{array}\right\}\ \text{Used to fit experimental data}$$

Finite Heat Release

A typical heat release curve consists of an initial spark ignition phase, followed by a rapid burning phase and ends with burning completion phase



The curve asymptotically approaches 1 so the end of combustion is defined by an arbitrary limit, such as 90% or 99% complete combustion where x_b = 0.90 or 0.99 corresponding values for efficiency factor *a* are 2.3 and 4.6

The rate of heat release as a function of crank angle is:

$$\frac{dQ}{d\theta} = \theta_{in}\frac{dx_b}{d\theta} = na\frac{\theta_{in}}{\theta_d}(1 - x_b)\left(\frac{\theta - \theta_s}{\theta_d}\right)^{n-1}$$

Finite Heat Release Model

Applying First Law to the closed system containing the gas in the cylinder for a small crank angle change, dθ,

$$dU = \delta Q - \delta W$$

assuming ideal gas $PV = mRT$ and $dU = mc_v dT$

$$\delta Q - PdV = \frac{c_v}{R}\left(PdV + VdP\right)$$

per unit crank angle

$$\frac{dQ}{d\theta} - P\frac{dV}{d\theta} = \frac{c_v}{R}\left(P\frac{dV}{d\theta} + V\frac{dP}{d\theta}\right)$$

$$\boxed{\frac{dP}{d\theta} = -k\frac{P}{V}\frac{dV}{d\theta} + \frac{k-1}{V}\left(\frac{dQ}{d\theta}\right)}$$

Finite Heat Release Model

The cylinder volume in terms of crank angle, V(θ), is

$$V(\theta) = \frac{V_d}{r-1} + \frac{V_d}{2}\left(R + 1 - \cos\theta - (R^2 - \sin^2\theta)^{1/2}\right)$$

Differentiating wrt θ

$$\frac{dV}{d\theta} = \frac{V_d}{2}\sin\theta\left(1 + \cos\theta(R^2 - \sin^2\theta)^{-1/2}\right)$$

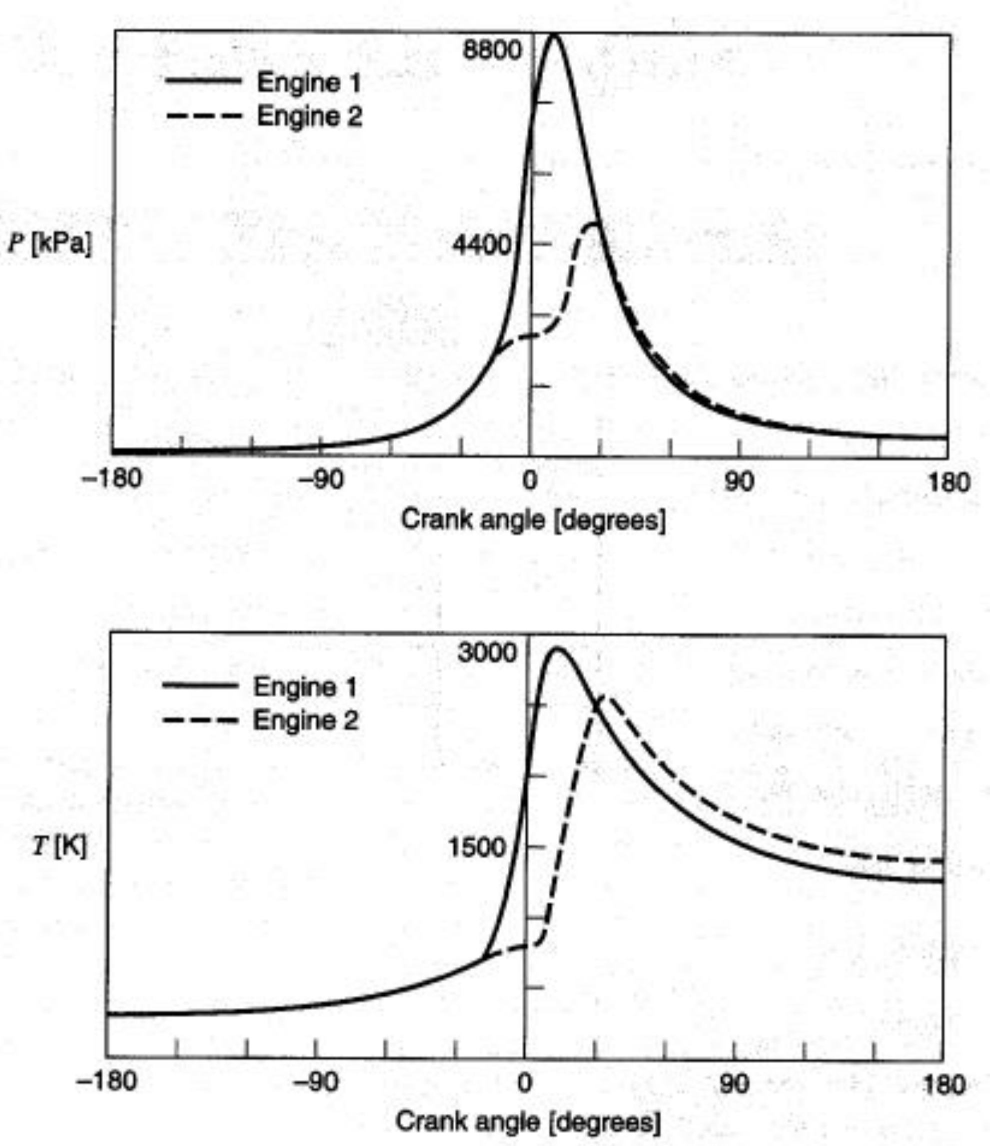
where $V_d = \frac{\pi}{4}B^2 S = \text{displacement volume}$

$r = \text{compression ratio}$

$\mathrm{R} = \frac{2l}{s}$

For the portion of the compression and expansion strokes with no heat Release, where $\theta < \theta_s$ and $\theta > \theta_s + \theta_d$ → $dQ/d\theta = 0$ and

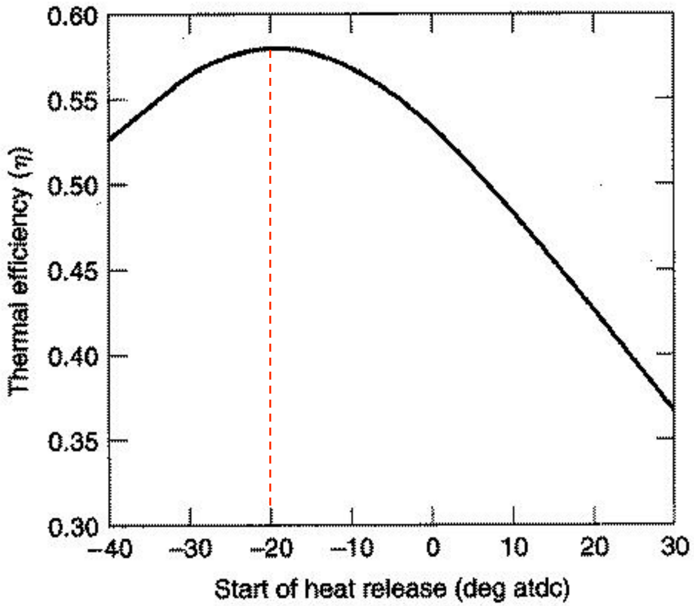
Finite Heat Release Model Results



Start of heat release:
Engine 1 - 20^o bTC
Engine 2 - TC

Duration 40^o

Finite Heat Release Model Results



Start of heat release θ_s (deg atdc)	Imep (bar)	Peak pressure crank angle θ_{max} (deg atdc)
−40	12.09	0
−30	12.96	5
−25	13.20	7
−20	13.29	10
−15	13.23	12
−10	13.02	17
−5	12.68	21
0	12.23	27
10	11.09	37
20	9.79	50
30	8.44	60