Combined Fatigue Loading Mode

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To begin a combined loading fatigue analysis you must first start with a stress analysis at the point of interest. This analysis will yield a maximum and minimum stress for each type of stress: axial, bending, and torsion. This process might involve moving an applied force to the point of interest and creating resulting moments and torques.

After calculating the maximum and minimum for each stresses the alternating and mean effective stresses must be calculated. The following equations can be used.

$$\sigma_{alt\_} = (\sigma_{max}-\sigma_{min})/2$$
$$\sigma_{mean} = (\sigma_{max}+\sigma_{min})/2$$

The alternating stress must then have various size, load, and stress concentration factors applied to it. This is necessary because these values are different for each loading mode. In addition, because these factors are applied to each stress they are not factored into endurance limit in the Marin equation. The following table shows how to apply these factors to each type of stress.

Axial	$\sigma_{alt}=K_{f,a}*\sigma_{alt,a}/k_{c,a}$
Bending	$\sigma_{alt=}\ K_{f,b}*\sigma_{alt,b}/k_b$
Torsion	$\tau_{alt=}\ K_{f\,t}*\tau_{alt\,t}/k_b$

The K_f in the above equations in the fatigue stress concentration factor incorporates the the notch sensitivity, q, which is a material property. The equation is $K_f=1+q(K_t-1)$. The effective alternating stresses adjusted for stress concentrations, size and load factor, are then used along with the effective mean stress to construct two Mohr's circles.

Mohr's Circle Construction

Given the axial, bending and shear stresses from our alternating and mean calculations, we can then construct a separate Mohr's circle for the alternating and the mean stresses. The alternating axial and bending stresses are combined and plotted on the alternating Mohr's circle σ axis. Then the alternating shear is plotted on the τ axis. For uni-axial stress elements combined stress plotted point will be plotted with shear and the other stress face will be plotted with zero on the σ axis and the opposite sign shear stress. For biaxial and tri-axial stress elements, there will be a combine σ stress for each faces of the stress element. Then you will plot the points on the Mohr's circle accordingly. From this we can then trace out the complete Mohr's circle. The maximum shear is found from radius equation of a Mohr's circle:

$$R = \sqrt{\left[\frac{\sigma\_alt1 - \sigma\_alt2}{2}\right]^2 + \tau\_alt^2}$$

σ_alt1 is the plotted point with combined axial and bending stresses. σ_alt2 is the stress plotted on the other stress face. For biaxial and tri-axial stress elements this will include the stress placed on the other faces of the stress element. τ_alt is the shear stress calculated from before. The principle stresses are found by first finding the center of the Mohr's circle. Then the radius of the Mohr's circle is added and/or subtracted to find the principle stresses. These principle stresses will then be entered into the equation for von Mises stresses. In this cases we will find the alternating von Mises stress or $\sigma`_a$. The same process is followed from start to finish to find the mean von Mises stress or $\sigma`_m$. These stresses will then be used to find the factors of safety or finite cycle life.

Fatigue Analysis of Combined Loading Mode

Once the von Mises stresses will have been calculated, there are two directions you can go with these values. Depending on what kind of design life the component has: you can find the life of the part, or the factor of safety used.

The first use of the von Mises stresses is for a life prediction for a finite life design. A finite life design means that the part will fail at a certain amount of cycles, given an alternating and mean stress. In this case, we will use the von Mises alternating and mean stress, since those stresses represent failure of the part. The number of cycles is determined by plotting σ_a` and $\sigma_m$` on a graph with the x-axis representing σ_{mean} and the y-axis represented by $\sigma_{alt.}$ Then the ultimate tensile strength is found and that value is plotted on the x-axis (y=0). From this point on the x-axis, a line is drawn to the von Mises plotted point, and is extended to the y-axis. This intersection on the y-axis is σ_f the z-axis on this graph represents N_f, or the number of cycles to failure. On the z-y plane is a graph of a stress-life (S-N) Basquin curve. From $\sigma_{f,}$ extend the line horizontally until it intersects the Basquin curve. Then, the corresponding z-value will give you the number of cycles that the part will experience before failure. This value can be determined graphically or through calculations using the following equations:

$$N_f= [\sigma_a`/a]^{1/b}$$

$$\text{where } a=(0.9*S_{ut})^2/S_e$$

$$\text{and } b=-1/3\log[0.9*S_{ut}/S_e]$$

The second use of the von Mises stresses is to determine the factor of safety for a component that is designed for infinite life. Infinite life means that the part will last, theoretically, for an infinite number of cycles at particular stress amplitude. The infinite life region is determined by the region below the line represented by the equation:

$$\sigma_a`/S_e + \sigma_m`/S_{ut} = 1/n$$

So once the von Mises stresses and the modified endurance limit have been determined, the ultimate tensile strength can be found in material property tables. Then the factor of safety is easily found by solving the modified Goodman equation above.