Moment Of Inertia

University of Idaho — mechanical engineering

1. *Divide the cross-sectional area into elements.* Cut up the cross-section into simple shapes, such as rectangles, triangles, circles, etc.

2. *Determine the area of each element.* After finding these, add them all together to find the total area of the shape.

3. *Select a convenient system of coordinate axes.* It is best if the coordinate axes lay outside of the cross-section.

4. *Find the centroids of each element relative to the coordinate axes.* Record distance from the centroid to the x axes as x^1, and distance from the centroid to the y axes as y^1.

5. *Find the static moment of each element.* Multiply the Area of each element with the distance from the centroid of each element to the reference axes(x and y). Add them all together to get total static moments Ai_x and Ai_y.

6. *Locate the centriod of the entire cross-section relative to the reference axes.* This is done by dividing the Ai_x and Ai_y by the total area.

7. *Calculate the 2nd moment of area for each element about the reference axes.* Multiply the Area of each element by the square of the distance from the centroid of each element to the centroid of the cross-section(x^1 and y^1).

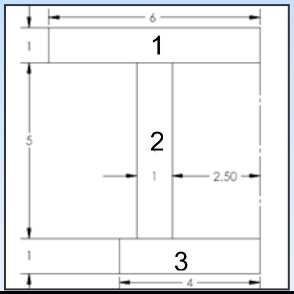
8. *Calculate moments of Inertia of each element about it's own centroid.* Check the basic shapes at the bottom of poster for help.

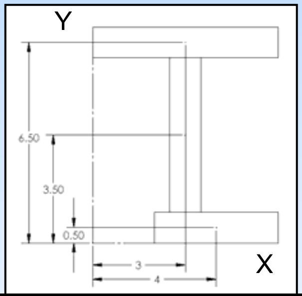
9. *Calculate Ix and Iy.* Do this by adding the results from step 7 and 8.

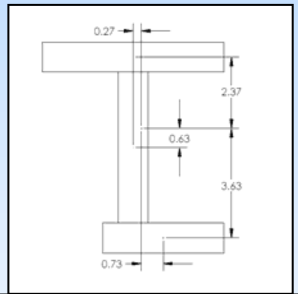
10. *Calculate Ixy.* Add together the Area for each element multiplied by x^1 and y^1 with I_{oxy} for each element, and sum the results for all the elements.

11. *Find the principal moments of inertia.* Use this equation.

$$Im, n = \frac{Ix + Iy}{2} + \left[\left(\frac{Ix - Iy}{2}\right)^2 + Ixy^2\right]^{\frac{1}{2}}$$



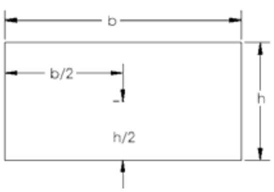




MOI's		Principal MOI's	
Iy	26.6835	I_1	102.09
Ix	99.64	I_2	24.23
Ixy	-13.59		

Element	b	h	A_i	x	y	Ai x	Ai y	x'	y'	A_i x'	A_i y'	$A_i x'^2$	I_{y0}	$A_i y'^2$	Ix_0	A_i x'y'
1	6	1	6	3	6.5	18	39	-0.27	2.37	-1.62	14.2	0.4374	18.0	34	0.500	-3.8394
2	1	5	5	3	3.5	15	17.5	-0.27	-0.63	-1.35	-3.2	0.3645	0.417	2	10.4	0.8505
3	4	1	4	4	0.5	16	2	0.73	-3.63	2.92	-14.5	2.1316	5.33	53	0.333	-10.5996
Sum			15			49	58.5			-0.05	-3.45	2.9335	23.75	88.3935	11.25	-13.5885

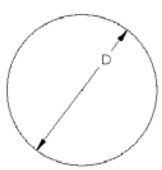
Rectangle



$$Ix = \frac{b \cdot h^3}{12} \quad Iy = \frac{h \cdot b^3}{12}$$

$Ixy = 0$

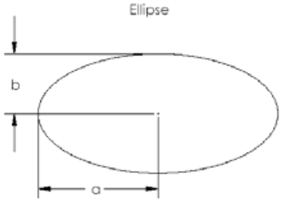
Circle



$$Ix = Iy = \frac{\pi \cdot r^4}{4}$$

$Ixy = 0$

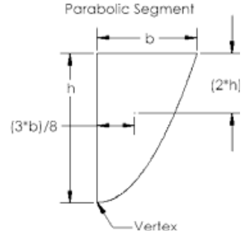
Ellipse



$$Ix = \frac{\pi \cdot a \cdot b^3}{4} \quad Iy = \frac{\pi \cdot b \cdot a^3}{4}$$

$Ixy = 0$

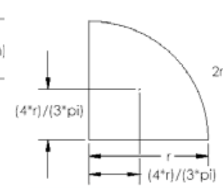
Parabolic Segment



$$Ix = \frac{16 \cdot b \cdot h^3}{105} \quad Iy = \frac{2 \cdot h \cdot b^3}{15}$$

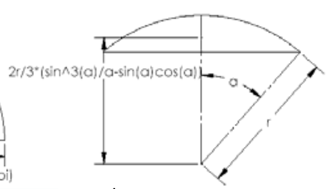
$$Ixy = \frac{b^2 \cdot h^2}{12}$$

Quarter Circle



$$Ix = Iy = \frac{\pi \cdot r^4}{16} \quad Ixy = \frac{r^4}{8}$$

Circular Segment

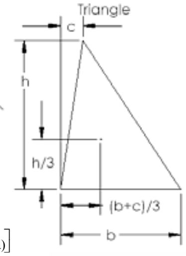


$$Ix = \frac{r^4}{4} \cdot \left[a - \sin(a) \cdot \cos(a) + 2 \cdot \sin^3(a) \cdot \cos(a)\right]$$

$$Iy = \frac{r^4}{12} \cdot \left[3a - 3\sin(a) \cdot \cos(a) - 2 \cdot \sin^3(a) \cdot \cos(a)\right]$$

$Ixy = 0$

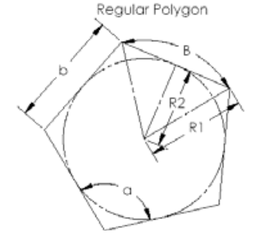
Triangle



$$Ix = \frac{b \cdot h^3}{36} \quad Ixy = \frac{b \cdot h^2}{72} \cdot (b - 2 \cdot c)$$

$$Iy = \frac{b \cdot h}{36} \cdot \left(b^2 - b \cdot c + c^2\right)$$

Regular Polygon



$$I = \frac{n \cdot b^4}{192} \cdot \cot\left(\frac{B}{2}\right) \cdot \left[1 + \left(3 \cdot \cot^2 \frac{B}{2}\right)\right]$$

$$A = \frac{n \cdot b^2}{4} \cdot \cot\left(\frac{B}{2}\right)$$

n=number of sides