

Analysis of Pure Torsion

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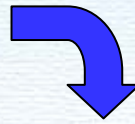
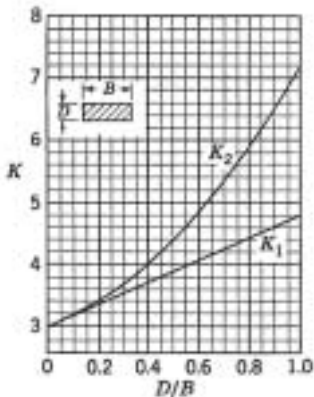
Common Torsion Analysis:

General Assumptions:
Uniform Material Properties
Uniform Cross-Sectional Area
Static Loading



Shape	Area A	Maximum Shear Stress τ_{max}	Rate of Twist $\theta = \phi/L$ $\phi = \text{radians (57.3}^\circ)$
	$\frac{\pi \cdot d^2}{4}$	$\frac{16 \cdot T}{\pi \cdot d^3}$	$\frac{32 \cdot T}{\pi \cdot d^4 \cdot G}$
	$\frac{\pi \cdot (d_o^2 - d_i^2)}{4}$	$\frac{16 \cdot T \cdot D}{\pi \cdot (d_o^4 - d_i^4)}$	$\frac{32 \cdot T}{\pi \cdot (d_o^4 - d_i^4) \cdot G}$
	$\pi \cdot d \cdot t$	$\frac{2 \cdot T}{\pi \cdot d^2 \cdot t}$	$\frac{4 \cdot T}{\pi \cdot d^3 \cdot t \cdot G}$
	$\frac{\pi \cdot L \cdot W}{4}$	$\frac{16 \cdot T}{\pi \cdot L \cdot W^2}$	$\frac{(L^2 + W^2) \cdot T}{16 \cdot \pi \cdot L^3 \cdot W^3 \cdot G}$
	L^2	$4.8 \cdot \frac{T}{L^3}$	$7.2 \cdot \frac{T}{L^4 \cdot G}$
	$L \cdot W$	$K_1 \cdot \frac{T}{L \cdot W^2}$	$K_2 \cdot \frac{T}{L \cdot W^4}$
	$0.433 \cdot L^2$	$20 \cdot \frac{T}{L^3}$	$8.8 \cdot \frac{T}{L^4 \cdot G}$
	$0.866 \cdot L^2$	$20 \cdot \frac{T}{L^3}$	$8.8 \cdot \frac{T}{L^4 \cdot G}$

Rectangular Cross Section Correction Factor (K)



Complex Torsion Analysis:

Common Situations:
Thin walled members
Irregular cross sections
Non-prismatic members



Round Sections, variable thickness ($t = \text{const}$)	Thin-walled sections, any shape ($t = \text{variable}$)	Noncircular thick sections (open/closed geometry)
Distributed Torque: (Not used in this case)	Distributed Torque: $\tau = \frac{M}{t \cdot [A]}$ [A] = area enclosed by median line	Distributed Torque: $\tau = \frac{M \cdot k}{t \cdot [A]}$ [A] = volume enclosed by an element k = distance between planes
Shear Stress: $\tau = \frac{M \cdot r}{J}$ $J_p = \frac{1}{2} \pi \cdot r^4 - \frac{1}{2} \pi \cdot r_i^4$ $J_{\text{net}} = \frac{16 \cdot M}{\pi \cdot t^3}$ (solid) $J_{\text{net}} = \frac{16 \cdot M}{\pi \cdot t^3}$ (annular)	Shear Stress: $\tau = \frac{M}{t \cdot [A]}$ + = distance to a given point	Shear Stress: $\tau_{\text{max}} = \frac{M \cdot k}{t \cdot [A]}$ $\tau_{\text{min}} = \frac{M \cdot k}{t \cdot [A]}$ Approximate formula * $\tau_{\text{max}} = \frac{M \cdot k}{t \cdot [A]}$
Deflection (twist), radians		
$\theta = \phi = \frac{M \cdot L}{G \cdot J}$	$\theta = \phi = \frac{M \cdot L}{G \cdot t \cdot [A]}$ (cont. t)	$\theta = \phi = \frac{M \cdot L}{G \cdot [A]}$
$J = J_p$	$J = \frac{1}{2} \int_0^L t^3 ds$	$J = \frac{1}{2} \int_0^L t^3 ds$
$\theta = \frac{M \cdot L}{G \cdot J}$	For constant t $\theta = \frac{M \cdot L}{G \cdot t \cdot [A]}$	
Torsion Energy		
$U = \frac{1}{2} M \cdot \theta$	$U = \frac{1}{2} M \cdot \theta$	$U = \frac{1}{2} M \cdot \theta$

Poster supplemented by handout. See Dr. Odom for more information
Supplement includes: Examples and Complex Analysis Theory

$$* B_e = \sqrt{\frac{\pi}{4}} B$$