SHAFT DESIGN

Shaft Equations:

All of the following equations are general equations; you may need to use modifying factors such as: loading factors, pulsating power source factors, safety factors, and stress concentration factors.

Basic equations in torsion:
Solid round shaft:

$$\tau = \frac{16 \cdot T}{\pi \cdot D^3}$$

Hollow round shaft:

$$\tau = \frac{16 \cdot T \cdot D_0}{\pi \cdot (D_0^4 - D_i^4)}$$

Basic equation in bending:
Solid shaft:

$$\sigma = \frac{32 \cdot M}{\pi \cdot D^3}$$

Hollow shaft:

$$\sigma = \frac{32 \cdot M \cdot D_o}{\pi \cdot (D_o^4 - D_i^4)}$$

Combined loading (solid shaft):

$$\tau_{max} = \frac{2}{\pi \cdot D^3} \sqrt{(8 \cdot M + F \cdot D)^2 + (8 \cdot T)^2}$$

(18-4) max sheer stress†

$$\sigma' = \frac{4}{\pi \cdot D^3} \sqrt{(8 \cdot M + F \cdot D)^2 + 48 \cdot T^2}$$

(18-5) Von Mises stress†

Torsional deflection:

$$\theta = \frac{32 \cdot T \cdot L}{\pi \cdot G \cdot D^4}$$

Radians

Factors of Safety:

$$\tau_{all} = \frac{S_y}{2 \cdot n}$$

(18-6) Max sheer stress theory†

$$\sigma' = \frac{S_y}{n}$$

(18-7) Distortion energy†

T= torque (lb-in, N-m)
F= axial load (lb, N)
Sy= yield strength
n = factor of safety
τ= sheer stress (psi, Pa)
D= diameter of solid shaft (in, m)
D_o = outside diameter of solid shaft (in, m)
D_i= inside diameter of solid shaft (in, m)
M= bending moment (lb-in, N-m)
L= length of shaft (in, m)
G= sheer modulus (psi, Pa)

Keyed Shafts

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Use of Shafts

A machine is a device that converts some sort of energy into work. In many machines transfer of power (energy with respect to time) is needed in order to perform this task. Shafts are efficient devices for transferring power and can commonly be found in machines world wide.

Shaft Definitions

Shaft- *A rotating member used to transmit power.*
Axle- *A stationary member used as support for rotating elements such as wheels, idler gears, etc.*
Spindle- *A short shaft or axle (e.g., head-stock spindle of a lathe).*
Stub shaft- *A shaft that is integral with a motor, engine or prime mover and is of a size, shape, and projection as to permit easy connection to other shafts*
Line shaft- *A shaft connected to a prime mover and used to transmit power to one or several machines*
Jackshaft- *(Sometimes called countershaft). A short shaft that connects a prime mover with a line shaft or a machine*
Flexible shaft- *A connector which permits transmission of motion between two members whose axes are at an angle with each other*

Shapes

Most shafts are round but they can come in many different shapes including square and octagonal. Keys and notches can also result in some unique shapes.

Hollow Versus Solid Shafts

Hollow shafts are lighter than solid shafts of comparable strength but are more expensive to manufacture. Thusly hollow shafts are primarily only used when weight is critical. For example the propeller shafts on rear wheel drive cars require lightweight shafts in order to handle speeds within the operating range of the vehicle.

General Principles

- Keep shafts short, with bearings close to the applied loads. This will reduce deflections and bending moments, and increases critical speeds.
- Place necessary stress raisers away from highly stressed shaft regions if possible. If unavoidable, use generous radii and good surface finishes. Consider local surface-strengthening processes (shot-peening or cold-rolling).
- Use inexpensive steels for deflection-critical shafts because all steels have essentially the same modulus of elasticity.
- Early in the design of any given shaft, an estimate is usually made of whether strength or deflection will be the critical factor. A preliminary design is based on that criterion; then, the remaining factor is checked.

Material Processing Tips

- To resist wear, case-hardening methods such as, nitriding, cyaniding, flame and induction hardening can be used.
- Cold-drawn steel bars have better physical properties than hot-rolled bars of similar steels. Cold-drawing causes residual surface stresses that offset higher endurance strength due to hot-rolling.
- Cutting keyways and slots in the shaft may cause warping due to the relief of surface stresses.
- Peening and other processes that produce surface compressive stresses counteract the effect of fatigue stress.

Material Selection:

The most common shafting material is cold-drawn carbon steel. When toughness shock resistance and greater strength are required alloy steel bars (e.g. 1347, 3140, 4150, 4340, 5145, 8650) are commercially available.

If surface wear resistance is a dominant factor, a carburizing grade steel (e.g. 1020, 1117, 2315, 4320, 4820, 8620) can be used.

The following is a table of properties of the steels listed above. Please note that the properties in this table are based on normalized or as-rolled steels, *not* cold drawn steels, so it is possible that different properties and/or prices can be attained. The purpose of this table is to give you a ballpark estimate of material properties and relative prices to aid you in your material selection process. For actual prices and properties contact a steel distributor.

Steel Alloy	Tensile Strength (kpsi)	Density (lb/in^3)	Modulus of Elasticity (Mpsi)	Sheer Modulus (Mpsi)	Fracture Toughness (Ksi in^1/2)	Price ($/lb)	Hardness (Vickers)
1347	373.00	0.28	32.05	12.47	14.11	$5.00	840.00
3140*	116.00	0.21	29.15	11.17	60.06	$0.37	240.00
4150*	150.00	0.28	29.15	11.17	60.00	$0.37	292.50
4340*	166.80	0.28	29.15	11.17	51.87	$0.37	325.00
8650*	132.70	0.28	29.88	11.46	56.42	$0.37	275.00
1020**	57.30	0.28	29.73	11.46	37.31	$0.37	135.00
1020*	57.30	0.28	29.73	11.46	39.13	$0.37	125.00
1117**	63.00	0.28	29.73	11.46	45.50	$0.34	135.00
1117*	60.00	0.28	29.73	11.46	43.68	$0.34	130.00
4320*	103.00	0.28	29.15	11.17	71.80	$0.54	215.00
4820*	98.00	0.28	29.15	11.17	65.50	$0.37	210.00
8620*	82.70	0.28	29.15	11.17	78.26	$0.45	170.00

*-normalized, **-as-rolled,
All mechanical properties are based on minimum values, higher properties can be attained
Prices quotes are based on maximum prices, prices vary greatly depending on the steel market and the quantity/ quality of the steel you are ordering

Rules of Deflection

- Deflections should not cause mating gear teeth to separate more than about .005 in. They should also not cause the relative slope of the gear axes to change more than .03 deg.
- The shaft deflection across a plain bearing must be small compared to the oil film thickness.
- The shaft angular deflection at a ball or roller bearing should generally not exceed .04 deg. unless the bearing is self-aligning.
- Rule Of Thumb: Restrict the torsional deflection to 1° for every 20 diameters of length, sometimes less.
- Rule Of Thumb: In bending the deflection should be limited to .01 in. per foot of length between supports.

Common Elements Used To Transfer Torque†

- Keys
- Splines
- Setscrews
- Pins
- Press or shrink fits
- Tapered fits

Common Means of Securing Shafts†

- Press and shrink fits
- Cotter and washer
- Nut and washer
- Sleeve
- Shaft shoulder
- Ring and groove
- Setscrew
- Split hub or tapered two-piece hub
- Collar and screw
- Pins

Common Shaft Sizes (English in.)

1/2	1 1/2	2 7/16	3 15/16
9/16	1 9/16	2 1/2	4
5/8	1 5/8	2 5/8	4 1/4
11/16	1 11/16	2 3/4	4 7/16
3/4	1 3/4	2 7/8	4 1/2
13/16	1 13/16	2 15/16	4 3/4
7/8	1 7/8	3	4 15/16
1	1 15/16	3 1/8	5
1 1/16	2	3 1/4	5 1/4
1 1/8	2 1/16	3 3/8	5 7/16
1 3/16	2 1/8	3 7/16	5 1/2
1 1/4	2 3/16	3 1/2	5 3/4
1 5/16	2 1/4	3 5/8	5 15/16
1 3/8	2 5/16	3 3/4	6
1 7/16	2 3/8	3 7/8	

Common Shaft Sizes (Metric mm)

5	15	35	80
6	16	40	90
8	17	45	100
10	18	50	110
11	19	55	120
12	20	60	80
13	25	65	90
14	30	70	100
			110
			120

*** It is advisable to consult the supplier for the availability of desired shaft sizes.**

www.theadamscompany.com/ shafts.htm

REFERENCE:

† Shigley, Joseph Edward, and Charles R. Mischke. Mechanical Engineering Design. Fifth Edition. Boston: McGraw Hill, 2002.

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