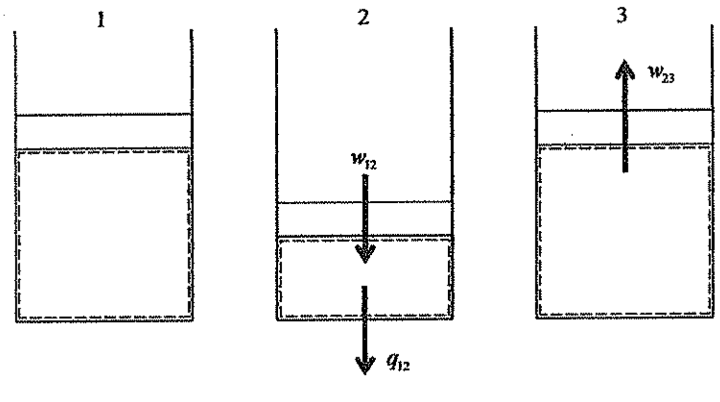
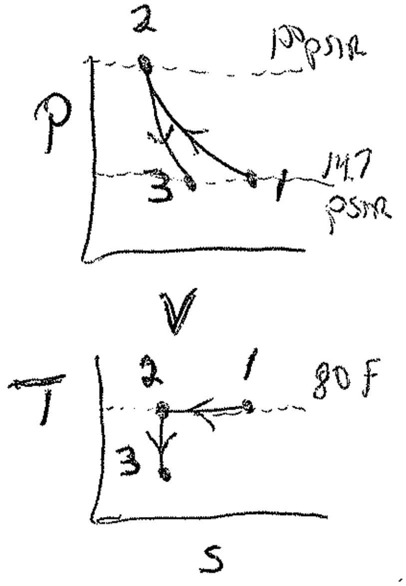
Piston-Cylinder Problem (w/Ideal Gas)

Neon undergoes a two-step process in a piston-cylinder assembly. The first process is isothermal compression from (1 atm, 80 F) to 100 psia. The second process is reversible and adiabatic expansion back to the initial pressure (1 atm). The gas constant for Neon is .0984 Btu/lbm-R. The isochoric heat capacity for Neon is .246 Btu/lbm-R.

a) Sketch this sequence of processes on Pv and Ts diagrams. Show how to use these diagrams to estimate the sign and relative magnitude
b) of heat transfer and work transfer for each process.
c) Find the final temperature.
d) Find the heat transfer and work transfer for each process.
e) Find the overall heat transfer and work transfer.

GIVEN: Neon undergoing a two-step process in a piston-cylinder assembly.





FIND: (a) The specific entropy change for the reversible, isothermal process

(b) The specific entropy change for the reversible, adiabatic process

(c) The work and heat transferred per lbm of neon for the reversible, isothermal process

(d) The work and heat transferred per lbm of neon for the reversible, adiabatic process

SOLUTION: Neon is an inert gas, therefore the heat capacities are constant. Therefore, for the reversible, isothermal process,

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = -R\ln\frac{P_2}{P_1}$$

$$\therefore \quad s_2 - s_1 = -\left(0.09841\frac{\text{Btu}}{\text{lbm-R}}\right)\ln\frac{100\text{ psia}}{14.7\text{ psia}} = \underline{\underline{-0.1887\frac{\text{Btu}}{\text{lbm-R}}}} \quad \leftarrow \quad \text{(a)}$$

A reversible and adiabatic process for a close system is isentropic. Therefore,

$$s_3 - s_2 = \underline{\underline{0\frac{\text{Btu}}{\text{lbm-R}}}} \quad \leftarrow \quad \text{(b)}$$

For the reversible, isothermal process, the First Law for the system (1-2) is,

$$q_{12} - w_{12} = u_2 - u_1$$

The process is isothermal and the neon is being modeled as an ideal gas. Therefore, the First Law reduces to,

$$q_{12} - w_{12} = 0 \quad \rightarrow \quad q_{12} = w_{12}$$

For an ideal gas undergoing an isothermal process, the *P-v* relationship can be written as,

$$Pv^n = \text{constant} \qquad n = 1 \qquad \therefore \quad Pv = \text{constant}$$

For a polytropic process where the polytropic exponent, $n = 1$, the work is determined by Eq. (4.28),

$$w_{12} = P_1 v_1 \ln \frac{v_2}{v_1}$$

Substituting the ideal gas EOS into the above expression,

$$w_{12} = RT_1 \ln \frac{RT_2}{P_2} \frac{P_1}{RT_1} = RT_1 \ln \frac{P_1}{P_2}$$

$$w_{12} = \left(0.09841 \frac{\text{Btu}}{\text{lbm-R}}\right)(80 + 459.67)\text{R} \ln \frac{14.7 \text{ psia}}{100 \text{ psia}} = \underline{\underline{-101.8 \frac{\text{Btu}}{\text{lbm}}}} \quad \leftarrow$$

From the First Law,

$$q_{12} = w_{12} = \underline{\underline{-101.8 \frac{\text{Btu}}{\text{lbm}}}} \quad \leftarrow \quad \text{(c)}$$

For the reversible, adiabatic process, the First Law is,

$$q_{23} - w_{23} = u_3 - u_2$$

However, since the process is adiabatic,

$$q_{23} = \underline{\underline{0 \frac{\text{Btu}}{\text{lbm}}}} \quad \leftarrow \quad \text{(d)}$$

Therefore, the First Law for this process is,

$$w_{23} = u_2 - u_3 = c_v (T_2 - T_3)$$

The final temperature, T_3, needs to be found. Since the neon is an ideal gas and the process is isentropic (reversible + adiabatic),

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{(k-1)/k} \quad \rightarrow \quad T_3 = T_2 \left(\frac{P_3}{P_2}\right)^{(k-1)/k}$$

The value of *k* can be found from the given information,

$$k = \frac{c_p}{c_v} \qquad c_v = c_p - R = (0.246 - 0.09841)\frac{\text{Btu}}{\text{lbm-R}} = 0.148\frac{\text{Btu}}{\text{lbm-R}}$$

$$\therefore \quad k = \frac{0.246 \text{ Btu/lbm-R}}{0.148 \text{ Btu/lbm-R}} = 1.667$$

Then,

$$T_3 = T_2\left(\frac{P_3}{P_2}\right)^{(k-1)/k} = (80 + 459.67)\text{R}\left(\frac{14.7 \text{ psia}}{100 \text{ psia}}\right)^{(1.667-1)/1.667} = 250.59\text{R} = -209.08°\text{F}$$

Now, the work transferred can be found,

$$w_{23} = c_v(T_2 - T_3) = \left(0.148\frac{\text{Btu}}{\text{lbm-R}}\right)\left[80 - (-209.08)\right]\text{R} = \underline{\underline{42.8\frac{\text{Btu}}{\text{lbm}}}} \quad \leftarrow \quad \text{(d)}$$

REFLECTION:

- The entropy decreases during the compression process. This is consistent with entropy being a measure of molecular disorder.
- Reversible + adiabatic = isentropic!
- This two-step scheme is quite useful if the goal is to decrease the temperature of the neon. The final temperature is very cold!
- An alternative way to find the work done from 2-3 would be from the integral of Pdv. Recall, that for an isentropic process, this results in,

$$w_{23} = \frac{P_3v_3 - P_2v_2}{1-n} = \frac{R(T_3 - T_2)}{1-n}$$

Calculating work this way results in the exact same answer as seen above!