VORTEX TUBE ANALYSIS

GIVEN: A vortex tube operating according to manufacturer's performance data as shown,

Air
$T_1 = 70°\text{F}$
$P_1 = 100$ psig
1

$T_2 = (T_1 - 71)°\text{F} = -1°\text{F}$
$P_2 = 0$ psig
$\dot{m}_2 / \dot{m}_1 = 0.70$
2

$T_3 = (T_1 + 151)°\text{F} = 221°\text{F}$
$P_3 = 0$ psig
3

FIND: If this is possible?

To determine whether the vortex tube can operate as shown, we need to see if it violates any of the Laws of the Universe.

<u>Conservation of Mass</u>

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \quad \rightarrow \quad \frac{\dot{m}_2}{\dot{m}_1} + \frac{\dot{m}_3}{\dot{m}_1} = 1 \quad \rightarrow \quad \alpha_2 + \alpha_3 = 1 \quad \alpha_n = \frac{\dot{m}_n}{\dot{m}_1}$$

We have no data on the outlet mass flow rate, but that's OK. Let's force the flow at the outlet to obey the conservation of mass. Then,

$$\alpha_1 = 1 \qquad \alpha_2 = 0.70 \qquad \alpha_3 = 1 - 0.70 = 0.30$$

<u>The First Law of Thermodynamics</u>

$$\dot{m}_1 h_1 - \left(\dot{m}_2 h_2 + \dot{m}_3 h_3\right) = 0$$
$$h_1 - \alpha_2 h_2 - \alpha_3 h_3 = 0$$

Now, we need a fluid property model. This can be estimated found from the air tables in your supplement. For an exact calculation, EES can calculate real-fluid air properties using fluid 'air_ha'. The following enthalpy values are found using EES which closely resemble the air table results.

$$h_1 = 125.9 \text{ Btu/lbm} \quad h_2 = 109.6 \text{ Btu/lbm} \quad h_3 = 163.0 \text{ Btu/lbm}$$

Now, we can check the First Law …

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$$h_1 - \alpha_2 h_2 - \alpha_3 h_3 = 0$$

$$\left[125.9 - (0.70)(109.6) - (0.30)(163.0)\right]\frac{\text{Btu}}{\text{lbm}} = 0$$

$$0.28\frac{\text{Btu}}{\text{lbm}} = 0\frac{\text{Btu}}{\text{lbm}}$$

This is not *exactly* zero, but certainly within round-off and measurement error. So, assuming that the First Law is obeyed, let's move on to the Second Law.

<u>The Second Law of Thermodynamics</u>
Assuming that the vortex tube is adiabatic,

$$\dot{m}_1 s_1 - \left(\dot{m}_2 s_2 + \dot{m}_3 s_3\right) + \dot{S}_P = 0$$

$$\dot{S}_P = \left(\dot{m}_2 s_2 + \dot{m}_3 s_3\right) - \dot{m}_1 s_1$$

$$\frac{\dot{S}_P}{\dot{m}_1} = \alpha_2 s_2 + \alpha_3 s_3 - s_1$$

Again, using EES to find the real-fluid entropy values,

$$s_1 = 1.507 \text{ Btu/lbm} \quad s_2 = 1.601 \text{ Btu/lbm-R} \quad s_3 = 1.696 \text{ Btu/lbm-R}$$

Therefore, the entropy production per mass of air entering the tube is,

$$\frac{\dot{S}_P}{\dot{m}_1} = \alpha_2 s_2 + \alpha_3 s_3 - s_1 = \left[(0.7)(1.601) + (0.3)(1.696) - 1.507\right]\frac{\text{Btu}}{\text{lbm-R}} = 0.1225\frac{\text{Btu}}{\text{lbm-R}}$$

This is definitely a positive number! The device is irreversible which implies this process CAN happen!

The obvious question is … "OK. Now, I've got thermodynamics telling me that this thing is possible. HOW DOES IT WORK!?!?"

Lesson learned … Thermodynamics does not tell us what is going on *internally* other than an indication of irreversibility through the entropy production.