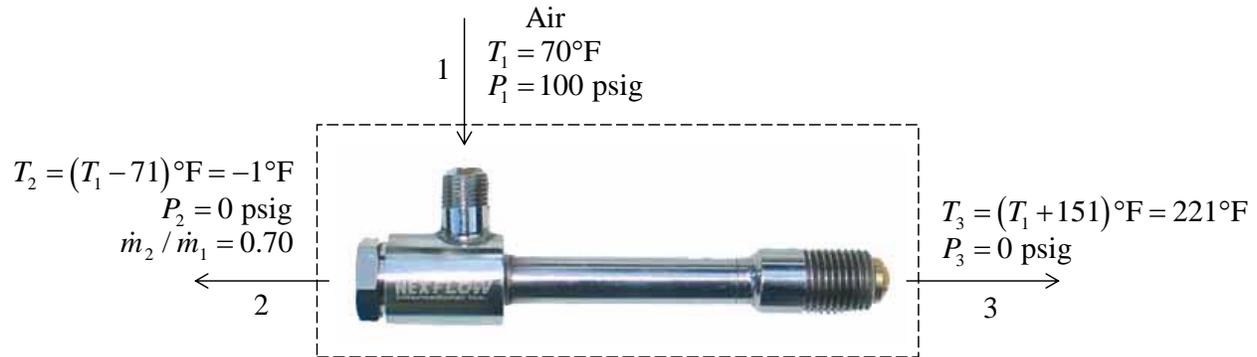


VORTEX TUBE ANALYSIS

GIVEN: A vortex tube operating according to manufacturer's performance data as shown,



FIND: If this is possible?

To determine whether the vortex tube can operate as shown, we need to see if it violates any of the Laws of the Universe.

Conservation of Mass

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \quad \rightarrow \quad \frac{\dot{m}_2}{\dot{m}_1} + \frac{\dot{m}_3}{\dot{m}_1} = 1 \quad \rightarrow \quad \alpha_2 + \alpha_3 = 1 \quad \alpha_n = \frac{\dot{m}_n}{\dot{m}_1}$$

We have no data on the outlet mass flow rate, but that's OK. Let's force the flow at the outlet to obey the conservation of mass. Then,

$$\alpha_1 = 1 \quad \alpha_2 = 0.70 \quad \alpha_3 = 1 - 0.70 = 0.30$$

The First Law of Thermodynamics

$$\dot{m}_1 h_1 - (\dot{m}_2 h_2 + \dot{m}_3 h_3) = 0$$

$$h_1 - \alpha_2 h_2 - \alpha_3 h_3 = 0$$

Now, we need a fluid property model. This can be estimated found from the air tables in your supplement. For an exact calculation, EES can calculate real-fluid air properties using fluid 'air_ha'. The following enthalpy values are found using EES which closely resemble the air table results.

$$h_1 = 125.9 \text{ Btu/lbm} \quad h_2 = 109.6 \text{ Btu/lbm} \quad h_3 = 163.0 \text{ Btu/lbm}$$

Now, we can check the First Law ...

VORTEX TUBE ANALYSIS

$$h_1 - \alpha_2 h_2 - \alpha_3 h_3 = 0$$

$$\left[125.9 - (0.70)(109.6) - (0.30)(163.0) \right] \frac{\text{Btu}}{\text{lbm}} = 0$$

$$0.28 \frac{\text{Btu}}{\text{lbm}} = 0 \frac{\text{Btu}}{\text{lbm}}$$

This is not *exactly* zero, but certainly within round-off and measurement error. So, assuming that the First Law is obeyed, let's move on to the Second Law.

The Second Law of Thermodynamics

Assuming that the vortex tube is adiabatic,

$$\dot{m}_1 s_1 - (\dot{m}_2 s_2 + \dot{m}_3 s_3) + \dot{S}_P = 0$$

$$\dot{S}_P = (\dot{m}_2 s_2 + \dot{m}_3 s_3) - \dot{m}_1 s_1$$

$$\frac{\dot{S}_P}{\dot{m}_1} = \alpha_2 s_2 + \alpha_3 s_3 - s_1$$

Again, using EES to find the real-fluid entropy values,

$$s_1 = 1.507 \text{ Btu/lbm} \quad s_2 = 1.601 \text{ Btu/lbm-R} \quad s_3 = 1.696 \text{ Btu/lbm-R}$$

Therefore, the entropy production per mass of air entering the tube is,

$$\frac{\dot{S}_P}{\dot{m}_1} = \alpha_2 s_2 + \alpha_3 s_3 - s_1 = \left[(0.7)(1.601) + (0.3)(1.696) - 1.507 \right] \frac{\text{Btu}}{\text{lbm-R}} = 0.1225 \frac{\text{Btu}}{\text{lbm-R}}$$

This is definitely a positive number! The device is irreversible which implies this process CAN happen!

The obvious question is ... "OK. Now, I've got thermodynamics telling me that this thing is possible. HOW DOES IT WORK!?!?"

Lesson learned ... Thermodynamics does not tell us what is going on *internally* other than an indication of irreversibility through the entropy production.