Risk and Uncertainty

- We’ve already mentioned that interest rates reflect the risk involved in an investment.
- Risk and uncertainty can affect an investment in a variety of ways.
  - In some situations, it is helpful to distinguish between risk and uncertainty; it won’t be necessary in our discussion.
- We will discuss tools that have been developed to help quantify the role of risk and uncertainty in an economic analysis.

Sources of Uncertainty

- Inaccuracy in the estimates used in the study.
  - Income estimates,
  - Operating expense estimates.
  - Uncertainty about both decreases as experience is gained.
- Uncertainty due to the type of business and future health of the economy.
  - Mining operations are risky because metal prices have always been volatile.
  - Construction is very sensitive to interest rates.

Sources of Uncertainty (cont.)

- Type of physical plant and equipment.
  - General-purpose machinery is usually fairly stable in price.
  - Specialized equipment depends on the demand for the particular operation provided by the machine, and will be more uncertain.
- Length of the study period.
  - The length of the study period is usually tied to the life-time of the equipment, which is rarely known in advance.

Methods for Dealing with Uncertainty

- Breakeven analysis
  - Determines the value required for a key parameter (e.g. M.A.R.R.) in order for the project to show a profit.
- Sensitivity analysis
  - Calculates changes in, say, net present worth due to changes in various parameters.
- Optimistic-pessimistic estimation
- Risk-Adjusted M.A.R.R.
- Monte-Carlo simulation (and other probabilistic methods.)

Monte Carlo Simulation

- Monte Carlo simulation involves.
  - Randomly selecting values for the various parameters based on the probabilities assigned to them.
  - Incomes, operating expenses, interest rates, etc.
  - Determining the outcome of the study for that particular combination of parameters.
  - Repeating the process for a very large number of iterations.
  - Calculating the likelihood of a particular outcome.
    - E.g., what proportion of the cases produced a profit?
    - What was the expected profit?
Advantages and Disadvantages

- Monte Carlo simulation can provide a great deal of insight into very complicated situations involving uncertainty and probability.
- It is relatively easy to put together a Monte Carlo simulation with Mathcad or other specialized software.
- But the results are meaningless unless the probabilities are based on sound information.

Probability

- The probability of an event occurring is described by a number between zero and one.
  - Zero: Complete impossibility
  - One: Absolute certainty.
- There is also a rigorous set of rules for the manipulation of probabilities.
- However, the definition of a probability is the subject of some debate.

Frequentist

- A probability describes the frequency with which an event has occurred in the past.
- For example, concrete manufacturers are required to take several samples of every batch of concrete they make and cure and test those samples for strength. As a result they have hundreds of data points for the strength their standard mixes.

Bayesian

- A probability can describe one’s a priori knowledge about the likelihood of an event.
  - Sometimes this is an “intuitive” estimate of the likelihood.

Economic Example

- The operating expenses for Structure N are equally likely to be anywhere between $800 and $1200/year.
Uniform Probability Distribution

<table>
<thead>
<tr>
<th>Annual Operating Expense</th>
<th>Probability Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$600</td>
<td>0.005</td>
</tr>
<tr>
<td>$700</td>
<td>0.005</td>
</tr>
<tr>
<td>$800</td>
<td>0.005</td>
</tr>
<tr>
<td>$900</td>
<td>0.005</td>
</tr>
<tr>
<td>$1,000</td>
<td>0.005</td>
</tr>
<tr>
<td>$1,100</td>
<td>0.005</td>
</tr>
<tr>
<td>$1,200</td>
<td>0.005</td>
</tr>
<tr>
<td>$1,300</td>
<td>0.005</td>
</tr>
<tr>
<td>$1,400</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Probability Distributions

- Roughly speaking, a probability distribution describes the likelihood that a random variable will take a certain value.
- In general, probability distributions can be classified as:
  - Discrete distributions, or
  - Continuous distributions.

Discrete Probability Distributions

- The probability that a fair die will land on any of its six faces is one-in-six.

<table>
<thead>
<tr>
<th>Die Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.16</td>
</tr>
<tr>
<td>2</td>
<td>0.16</td>
</tr>
<tr>
<td>3</td>
<td>0.16</td>
</tr>
<tr>
<td>4</td>
<td>0.16</td>
</tr>
<tr>
<td>5</td>
<td>0.16</td>
</tr>
<tr>
<td>6</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Continuous Probability Distributions

- Continuous distributions assign a probability rate to a continuous range of values.

Normal Probability Distribution

- The normal probability distribution is the so-called “bell curve.”
- The center or peak of the bell curve is known as the mean, average, or expected value of the distribution.
- The spread or scatter of the bell around the mean is described by the standard deviation \( \sigma \).
  - 95.45% of the time, the value of the random variable will be within \( \pm 2\sigma \) of the mean.

Example

- The expected value for the concrete strength in Slide 10 is 5000 psi.
- The standard deviation 750 psi
- We are 95% confident that it will be between 3500 psi and 6500 psi.
  - I.e., \( 2\sigma = 1500 \) psi; therefore between, 5000 \( \pm 1500 \)
Normal Distribution

Previous Example

- We could assume the following random variables in the previous economic comparison problem to perform a Monte Carlo simulation.

<table>
<thead>
<tr>
<th>Structure M Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Cost</td>
</tr>
<tr>
<td>Interest rate $\mu = 15%$, $\sigma = 2.5%$</td>
</tr>
<tr>
<td>Useful Life $\mu = 10$ years, $\sigma = 1.5$ years</td>
</tr>
<tr>
<td>Annual O&amp;M uniform between $1,900 and $2,500</td>
</tr>
</tbody>
</table>

Structure N

- For Structure N, we could assume

<table>
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</thead>
<tbody>
<tr>
<td>First Cost</td>
</tr>
<tr>
<td>Interest rate $\mu = 15%$, $\sigma = 2.5%$</td>
</tr>
<tr>
<td>Salvage Value $\mu = 10,000$, $\sigma = 4,000$</td>
</tr>
<tr>
<td>Useful Life $\mu = 25$ years, $\sigma = 3.75$ years</td>
</tr>
<tr>
<td>Annual O&amp;M uniform between $900 and $1,100</td>
</tr>
</tbody>
</table>

- Since the life spans of both alternatives are unknown, we must use the annual cost method to ensure the same study period for both alternatives.

Comparison

- Based on these parameters, a Monte Carlo simulation was run with 10,000 iterations. The average annual cost for each alternative was
  - Structure M: $4,627/year
  - Structure N: $7,178/year

- Which favors Structure M.

  Of the 10,000 simulations only three iterations showed Structure N to be preferred over M.
  - The probability that Structure M is preferred is 0.9997.

Effects of Uncertainty

- Another simulation was run assuming the mean values of all the random variables remained the same but Structure M had more scatter in its random variables.

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<td>Interest rate $\mu = 15%$, $\sigma = 2.5%$</td>
</tr>
<tr>
<td>Useful Life $\mu = 10$ years, $\sigma = 3.0$ years</td>
</tr>
<tr>
<td>Annual O&amp;M uniform between $1,400 and $3,000</td>
</tr>
</tbody>
</table>

Results

- A Monte Carlo simulation was run with 10,000 iterations. The average annual cost for each alternative was
  - Structure M: $4,799/year
  - Structure N: $7,178/year

- The mean cost for Structure M has increased slightly (Structure N was not changed), but
- Structure M is still the preferred alternative.
Results (cont.)

- Of the 10,000 simulations now 189 iterations showed Structure N to be preferred over M.
  - The probability that Structure M is preferred has decreased to 0.9811.
- Increasing the uncertainty increases the probability that Structure M will cost more than Structure N.

Sensitivity to Interest Rate

- Returning to the original parameters for Structure M but increasing the uncertainty on the interest rate by letting $\sigma = 5\%$ leads to average annual costs for each alternative of
  - Structure M: $4642/\text{year}$
  - Structure N: $7222/\text{year}$
  - Which are still nearly the same,
- But the probability that Structure M is preferred has decreased to 0.9654.
  - Indicating that the analysis is sensitive to the interest rate.

Summary

- There are many sources of uncertainty in the economic predictions we make.
- The uncertainty can be accounted for in a variety of different ways.
- Monte Carlo simulations account for the uncertainty about many of the parameters.
- The quality of the results depends on the quality of the probabilistic modeling.