

Chapter 8

The Ontology of Determinant Judgments

And what do you suppose a man must know to know himself?

Socrates

§ 1. Imagination

Cognition is the conscious representation of objective knowledge in an intuition with understanding in concepts. In our descriptions so far we have discussed separately two mental distinctions in the theory of the formation of objective representations: imagination and understanding. We must now discuss the interplay that takes place between these two in the making of conscious objective representations.

Perhaps the first question we should address in this discussion is the nature of this distinction we draw between the something we call imagination and the something we call understanding. How shall we view these? Are we justified in viewing them in the plural – as two phenomena rather than one phenomenon – despite the fact that we are about to make an argument that rests on the on-going reciprocity between ‘imaginative events’ and ‘events of understanding’? Is this division of our cognitive faculty merely a logical division or should we regard it as a real division? These are the sorts of introductory issues and questions we should first settle since both ‘imagination’ and ‘understanding’ are terms different writers have employed in quite different ways in various mind theories. Kant himself was notoriously vague in his usage these terms and thereby sowed the seeds for many later controversies over interpretation the Critical Philosophy. We begin with the idea of imagination.

§ 1.1 Imagination from the Classical Viewpoint

Our usage – and Kant’s – of the term ‘imagination’ is somewhat close to the first connotation of the following dictionary definitions:

imagination: n. [L. *imaginatio* (-onis), imagination, from *imaginari*, to imagine, from *imago* (-inis), an image]

1. (a) the act or power of forming mental images of what is not actually present; (b) the act or power of creating mental images of what has never been actually experienced, or creating new images or ideas by combining previous experiences; creative power. *Imagination* is often regarded as the more serious and deeply creative faculty, which perceives the basic resemblances between things, as distinguished from *fancy*, the lighter and more decorative faculty, which perceives superficial resemblances.
2. image in the mind; conception; idea
3. a foolish notion; empty fancy
4. the ability to understand and appreciate imaginative creations of others.

By now it is perhaps somewhat obvious that such ideas as “what is not actually present” and “mental images” and the like require from us more than a passing glance if we are to understand this idea of ‘imagination’ in a detailed technical sense. It is clear that the phenomenon of imagination is a mental phenomenon. Plato described it in his *Sophist* [263d-264a]¹ in the following way:

STRANGER: And therefore thought, opinion, and imagination are now proved to exist in our minds as both true and false.

THEAETETUS: How so?

STR: You will know better if you first gain a knowledge of what they are, and in what they severally differ from one another.

THEAE: Give me the knowledge which you would wish me to gain.

STR: Are not thought and speech the same, with this exception, that what is called thought is the unuttered conversation of the soul with herself?

THEAE: Quite true.

STR: But the stream of thought which flows through the lips and is audible is called speech?

THEAE: True.

STR: And we know there exists in speech . . .

THEAE: What exists?

STR: Affirmation.

THEAE: Yes, we know it.

STR: When affirmation or denial takes place in silence and in the mind only, have you any other name by which to call it but opinion?

THEAE: There can be no other name.

STR: And when opinion is presented, not simply, but in some form of sense, would you not call it imagination?

THEAE: Certainly.

Plato linked thinking with speech, as we see above, and held imagination to be a kind of thinking “in some form” of sensation. We argued against such an equation of thinking with speech earlier in this treatise on the basis of Piaget’s work; but what of Plato’s idea that imagination is merely “opinion” (*dóxa*) presented “in some form of sense”? Given their serious differences in fundamental ontological ideas, it is perhaps unsurprising that Aristotle disagreed with his former master on this point:

That perceiving and understanding are not identical is therefore obvious; for the former is

¹ The translation presented here is by Benjamin Jowett from the Oxford University Press edition of *The Dialogues of Plato*.

universal in the animal world, the latter is found in only a small division of it. Further, thinking is also distinct from perceiving . . . for perception of the special objects of sense is always free from error, and is found in all animals, while it is possible to think falsely as well as truly, and thought is found only where there is discourse of reason. For imagination is different from either perceiving or discursive thinking, though it is not found without sensation, or judgment without it. That this activity is not the same kind of thinking as judgment is obvious. For imagining lies within our power whenever we wish . . . but in forming opinions we are not free: we cannot escape the alternative of falsehood or truth. Further, when we think something to be fearful or threatening, emotion is immediately produced, and so too with what is encouraging; but when we merely imagine we remain as unaffected as persons who are looking at a painting of some dreadful or encouraging scene . . .

Thinking is different from perceiving and is held to be in part imagination, in part judgment: we must therefore first mark off the sphere of imagination and then speak of judgment. If then imagination is that in virtue of which an image arises for us, excluding metaphorical uses of the term, is it a single faculty or disposition relative to images, in virtue of which we discriminate and are either in error or not? The faculties in virtue of which we do this are sense, opinion, knowledge, thought.

That imagination is not sense is clear from the following considerations: Sense is either a faculty or an activity, e.g., sight or seeing: imagination takes place in the absence of both, as e.g. in dreams. Again, sense is always present, imagination not . . . Again, sensations are always true, imaginations are for the most part false . . .

It remains therefore to see if it is opinion, for opinion may be either true or false. But opinion involves belief . . . and in the brutes though we often find imagination we never find belief . . . It is clear then that imagination cannot, again, be opinion *plus* sensation, or opinion mediated by sensation, or a blend of opinion and sensation . . .

Imagination is therefore neither any one of the states enumerated, nor compounded out of them. But since when one thing has been set in motion [*kinesis*] another thing may be moved by it, and imagination is held to be a movement and to be impossible without sensation, i.e. to occur in beings that are percipient and to have for its content what can be perceived, and since movement may be produced by actual sensation and that movement is necessarily similar in character to the sensation itself, this movement cannot exist apart from sensation or in creatures that do not perceive . . .

If then imagination presents no other features than those enumerated and is what we have described, then imagination must be a movement resulting from an actual exercise of a power of sense.

As sight is the most highly developed sense, the name *phantasia* [imagination] has been formed from *pháous* [light] because it is not possible to see without light.

And because imaginations remain in the organs of sense and resemble sensations, animals in their actions are largely guided by them, some (i.e. the brutes) because of the non-existence in them of thought, others (i.e. men) because of the temporary eclipse in them of thought by feeling or disease or sleep [ARIS9: 680-682 (427^b5-429^a9)].

For Aristotle imagination “resembles” but is not the same as sensation. He places imagination “in” the “organs of sense” and since, in his view, the senses are never in error, imagination cannot be and is not part of “thinking” or of “judgment.” That imagination may be “false” is due to the fact that it is a “movement” produced by the activity of sense and is not itself a sense.

That we may question the usefulness, and even the correctness, of either of the above viewpoints is rather obvious from the qualitative and, if I may use the word, “intuitive” nature of the grounds of the arguments Plato and Aristotle employ. Have we no better factual evidence to go on than this? Let us examine this idea of “imagination” from the viewpoint of psychology.

§ 1.2 James' View of Imagination

William James described imagination as the forming of “mental pictures” from combinations of “past data.” In looking at James’ theory, we should not take the idea of “mental pictures” too literally. James did not hold these “pictures” to be exclusively *visual* “images” any more than did Aristotle. He did, however, reject utterly the wax tablet and “atomistic” views of Aristotle and Locke. Nonetheless, his theory of imagination is fundamentally empiricist.

Sensations, once experienced, modify the nervous organism, so that copies of them arise again after the original outward stimulus is gone. No mental copy, however, can arise in the mind of any kind of sensation which has never been directly excited from without . . .

Fantasy, or imagination, are the names given to the faculty of reproducing copies of originals once felt. The imagination is called "reproductive" when the copies are literal; "productive" when elements from different originals are recombined so as to make new wholes . . . The phenomena ordinarily ascribed to imagination, however, are those mental pictures of possible sensible experiences, to which the ordinary processes of associative thought give rise.

When represented with surroundings concrete enough to constitute a *date*,² these pictures, when they revive, form *recollections* . . . When the mental pictures are of data freely combined, and reproducing no past combination exactly, we have acts of imagination properly so called [JAME2: 480].

James goes on to cite evidence that these “images” are usually “vague” in the sense that imagination seems not to give all the details of the originals with perfect accuracy. Like Aristotle, he rejects the idea that these vague images are “abstract ideas.” He also rejects the idea of a “faculty of the Imagination” – by which he seems to take “the Imagination” to mean some sort of central mental structure or perhaps some sort of brain “organism.”

Until very recent years it was supposed by all philosophers that there was a typical human mind which all individuals' minds were like, and that propositions of universal validity could be laid down about such faculties as "the Imagination." Lately, however, a mass of revelations have poured in, which make us see how false a view this is. There are imaginations, not "the Imagination," and they must be studied in detail [JAME2: 484].

These “revelations” come from the factual documentation that there is a great deal of diversity among human beings as to the nature of how we imagine things. This diversity shows itself in differences found by case studies that point to a plurality of what we might call “modes” of imagination.

James cites studies which show that different specific “modes” seem to predominant in specific individuals. Broadly, these modes can be classified as: 1) visual, 2) articulatory, 3) auditory, 4) tactile, and 5) motor-gesticular. This is not to say that a specific person possesses only one of these modes to the exclusion of all others. Rather, the findings are indicative of individuals developing a primary mode of imagination at the expense of the other modes. We

² Presumably James means recognition of some actual past event.

might do well to call a person's primary mode his "habitual mode" of imagination since it is a fact some individuals cultivate and use more than one mode of imagination. For example, students (both past and present) of electromagnetic theory will recall the famous "right hand rule" – a gesture made with the right hand that serves to help "visualize" the force acting on a current-carrying wire in the presence of a magnetic field. In his biography of Richard Feynman, Gleick noted:³

Those who watched Feynman in moments of intense concentration came away with a strong, even disturbing sense of the physicality of the process, as though his brain did not stop with the gray matter but extended through every muscle in his body. A Cornell dormitory neighbor opened Feynman's door to find him rolling on the floor beside his bed as he worked on a problem. When he was not rolling about, he was at least murmuring rhythmically or drumming with his fingertips . . . For Feynman it was a nature whose elements interacted with palpable, variegated, fluttering rhythms.

Furthermore, studies of patients suffering from aphasia⁴ show that it is possible for an individual to suffer the loss of function of their habitual mode of imagination and to compensate for this loss to some degree by developing one of the heretofore lesser modes of imagination [JAME2: 490-491].

James was naturally interested in the question of what neural process "underlies" imagination. In particular, he asked if the seat of imagination is "purely cerebral" to the exclusion of the sense-organ, or if imagination lies vested in the neural processes of sensation, or if imagination "overlaps" the two. On the basis of the studies available at the time, he concludes:

Taken together, all these facts would force us to admit that the subjective difference between imagined and felt objects is less absolute than has been claimed, and that the cortical processes which underlie imagination and sensation are not quite as discrete as one is tempted at first glance to suppose . . .

The imagination-process CAN then pass over into the sensation-process. When we come to the study of hallucinations . . . we shall see that this is by no means a rare occurrence. At present, however, we must admit that *normally the two processes do NOT pass over into each other* [JAME2: 499-500].

James then asks if the "sensation-process" and the "imagination-process" occupy different localities in the brain or whether they occupy the same localities but differ merely in their "intensity" or capability to arouse other cortical activities. He decides this question in favor of the latter:

³ James Gleick, *Genius*, N.Y.: Pantheon Books, 1992, pg. 244.

⁴ Aphasias are disorders which affect language, and there are several different types of disorders under the general heading of "aphasias." The effect of aphasia on imagination is a kind of by-product of the disorder (see [KAND: 839-851]).

It seems almost certain . . . that the imagination-process differs from the sensation-process by its intensity rather than by its locality [JAME2: 500].

Thus, in the final analysis, James' view and that of Aristotle seem to be not as far apart as one might have expected. Indeed, it is not too difficult to see James' "intensity" idea in terms of Aristotle's "movement" idea and vice versa.

§ 1.3 Piaget's View of Imagination

One cautionary note with regard to James' theory demands our attention. The studies and facts upon which James based his conclusions all involved the phenomenon of imagination as it occurs in adults. If, however, we are to explore this phenomenon at its roots, we must guard against what Piaget sometimes called the tendency to 'adultomorphize' the theory – i.e. to view and interpret childish behaviors in terms of adult behaviors and suppositions. The need for this caution is clear from the findings James cites regarding the individual's habitual mode of imagining since we must suppose that any mental habits a person develops are the result of a lifetime of the 'practice of thinking.' If there is any *a priori* organization of *nous* that underlies the phenomenon of imagination – and our principle of reciprocity between biological structure and mental structure in the sensorimotor idea requires us to assume that this is indeed the case – then the study of the development of childish imagination takes on a place of pronounced importance in our understanding of the phenomenon.

In undertaking such a study, one problem arises directly from the common dictionary definition of 'imagination.' When we speak of a 'mental image' we most often mean 'a mental image of an *object*.' Furthermore, by 'object' in this connotation we usually mean a 'concrete object' – i.e. a real person, thing, place, etc. The problem with this way of thinking about imagination is that a new-born infant has not yet made any elaboration of concrete objects and, indeed, must go through a lengthy process of elaborating a mental 'framework' within which the elaboration of real objects – in terms of *Existenz* – becomes possible. This process constitutes what Piaget called "the construction of reality in the child."

It is obvious that we cannot know directly and with any great degree of confidence whether or not a six-month-old baby "has any imagination" in the dictionary sense of that word. Accordingly, the approach taken by Piaget and his co-workers is based upon what can be inferred from the infant's behavior. Approached from this direction, the roles played by the child's various senses are more clearly visible, and sensory modalities other than sight have a far greater role in early development than is the case for adults in everyday life. Perhaps to avoid unintentionally overemphasizing the role of vision in imagination, Piaget favors the word "representation" over

“imagination” in his theory (although it is completely clear that he takes these two as synonymous).

Perception is the knowledge of objects resulting from direct contact with them. As against this, representation or imagination involves the evocation of objects in their absence or, when it runs parallel to perception, in their presence. It completes perceptual knowledge by reference to objects not actually perceived . . .

Hence if representation can be said to extend perception, it can also be said to introduce a new element peculiar to itself. What is distinctive of representation is a system of meanings or *significations* embodying a distinction between that which signifies and that which is signified [PIAG5: 17].

Described in this way imagination (Piagetian representation) is a phenomenon that first manifests itself, to a high degree of confidence from the psychologist’s viewpoint, in the sixth stage of the development of sensorimotor intelligence.

From the point of view of object formation each of our observations thus leads to the same conclusion: the object is no longer, as it was during the first four stages, merely the extension of various accommodations, nor is it, as in the fifth stage, merely a permanent body in motion whose movements have become independent of the self but solely to the extent to which they have been perceived; instead, the object is now definitely freed from perception and action alike and obeys entirely autonomous laws of displacement. In effect, by virtue of the very fact that it enters the system of abstract or indirect images and relations, the object acquires in the subject’s consciousness, a new and final degree of liberty. It is conceived as remaining identical to itself whatever may be its invisible displacements or the complexity of screens which mask it. Doubtless this representation of the object which we call the characteristic of the sixth stage is already budding in the previous stages. As soon as the child in the fourth stage begins to search actively for the vanished object it can be claimed that there exists a sort of evolution of the absent object. But never until the present stage has this behavior led to real evocation, because it merely utilized a system of signs linked with the action; searching for an object under a screen when the subject has seen it disappear (stages IV and V) does not necessarily presuppose that the subject “imagines” the object under the screen but simply that he has understood the relation of the two objects at the moment he perceived it (at the moment when the object was covered) and that he therefore interprets the screen as a sign of the actual presence of the object. It is one thing to assume the permanence of an object when one has just seen it and when some other object now in sight recalls its presence, and it is quite another thing to imagine the first object when there is nothing in sight to attest its hidden existence. True representation therefore begins only when no perceived sign commands belief in permanency, that is to say, from the moment when the vanished object is displaced according to an itinerary which the subject may deduce but not perceive [PIAG2: 84-85].

Thus, Piaget holds that it is not until the sixth and final stage of sensorimotor intelligence that the existence of childish imagination (in the usual context of that word) is established *as a fact*.

Yet this development – the behaviorally observable manifestations of imagination – must be grounded in something. Piaget uses the phrase “true representation” to mean that the child’s cognition of the object has been severed from the child’s own actions, i.e. that “independent” *Existenz* has been attributed to the object.

Piaget elsewhere refers to this “independence” of an objective representation (where here the term ‘representation’ is our usual terminology of this treatise) as a ‘mobile scheme.’ By this he means a scheme that has become “available” for incorporation into other schemes outside the one in which the scheme was initially constructed. Thus, in viewing Piaget’s theory, we are obliged to keep in mind the more or less *de facto* meaning Piaget assigns to the words ‘real’ and ‘reality’ in his writings, and to take note that this usage of the term ‘reality’ differs fundamentally from the way in which we use that term in the Critical Philosophy.

Bearing this in mind, let us look at the idea of ‘signification.’ The hallmark of imagination in Piaget’s theory is the attribution of some sort of ‘significance’ to a scheme, as we saw in the quote given above from *The Child’s Conception of Space*. However, ‘perceptions’ are also said to contain ‘significations’ in Piaget’s theory and so we must consider what constitutes the essential distinction between Piagetian perception and Piagetian ‘representation’ (imagination).

Admittedly, perception itself contains significations (for example, forms seen in perspective are related back to the constant form) but in this case they are merely signs or pointers, part and parcel of the sensorimotor scheme. In contrast to this, representational signification draws a clear distinction between the *significants* or signifiers which consist of signs . . . and symbols (images, imitative gestures, sketches), and the things they signify (in the case of spatial representation; spatial transformations, spatial states, etc.). The transition from perception to representation is a twofold problem, embracing that which signifies and that which is signified; that is to say, both image and thought [PIAG5: 17].

Earlier it was pointed out that Piaget’s terminology and that of this treatise frequently use the same words to mean quite different things. In the quote given above we have an important instance of this. Piagetian “perceptions” are said to “contain significations” that are “mere signs or pointers” of some sort within a sensorimotor scheme. In the terminology of this treatise, this does not describe an intuition – which is a singular representation – but, rather, mixes together (or at least seems to mix together) elements from both intuition and concepts. It is clear that, viewed as a representation, we can speak of an intuition as “containing” such “signs and pointers” but only in an unconscious (i.e. potential) way. But when one speaks of “forms seen in perspective related back to the constant form,” this can mean nothing else than a full-fledged *cognition* involving both intuition and concepts. This we could have anticipated from Piaget’s description of “perception” as “the knowledge of objects resulting from direct contact with them.”

In Piaget’s system ‘perception’ antecedes ‘representation’; however, it is clear from what we have just said that ‘perception’ in this sense is not merely a product of the synthesis of apprehension but obviously must also involve the synthesis of imagination and the synthesis of re-cognition in a concept (which we discussed in Chapter 3). It follows that “imagination” as this term is used in this treatise and Piagetian “representation” (or “imagination”) refer to two quite different ideas.

The phenomenon Piaget calls “representation” is, in this sense, a more highly developed and evolved mental act than that of what Kant called the synthesis of imagination or even what James called the “imagination-process.” Piaget views “imagination” as the ability to evoke the cognition of a Piagetian object in the absence of direct sensory stimulation attributable to that object. In regard to this the “image” plays the central role.

In this chapter it is the image we particularly want to discuss, although we cannot entirely dissociate it from the concepts it serves to indicate. But no sooner do we begin to study the spatial image than we are once again confronted with the problem of movement and its relation to the sensory or figural element, a problem we have just discussed in connection with perception. Now in all probability the image is an internalized imitation (as we have elsewhere tried to show¹), and is consequently derived from motor activity, even though its final form is that of a figural pattern traced on the sensory data. Thus because of its very nature the mental image tends to oscillate between purely motor and figural characteristics, just as did sense perception at an earlier stage. The intellectual relationships which constitute the beginnings of representational space are at first linked to the image as a means of support. But as they attain to spatial transformations, as opposed to static forms, these relationships separate the figural from the motor elements of the image, and at the same time free themselves from the figural elements to such an extent that the latter are henceforth used simply as auxiliary symbols [PIAG5: 17-18].

The construction and elaboration of “images” is so obviously a part of what we usually mean in the everyday use of the word “imagination” that the study of how “images” develop provides important evidence pertaining to the mental physics of the phenomenon of imagination. Let us recall that the heart of Piaget’s system is the on-going development of a progressively more complex system of interconnected schemes through the basic processes of assimilation, accommodation, and equilibration. It is to these fundamental processes of the faculty of pure consciousness that we will eventually tie our idea of imagination in this treatise – demoting it from the level where Piaget places ‘representation,’ which as we noted earlier is a placement based on the requirements of behavioral observation. Before undertaking this, however, let us round out our summary of Piaget’s theory by taking a look at the general idea that underlies the development of Piagetian images. This is the idea of the *semiotic function*.

At the end of the sensorimotor period, at about one and a half to two years, there appears a function that is fundamental to the development of later behavior patterns. It consists in the ability to represent something (a signified something: object, event, conceptual scheme, etc.) by means of a “signifier” which is differentiated and which serves only a representative purpose: language, mental image, symbolic gesture, and so on. Following H. Head and the specialists in aphasia, we generally refer to this function that gives rise to representation as “symbolic.” However, since linguists distinguish between “symbols” and “signs,” we would do better to adopt their term “semiotic function” to designate those activities having to do with the differentiated signifiers as a whole [PIAG15: 51].

It is clear that this idea of the semiotic function is not the idea of some innate and *a priori*

¹ The reference is to Piaget's *Plays, Dreams and Imitation in Childhood*, 1951.

power of mind. Rather, the semiotic function is a behavior – an activity – that becomes possible for the child only after a long elaboration of a system of schemes that can thereafter be called upon to provide the *materia circa quam* of this activity. From a behaviorist’s viewpoint (and, I speculate, perhaps from Piaget’s viewpoint as well), the semiotic function is an example of an *emergent behavior*, rather like the psychological counterpart of the “emergent properties” of biology. One important question for our theory is centered on this: Must we regard the semiotic function as an “emergent property” of mind or can we regard it, with objective validity, as a behavioral phenomenon grounded in more fundamental principles? Before taking up this question, we must gain a clearer understanding of what this ‘semiotic function’ describes. We begin this discussion with the stage of the infant’s development just prior to the appearance of the semiotic function:

The sensori-motor mechanisms are pre-representational, and behavior based on the evocation of the absent object is not observed until the second year. When the scheme of the permanent object is in process of being formed, from about nine to twelve months, there is certainly a search for an object that has disappeared; but since it has just been perceived, the search is part of an action already under way, and a series of clues remains to aid the child to find the object again.

Although representation does not yet exist, the baby forms and uses significations, since every sensori-motor assimilation (including perceptual assimilations) already implies the attribution of a signification, of a meaning. Significations and consequently also a duality between "signified" (the schemes themselves with their content; that is, the action) and "signifiers" are already present. However, these "signifiers" remain perceptual and are not yet differentiated from the "signified." This makes it impossible to talk about the semiotic function at this level. An undifferentiated signifier is, in fact, as yet neither a "symbol" nor a "sign" (in the sense of verbal signs). It is by definition an "indicator" (including the "signals" occurring in conditioning, like the sound of the bell that announces food). An indicator is actually undifferentiated from its signified in that it constitutes an aspect of it (whiteness for milk), a part (the visible section for a semi-hidden object), a temporal antecedent (the door that opens for the arrival of Mama), a causal result (a stain), etc. [PIAG15: 52-53].

Here we see already in place an elaborate system of *meaningful* representations (the sensorimotor schemes) in our Kantian sense of the word representation. While the absence of Piagetian representation at this stage is of great importance from the viewpoint of the observing psychologist, from the viewpoint of the Copernican hypothesis this is a matter of no particularly great importance in the consideration of the first principles of mental physics. The reason for this is a simple one. The psychologist studies appearances and phenomenon; mental physics is concerned with the overall doctrine of the phenomenon of mind *as a systematic doctrine*. The definitions and terms Piaget introduces are concerned with the dissection of merely empirical mental constructs; the Critical Philosophy is additionally concerned with the rational grounds for the possibility of such constructs.

One other thing of note in the description given above lies in Piaget’s and Inhelder’s comment concerning the “search for an object that has just disappeared.” The Piagetian theory

proposes that the “disappearance” of the Piagetian object – i.e. the cessation of sensible stimuli accredited to the appearance of the object – need not be seen as evidence for supposing that the infant’s search for the thing involves any mental “image” of the vanished object. Rather, they argue, it is the “action already underway” that can account for the “search” phenomenon. Are we to suppose this action was initiated by the infant prior to the object’s disappearance? No, of course not, and neither of these researchers holds that it does. How, then, are we to understand the phenomenon of the infant’s search for an object that has disappeared (e.g. been hidden under a blanket by the psychologist)? The Piagetian theory holds that the infant is seeking to re-establish a kind of global state of being (my wording, not Piaget’s) in which the object is “present” but in which what is important to the infant is not the object *per se* but the general syncretic appearance in which the appearance of the object is an undifferentiated part. Put another way, it is the *totality of the experience* that is ‘significant’ for the infant and not the Piagetian object, which the infant as of yet has not clearly differentiated from the scheme.

The facts tend to support this interpretation of infantile behavior prior to the behavioral evidence for the semiotic function. However, even if it is this totality of experience that is the *object* and goal of the infant’s actions, it is difficult to see the phenomenon of the infant’s search behavior in any other way than by supposing the infant is *capable of mentally representing* in some fashion the object of his actions. Under the Copernican hypothesis it is a matter of complete indifference whether this representation is that of some undifferentiated global scheme or of a thing, as the adult observer of the infant’s behavior perceives things, because such a distinction is merely one of the infant’s (or the adult’s) degree of logical perfection in cognition. Put another way, Piaget’s distinctions speak to *Dasein* in mental representation but it is the *Existenz* of mental representation which is the object of the Critical theory of empirical consciousness.

Having pointed out this important distinction between the philosophical viewpoints of Piaget and Kant, let us now look at the phenomenon of the semiotic function.

In the course of the second year (and continuing from Stage 6 of infancy) . . . certain behavior patterns appear which imply the representative evocation of an object or event not present and which consequently presuppose the formation or use of differentiated signifiers, since they must be able to refer to elements not perceptible at the time as well as those which are present. One can distinguish at least five of these behavior patterns whose appearance is almost simultaneous and which we shall list in order of increasing complexity [PIAG15: 52].

Piaget and Inhelder list the following patterns: 1) deferred imitation, i.e. imitation that starts after the disappearance of the model in which the child imitates in some way the presence of the model; 2) symbolic play, “or the game of pretending, which is unknown at the sensori-motor level”; 3) drawing; 4) the mental image which “appears as an internalized imitation”; and, 5) verbal evocation – such as a little girl saying “meow” after the cat has disappeared. Piaget and

Inhelder go on to describe each of these behaviors in more detail [PIAG15: 54-91] and the interested reader can refer to their book. For our present purposes, two points of special interest will suffice to summarize their findings.

First, each of these behaviors is based on *imitation* – the “monkey see, monkey do” behavior so evident in children.

Imitation constitutes both the sensori-motor prefiguration of representation and the transitional phase between the sensori-motor level and the level of behavior that may properly be called representative.

Imitation is first of all a prefiguration of representation. That is to say, it constitutes during the sensori-motor period a kind of representation in physical acts but not yet in thought [PIAG15: 55].

Imitative behavior can be observed at Stage 2 of the sensori-motor period (that is, the stage that immediately follows the “reflex behavior” stage) and continues to be an important factor all through childhood. Thus, the phenomenon of imitative behavior is of fundamental importance in its own right for our theory.

Second, the semiotic function, as it advances from the restrictions of sensorimotor activity to full-blown abstract thinking, provides the foundation upon which develops what we usually call the human intellect.

In spite of the astonishing diversity of its manifestations, the semiotic function presents a remarkable unity. Whether it is a question of deferred imitation, symbolic play, drawing, mental images and image-memories or language, this function allows the representative evocation of objects and events not perceived at that particular moment. The semiotic function makes thought possible by providing it with an unlimited field of application, in contrast to the restricted boundaries of sensori-motor action and perception. Reciprocally, it evolves under the guidance of thought, or representative intelligence. Neither imitation nor play nor drawing nor image nor language nor even memory (to which we might have attributed a capacity for spontaneous reproduction comparable to that of perception) can develop or be organized without the constant help of the structuration characteristic of intelligence [PIAG15: 91].

From this second aspect of the semiotic function we can see that the behavioral evidence concerning “imaginative behavior” points quite clearly to an on-going reciprocity between the performance of physical sensorimotor actions, sensible representation, and that cognitive phenomenon we call “understanding.”

§ 1.4 Transcendental Imagination

The views presented in the previous two subsections are empirical. From these we now turn to look at imagination from the view of the Copernican perspective. This we call the *transcendental* viewpoint of imagination.

We have thus far seen the idea of imagination portrayed as non-verbal opinion (Plato), as a “movement of the soul” (Aristotle), as a neurological process linked to a neurological sensation-

process (James), and as an emergent ability for bringing forth mental “images” that signify things (Piaget). The first two of these ideas we can obviously disregard as being either refuted by scientific fact (Plato’s idea) or too vague (Aristotle’s idea). The latter two ideas have the merit of being able to be linked to observable empirical phenomena but, just as obviously, they represent two seemingly incompatible models of that which we call imagination. Our task is, on the one hand, to see if these latter two views are capable of being reconciled and, on the other hand, to seek for a more fundamental principle in which the empirical findings of James and of Piaget have their ground. The character of this last objective is, of course, why we call this Copernican view “transcendental” since the grounding principle we seek is one which is necessary for the possibility of imaginative phenomena.

We begin, as is always the case when dealing with transcendental considerations, with the phenomenon of experience. Let us remind ourselves of three aspects of experience that everyone holds to be true. First, all our experiences come to us by way of the senses through perceptions. Second, we are able to recognize our perceptions – that is, we combine our perceptions in concepts of *objects*. Third, we are able to *think*, e.g. to summon forth mental “images” on the basis of past experience and to re-cognize from phenomenal concepts the ideas of supersensible objects by means of ratiocination. These evident abilities of *nous* are the empirical basis of the threefold synthesis of experience we discussed previously in Chapter 3 (§6.2) – i.e., the synthesis of apprehension in intuition, the synthesis of reproduction in imagination, and the synthesis of recognition in a concept. The idea of “synthesis” was described by Kant in the following way:

I understand by synthesis in the most general sense the act of adding² different representations to each other and comprehending their manifoldness in one cognition.

Synthesis in general is, as we shall come to see, the mere action of the power of imagination,³ a blind but indispensable function of the soul without which we would have no knowledge at all, but of which we are seldom even conscious [KANT1a: 210-211 (B: 103)].

Now among our various classes of representation only intuition stands in immediate relationship to appearance. “Comprehension” is the seventh (and highest) degree of knowledge in Kant’s classification (Chapter 3, §6.2), and *to comprehend* means “to have insight into something through reason, but in such a way that it is sufficient for a certain intention” [KANT8a: 104 (24: 133)]. Thus, the specific synthesis processes described above are not to be viewed as independent processes but, rather, as particular elements belonging to *general* synthesis.

² *hinzuzuthun*. ‘Adding’ in this sense of the word has nothing to do with arithmetic. Its connotation is ‘to join in a combination’.

³ *Einbildungskraft* - literally, “power of imagination.” Most English translations of Kant render this simply as “imagination” (or sometimes “the” imagination). I shall follow this practice in the interest of economy of speech, and the reader should take “imagination” to mean “power of imagination” unless it is specifically noted otherwise or when we are discussing some other viewpoint (e.g. Piaget’s or James’) of imagination.

Furthermore, since synthesis in general pertains to comprehension, it is equally clear that the “insight into something” must involve the representation in intuition of an object. It is here that the idea of imagination takes on a central role. Kant describes the power of imagination as an act of spontaneity in presenting an object in intuition.

Imagination is the ability⁴ to present an object in intuition, even without its presence [KANT1a: 256 (B: 151)].

This working definition of imagination is clearly compatible with the descriptions of imagination given by both James and Piaget. Kant lists imagination as one of the three fundamental subjective sources of knowledge:

The possibility of an experience in general and cognition of its objects rests on three subjective sources of knowledge⁵: *sense*, *imagination*, and *apperception*; each of these can be considered empirically, namely in application to given appearances, but they are also elements or foundations *a priori* that make this empirical use possible. *Sense* puts forth the appearances empirically in *perception*, the *power of imagination* in association (and reproduction), *apperception* in the *empirical consciousness* of the identity of these reproductive representations with the appearances, through which they were given, hence in *recognition*⁶ [KANT1a: 236 (A: 115)].

All three of these subjective sources of knowledge share a common point in intuition since intuition is the immediate representation of an object of appearance. The making of an intuition is, in turn, an act of synthesis. This is because we can never ascribe to the senses, by themselves, any *necessity* of form and yet we regard an intuition as *necessarily* representing the appearance. Our task at hand is to understand the role played by imagination in this synthetic act.

In exploring this question we must draw a distinction between what Kant called the *productive* power of imagination and the *reproductive* power of imagination. We have already discussed, briefly, the idea of the reproductive imagination when we discussed the synthesis of reproduction. Here imagination performs a synthesis solely according to empirical rules (that is, the synthesis is based upon empirical concepts – the rules for reproducing an intuition). This power of imagination is clearly tied to that phenomenon we call *memory* and the phenomenon we typically refer to as the *association* of memories. Strictly speaking, reproductive imagination is the phenomenon of re-presenting in intuition representations of objects and events which antecede the act of reproductive imagination. It is, as it were, an act of the renewal of representations. This act is entirely different in kind from an act of the productive imagination, where the intuition does not necessarily correspond to some actual prior appearance.

⁴ *Vermögen*.

⁵ *Erkenntnisquellen*.

⁶ *der Recognition*. This is ‘recognition’ in the ordinary sense of that word, e.g., “I recognized his face.”

Although reproductive imagination and the phenomenon of memory seem to have a close connection with one another, we are in no position to *equate* the two. Indeed, the phenomenon of memory merits an in-depth examination in its own right. For the present it is enough to merely point out that the phenomenon of memory *involves* reproductive imagination but that it involves a number of other factors as well.

Imagination and Apprehension

In our examination of the role of imagination, we will first explore how imagination and the synthesis of apprehension relate to each other through intuitions. We have seen that both are capacities for the making of intuitions. In what way do they differ? Now, receptivity is the power of mind to be affected by objects. The manner or mode in which the effect takes place we called sensibility. In turn,

Sensibility in the faculty of knowledge¹ (the capacity for representations² in intuition) contains two parts: sense and imagination. The first is the capacity for intuition³ in the presence of the object, the second without its presence [AK 7: 153].

The term ‘faculty of’ in the above quote does not imply a ‘faculty’ in the connotation of some particular ‘knowledge organ’ in the brain. Kant’s term is *Vermögen* which, as we have previously pointed out, refers to an ability, capacity or a potential power for doing something. The principal difference between the synthesis of sense apprehension and the synthesis carried out with reproductive imagination is, consequently, a distinction in regard to the *source* of the matter of representation. In the case of sense apprehension this determinable is matter of outer sense and we take its transcendental origin as being in *soma* or as being an effect registered in *soma* by the environment in our Organized Being model. In the case of reproductive imagination, we are dealing with the ability of *nous* to affect sensibility spontaneously.

This view might seem, at first sight, to flatly contradict James’ view that imagination and sensation share “the same locality” in the brain. On the other hand, James was not too specific by what he meant by this. Modern neuroscience can say much regarding ‘sensory modalities’ but we should note that ‘sensation’ is still a rather ill-defined idea from the perspective of both empirical neuroscience and psychology. Indeed, James’ “definition” of ‘sensation’ is quite vague [JAME2: 452-453] and difficult to dissociate from his idea of ‘perception.’ The description he does give is in reasonable accord with the following modern-day textbook description:

¹ *Erkenntnißvermögen*, the capacity for structuring knowledge through cognitions.

² *das Vermögen der Vorstellungen*; the phrase means the capacity for structuring representations.

³ *das Vermögen der Anschauung*; the phrase means the capacity for structuring intuitions.

Information processed by a sensory system may or may not lead to conscious awareness of the stimulus. Regardless of whether the information reaches consciousness, it is called **sensory information**. If the information does reach consciousness, it can also be called a **sensation**. The understanding of a sensation's meaning is called **perception** [VAND: 224].

No more detailed explanation of 'a sensation' than this is usually to be found in neuroscience. At best sensation is viewed as a 'something' having the attributes of "sensory modality (or 'quality'), intensity, duration, and location" [KAND: 331-333]. It is, consequently, very hard to make a case either for or against the thesis that Kant's theory runs counter to that of James. At best we may observe that present evidence suggests that awareness of sensory stimuli (i.e., 'sensation') takes place, at the earliest, in the primary sensory cortices of the brain (e.g., [KAND: 329-340]). However, there is also evidence which suggests that even this view may be too simplified. In simple point of fact, neuroscience does not yet speak with authority on James' idea of a "locality" of either an "imagination-process" or a "sensation-process."

What we *can* comment on is: to the extent it is true that sensation – as an effect that is registered not only in *soma* (the neuroscience view) but also in *nous* (a reciprocity demanded of our theory by Rational Physics) – can in some fashion be said to "take place in" some eventually definable region of the brain (such as the primary sensory cortices), it is also known that the brain has a great many feedback paths by which the higher brain centers can affect (through what are called "descending pathways") the transmission of afferent (i.e., "incoming") sensory signals. Therefore Kant's view of apprehension and imagination as being part of the same synthetic process (the making of an intuition) while, at the same time, having different sources for their respective *materia ex qua* can *not* be said to contradict James' brain-function model at our present level of knowledge in neuroscience.

Putting this another way, apprehension and imagination are powers of synthesis which differ only in that the former takes its *materia ex qua* from the data of the senses while the latter takes its *materia ex qua* from *concepts*. Taking Kant's definition of sensation as the *effect* of an object on the capacity for representation, apprehension is the synthesis of an intuition in which we regard *soma* as the agent, while strict imagination is the synthesis of an intuition in which *nous* is the originating agent. For those cases, then, where the intuition is a representation involving *both* factors (i.e., *cognition*), the interaction between the synthesis of apprehension and the synthesis of reproduction in imagination is none other than the synthetic combination of both subjective sources of cognitive matter in the same intuition.

In summary, then, the phenomenon of imagination and the phenomenon of apprehension have in common the synthesis of intuitions. They differ principally in the source of the matter that goes into this synthesis.

Imagination and Understanding

Next we take up the relationship of imagination and understanding beginning with the synthesis of re-cognition in a concept. Now, the *ground* for the synthesis of re-cognition is nothing else than our principle of transcendental apperception. This principle, we recall, is the foundation of the idea of empirical consciousness. That idea, in turn, is the idea leading to the determination of the *Existenz* of Nature through understanding. From the transcendental apperception we derive the principle of the apperception we call Self-consciousness and the requirement that cognitions belong to this *one* consciousness. This latter requirement is what we mean by the term ‘unity of apperception.’ Kant writes,

The unity of apperception in regard to the synthesis of imagination is understanding, and this very same unity, relatively to the transcendental synthesis of imagination, is pure understanding. There is therefore in understanding pure knowledge a priori that contains the necessary unity of the pure synthesis of imagination with respect to all possible appearances [KANT1a: 238 (A: 119)].

An individual intuition is a singular representation at a moment in time. As such, a plurality of such intuitions gives us no synthetic unity in empirical consciousness. What we have is, rather, merely an aggregation of representations of divers appearances. There is in this no determination of an *object* whatsoever. To obtain the required unity in empirical consciousness we therefore require a synthesis, which we can view in two steps. The *end product* of this synthesis is *that which we call understanding the object* through objective perception in a cognition.

Every intuition is a *magnitude*, which we explained earlier as a unity regarded as containing a multiplicity. The object, on the other hand, is the unity *in a manifold of appearances* represented by intuitions. To obtain such a unity we require three conditions. The first is the synthesis of re-cognition which transforms intuitions into *concepts* capable of being combined with each other in a determinant judgment. The second is the synthesis of different concepts under a rule of determining judgment. Such a rule, which by its very function must be pure and *a priori* since such a rule must apply to objects of appearance *universally*, Kant called a **category of understanding**. The third is the synthesis of reproduction of an intuition in imagination.

The synthesis of the manifold of sensible intuition, which is possible and necessary a priori, can be called figurative (synthesis speciosa), as distinct from that which would be thought in the mere category in consideration of the manifold of an intuition in general, and which is called combination of understanding (synthesis intellectualis); both are transcendental, not merely because they themselves a priori precede but also because they ground the possibility of other knowledge a priori [KANT1a: 256 (B: 151)].

An experiential concept (or even a connected manifold of concepts) is by itself nothing other than a rule for the re-production of sensible intuitions and cannot on its own be called a cognition. But since all experiential concepts originate from intuition, the synthesis of re-cognition is bound *a priori* to constraints or rules that necessarily enforce the possibility for a concept to be reproduced in intuition. In short, every concept produced through the synthesis of re-cognition must accord with *the condition of sensibility*. A rule that places such *pure and formal* conditions on concepts is called a *transcendental schema* [KANT1a: 272-273 (B: 177-179)].

The synthesis of re-cognition is *not* an act of determining judgment. An intuition, as the *materia ex qua* for the production of concepts, belongs to sensibility in representation, and in sensibility *there is no judgment*. (An intuition is not the matter of a judgment). The power of imagination is the power to synthesize intuitions in the absence of sensation and this it cannot do *unless imagination includes the power to limit the form of concepts such that these representations satisfy the condition of sensibility*. As Kant put it,

the *image* is a product of the empirical capacity of productive imagination; the *schema* of sensible concepts . . . is a product and, as it were, a monogram of pure imagination *a priori*, through which and in accordance with which the images first become possible [KANT1a: 273-274 (B: 181)].

Kant illustrated this point with one of his rare examples:

The schema as it is regarded in itself is always only a product of imagination; but since the synthesis of the latter has as its aim no individual intuition but rather only unity in the determination of sensibility, the schema is to be distinguished from an image. Thus, if I place five points in a row . . . this is an image of the number five. On the contrary, if I only think a number in general, whether it be five or a hundred, this thinking is more the representation in an image of a method to represent a multitude . . . in accordance with a certain concept than the image itself . . . Now this representation of a general procedure of imagination to provide a concept with its image is what I call the schema for this concept [KANT1a: 273 (B: 179-180)].

This task, i.e. “providing a concept with its image,” is not something that we usually associate with the idea of imagination, but a brief moment of reflection upon it shows that this ‘makes sense.’ It is a fundamental theorem in information theory that once information is ‘lost’ during any process of data transformation, this information can *never be recovered*. Yet an experiential concept that cannot be turned back into an image in sensibility is an utter contradiction in terms. It follows at once that the synthesis of re-cognition in a concept must therefore provide the information necessary for the concept to be reproduced in intuition.

Now let us turn to the synthesis of reproduction in imagination. In the idea of the transcendental schema we have our first look at the power of imagination as a bridge from sensibility to understanding. The synthesis of reproduction, in like manner, is another bridge.

Now since all of our intuition is sensible, imagination belongs to *sensibility* because of the subjective condition under which it alone can give a corresponding intuition to notions of understanding; but so far as its synthesis is still an exercise of spontaneity (which is determining and not, like sense, merely determinable and can thus determine the form of sense *a priori* in accordance with the unity of apperception), imagination is to this extent a capacity to determine sensibility *a priori*, and its synthesis of intuitions in accordance with the categories must be the transcendental synthesis of *imagination*, which is an effect of understanding on sensibility [KANT1a: 256-257 (B: 151-152)].

Kant's last remark in this quote is worth emphasizing. The synthesis of reproduction *is an effect of understanding on sensibility*. If we likewise view the synthesis of re-cognition as an effect of sensibility on understanding, these two synthetic processes form a "closed circle" in which imagination and understanding stand in a complete and total relation of reciprocity insofar as the respective representations in sensibility and in concepts are concerned.

'Understanding' is not itself to be regarded as a logically distinct power of mind. Rather, it is an effect of determining judgment in reciprocity with imagination upon the faculty of representations. We can look at determining judgment and imagination as two *complementary* processes of representation bound together by the syntheses of reproduction and re-cognition and 'fed' through the synthesis of apprehension. Judgment and imagination, taken together, could justly be called *the power of cognition*. And here in this idea we can see the foundation and ground for Piaget's empirical characterization of "imagination" described in the previous section.

This leaves us to consider the second aspect of the synthesis of the cognition of an object that we mentioned near the beginning of this subsection. The discussion of this second aspect was put off until now because it does not involve the power of imagination. The synthesis to which we refer is the synthesis of the *manifold of concepts* by the process of determining judgment. This we may, with Kant, call the *intellectual synthesis* of cognitions. It is in the elucidation of *this* type of synthesis that we shall encounter the primitive functions and notions of determining judgment – the fundamental elements of the ontology of determinant judgments.

§ 2. Transcendental Schematism and Judgment

Our path from sensible intuitions to concepts and the ontology of determinant judgments goes by way of the transcendental schemata produced in the synthesis of pure imagination. As we have just discussed above, a transcendental schema is the 'bridge' between intuitive and intellectual representation and serves to enforce the conformity of concepts with the conditions of sensibility. Kant himself called the doctrine of schematism "one of the most important" factors in the transcendental critique of reason [KANT1a: 728, note 51].

Given this pronouncement by Kant, a reasonable person could expect this topic to be treated in depth in Kant's theory. However, what we might expect and what is in fact the case need not coincide, and this is the case with the doctrine of the transcendental schemata. Unlike the other great 'bridge' in Kant's theory – the doctrine of reflective judgment which joins practical reason to the determining judgment of cognitions, a topic Kant uses an entire book to discuss (*Critique of Judgment*) – the doctrine of transcendental schematism is covered in only one brief chapter in *Critique of Pure Reason* and in a few other scattered remarks and fragments [AK18: 686]. Even in the *Critique* the discussion has an ephemeral character:

Rather than pausing now for a dry and tedious dissection of what is required for transcendental schemata of pure notions of understanding in general, we would rather present them according to the order of the categories and in connection with these [KANT1a: 274 (B: 181)].

Kant scholars have rued this decision for two centuries. As Palmquist remarked to me one time, Kant's brevity has made the doctrine of the transcendental schemata "one of the thorniest topics in all Kant-interpretation."

We really have no choice but to tackle the details Kant left out, and this is what we will undertake in this Section and the next. But here, at the beginning, it is only fair that the reader be forewarned: Most of what follows is your author's reconstruction of this doctrine. Naturally, I think it is the correct reconstruction, but this is an area one can also expect will be a matter of dispute within the community of Kant scholars.

At least Kant was unambiguous as to the task that imagination's synthesis of the transcendental schemata must accomplish:

In all subsumptions of an object under a concept, the representation of the former must be *homogeneous* with the latter, i.e. the concept must contain that which is represented in the object that is to be subsumed under it, for that is what is meant by the very expression "an object is contained under a concept." . . .

Now, however, pure notions of understanding in comparison with empirical (indeed, in general sensible) intuitions are entirely unhomogeneous, and can never be encountered in any intuition. How then is the *subsumption* of the latter under the former, thus the *application* of the category to appearances, possible since no one would say: the category, e.g. causality, could also be intuited through the senses and be contained in the appearance? This so natural and important question is really the reason which makes a transcendental doctrine of the power of judgment necessary in order, namely, to show the possibility of how pure notions of understanding can be applied to appearances in general [KANT1a: 271-272 (B: 176-177)].

Kant thus gives us the fundamental description of the task at hand. Our first order of business is therefore to scrutinize this job description and untie any Cartesian knots we find in the bulrushes within it. The most natural place to begin is with the idea of an object being 'contained' under a concept.

The idea of a something we call an “object” is a very broad one – almost rivaling the three troublesome words of philosophy (being, reality, existence) in scope and abstraction. Viewed from the perspective of and as a question for transcendental Logic, what do we *mean* by the term ‘object’? First, an object is ‘something’ that is real *in some sense of the word ‘real.’* Earlier, when we discussed this latter troublesome word, we attributed two basic characteristics to what it meant for something ‘to be real.’ In the first place, such a ‘something’ is that of which we have a concept *connected with other concepts in the manifold of concepts.* This is the transcendental character of an object since the object is that which we regard as necessarily the ground of the representation. In the second place, the object, as a ‘real something,’ is regarded in terms of a *limitation* placed on the sum total of that representation of *Existenz* we call “Nature.” In other words, within the unity of the manifold of concepts the object is represented by the power to distinguish and, so to speak, break out some portion of this manifold and, metaphorically speaking, declare “*this thing.*” For each of us *an object is what we think it is* and, circular as this may seem, there is no getting around it within the boundaries set by the Copernican hypothesis.

Now *thinking is cognition* through concepts. But since cognition requires the representation of the appearance of the object in intuition as well as the “mere concept” of the object – a concept being nothing other than a rule for the re-presentation of intuitions – the capacity for *structuring* the organization, i.e. the *faculty*, of representation (*Vorstellungsfähigkeit*) must be such that an accord or ‘harmony’ exists between imaginative intuition and judgment. The construction of concepts must therefore be carried out in a manner so as to provide for this possibility, and the transcendental schemata – which link intuition and concepts – must consequently provide the ground *necessary* for this possibility.

In making an exposition of the requirements this ground must satisfy, our procedure follows a course not unlike the problem of building a bridge to connect the roads on either side of a river. It is no use beginning the construction of this bridge *until we know where the two roads come down to the riverbank.* In our case, on the one bank we have the conditions of sensibility; on the other we have the conditions for determinant judgment. We will begin with judgment and examine two ambiguities in Kant’s introduction above, namely what it means for one concept to “stand under” another and what we mean by the phrase “pure notion of understanding” (*reine Verstandesbegriff*).

The *idea* of “concept” has for *its* “object” a *noumenon*. We do not “sense” our concepts; rather, we say we “have a concept” of something. Seen in terms of this fundamental character, the concept is a “theoretical object” and so if we say one concept “contains” another or one concept “stands under” another, we must recognize that we are expressing, for want of a better description, a logical relationship – a mere appearance of a characteristic of the phenomenon of thinking.

Looked at from this *logical perspective* (as Palmquist called it in [PALM1: 127-129]), we can describe the idea of one representation ‘standing under’ another using the language of classical logic in the context of determinant judgment. In his 1762 essay on the four syllogistic figures of classical logic Kant wrote,

To compare [*vergleichen*] something as a mark with a thing is called *judging*. The thing itself is the subject, the mark is the predicate. The comparison [*Vergleichung*] is expressed through the sign of combination *is* or *are*, which when used plainly indicates that the predicate is a mark of the subject, but when saddled with the sign of negation makes known that the predicate is a mark opposed to the subject [KANT21: 89 (2: 47)].

In this merely formal view, a mark is a concept that may apply to many different logical subjects and, consequently, we say the subject term “stands under” the predicate term and the predicate term “understands” the subject term. Viewed from the logical perspective, the relationship “stands under” is a *mathematical* relationship in Kant’s sense of the word “mathematics”¹:

Philosophical knowledge² is *rational knowledge*³ from *concepts*, mathematical from the *construction* of concepts [KANT1a: 630 (B: 741)].

This idea of a concept ‘standing under’ some other concept is an idea which speaks to the *organization of representations* rather than to the ‘nature’ of the thing (the object) represented by the concept of the subject term. Hence this perspective has an analytic flavor to it and, as it is expressed in such a way as to make abstraction of all empirical content from the ‘subject’ and ‘predicate’ terms, it also has an *a priori* flavor.

There is another and non-mathematical perspective (which Palmquist called the *hypothetical perspective*) from which we can look at the act of judging. In this perspective we look at judging in terms of the ideas of a manifold of concepts and the connective relationships characteristic of this manifold:

Judgment is . . . the mediate knowledge of an object, hence the representation of a representation of it. In every [determinant] judgment is a concept that holds of many, and from this many also comprehends a specified representation, which is then referred immediately to the object [KANT1a: 205 (B: 93)].

The distinction here is that in this hypothetical perspective our focus has shifted from representation alone to a context of representation in which the idea of the appearance of the object is involved.

Consequently we have *at least* two perspectives from which we need to view this idea of one representation ‘standing under’ another. The logical view pertains to what we can call the

¹ The Greek root of the word "mathematics" means "what is learned."

² *Erkenntniß*.

³ *Vernunftkenntniß* - which may also be rendered "cognitions of reason."

‘mathematical theory’ of judgment; the hypothetical view pertains to the ‘dynamical theory’ of the phenomenon of thinking.

Now let us consider Kant’s idea of a *reine Verstandesbegriff* or ‘pure notion of understanding.’ We have said that a concept (*Begriff*) is a rule for the re-presentation (re-production) of an intuition. Does the term *Verstandesbegriff* then mean a rule for the intuitive representation of the phenomenon of understanding? Such an interpretation would clearly imply that ‘understanding’ is something we can represent in sensible intuition and would make a *Verstandesbegriff* – as a rule for the synthesis of the intuition of ‘understanding’ by the imagination – an *empirical* rule because any imaginative synthesis of an intuitive representation of understanding would necessarily have to draw its ground from our experience with a phenomenon called “understanding.” However, Kant’s term is not *Verstandesbegriff* but *reine Verstandesbegriff* – *pure* notion of understanding. By definition a ‘pure’ *Begriff* cannot draw its ground from experience but only from a ground that contains *nothing empirical*. We must look elsewhere for an explanation of the idea of a pure notion of understanding.

We have already described ‘understanding’ as an outcome or *effect* of the process of determining judgment. It is therefore reasonable to seek our explanation of the idea of a pure notion of understanding from the starting point of judgment and, in particular, what it is that a determinant judgment does. Here we will find it useful to take a look at Kant’s idea of a **function** (*Function*):

Indeed I understand by ‘function’ the unity of the act of ordering different representations under a common one [KANT1a: 205 (B: 93)].⁴

Viewed from our aforementioned logical perspective, the idea of a function is, in the Kantian sense, a mathematical idea. The phrase ‘unity of the act’ means the act viewed as some one object – a usage that fits well with Piaget’s idea of a “scheme.” Function is a general term and we will not err by regarding a Piagetian “scheme” as a particular instance of a Kantian “function” from the logical perspective.

This idea of a function is central to what we might call Kant’s “operational description” of determinant judgments:

All judgments are accordingly functions of unity in our representations, since instead of an immediate representation a higher one, which comprehends this and sundry others under itself, is used for the cognition of the object, and many possible cognitions are thereby drawn together into

⁴ It is not out of place here to comment on Kant’s oft-used phrases which go: I understand by ‘X’ Y. Kant uses this phrase to establish terminology. Today we would probably phrase this as, “I define X to mean Y.” Kant, however, was very cautious in using the word “define” since he used the word definition to mean “a sufficiently distinct and precise concept.”

one . . . Thinking is cognition through concepts. Concepts, however, subsist as predicates of possible judgments on some other representation of a still undetermined object . . . The functions of understanding can therefore collectively be found if one can completely present the functions of unity in judgments [KANT1a: 205-206 (B: 94)].

Here Kant adopts the logical perspective in describing the place of concepts (as “predicates”) within a judgment. As for the judgment itself, it is an act which not only produces a new representation but does so in a particular way – namely, in a way that produces unity in one’s representations. This function of a judgment ties back to the principle of the transcendental apperception and our requirement that representations be connected in one consciousness. Viewed from a *transcendental perspective*, this ‘unity in one consciousness’ may be seen as the object of the process of determining judgment in general.⁵

Now let us look at the ‘functions of understanding’ in terms of Piagetian schemes. A *mental* scheme, as an *action*, has the idea of its unity in the accomplishment of the representation of the necessary unity in the manifold of objective perceptions by means of a determinant judgment. This is an idea of the unity of a judgment from a *practical Standpoint*. In terms of cognitions, this unity necessarily includes the unity of the intuitive representation and that of the intellectual (i.e., conceptual) representation. From this we can see that:

The same function which gives unity to the different representations *in a judgment* also gives the mere synthesis of different representations *in an intuition* unity which, expressed generally, is called the pure notion of understanding. The same understanding, therefore, and indeed through the very same acts whereby it requires there be analytical unity of the logical form of a judgment by means of concepts, also brings into its representations, by means of the synthetic unity of the manifold in intuition in general that goes *a priori* to Objects, a transcendental content, on account of which they are called pure notions of understanding [KANT1a: 211-212 (B: 105)].

This gives us no less than three perspectives from which to view the ‘pure notion of understanding.’ From the *logical* perspective, the pure notion of understanding is a *scheme of judgment* – an action undertaken in the process of determining judgment for the ordering of representations. From the *hypothetical* perspective, a pure notion of understanding *always refers to an object of reason* even though this reference is not through immediate representation of the object (for that is the job of the intuition). This mediate reference to an object is made via the form of the concept that is the rule for the re-presentation of the intuition of the object’s appearance. From the *transcendental* perspective, the effect of the judgment (understanding) is an effect registered in the faculty of pure consciousness *made in accordance* with the principles of Rational Psychology, which legislate *a priori* for apperception in the synthetic unity of representation in *one* consciousness.

This exposition through these three different perspectives tells us quite a lot about the idea of

⁵ When a process produces something, we will call that outcome the “object of the process.”

the pure notion of understanding, but not quite enough. There is one more factor that must enter into our exposition of this idea, namely *experience*. As schemes of thinking, the pure notions of understanding pertain to our idea of thinking as cognition through concepts. But the possibility of cognition – and therefore of experience – rests on the ground of the thorough-going reciprocity between conceptual representation and the conditions of sensibility in general. Put another way, it is one thing to say a pure notion of understanding is a function; it is another to say what the meaning given in this function must be. Whatever it is, it must match up with the requirement for subsuming a given intuition under concepts (synthesis of re-cognition) on the one hand, and the requirement for producing sensible intuitions from concepts (synthesis of reproduction) on the other. These dual requirements obviously speak to the possibility of *actual empirical knowledge* and its *synthetic a posteriori* representation in cognition. Thus we have need for a fourth perspective, which we will follow Palmquist in calling the *empirical perspective*. From this perspective we would be justified in calling the pure notions of understanding the *categories of Nature* but, instead, we will follow Kant and call them the *categories of understanding*.

It is at this point in our exposition where we must next return to Kant's transcendental schemata. We have marked out the roads on either side of the river which, figuratively, lies between sensible and intellectual representation. Our attention must now turn to the conditions of sensibility with which the categories of understanding must accord. We must build the bridge.

§ 3. The Transcendental Schemata

Kant used the term 'schema' to mean "the formal and pure condition of sensibility to which the use of the [corresponding] notion of understanding is restricted." He used the term 'schematism' to mean "the procedure of understanding with these schemata" [KANT1a: 273 (B: 179)]. The problem from the empirical perspective is to understand how the subsumption of intuitions under the categories of understanding is possible. At issue is the fact that a category is itself non-sensible while the opposite is true of an intuition. Thus, categories and intuitions are non-homogeneous and therefore how can the one 'contain' or 'form a unity of representation' with the other? This can only be possible, Kant wrote, if we posit a third mediating class of representation which is homogeneous with the category on the one side and with the intuition on the other.

Now how can such a 'bridge' be possible? Here we recall that what we require in the schematism of concepts is their conformity with and limitation to the conditions of sensibility. At the same time, this schematism must be one that allows the categories to fulfill their principal function, which is the synthetic unity of the manifold of cognitions in one consciousness.

The solution to this problem is found by recognizing that the *formal* condition of sensibility is its representation in what we have previously called the *form of inner sense* (Chapter 6, §3.3).

This formal condition, as we discussed previously, is the pure intuition of (subjective) time. Consequently, because a transcendental time-determination is homogeneous with categories on the one hand (because such a determination has a universal character and rests on a rule *a priori*) and with the representation of an appearance on the other (since time is a pure form in every intuition), “the application of the category to appearances becomes possible by means of the transcendental time-determination” [KANT1a: 272 (B: 178)]. A transcendental schema is, therefore, nothing else than a time-determination synthesized by the power of imagination.

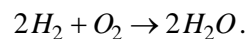
The intuition of subjective time has three *modi*: persistence in time, succession in time, and coexistence in time. These *modi*, when applied to the four titles of Quantity, Quality, Relation, and Modality in our general 2LAR, yield twelve distinct and fundamental ‘*momenta*’ or significations possible in making a time-determination. It follows from this that we will have twelve corresponding categories of understanding – one for each possible manner of time-determination – grouped according to the headings of Quantity, Quality, etc.

Now one sees from all this that the schema of each category contains and makes representable: as that for magnitude, the generation (synthesis) of time itself in the successive apprehension of an object; the schema for Quality, the synthesis of sensation (perception) with the representation of time, or the filling of time; for Relation, the relationship of the perceptions among one another in all time (i.e. according to a rule of time-determination); finally, the schema for Modality and its categories, time itself as the correlate of the determination of an object, whether and how it belongs to time [KANT1a: 275-276 (B: 184)].

§ 3.1 The Schemata of Quantity

In terms of Quantity, every intuition is an extensive magnitude – a unity composed of a homogeneous multiplicity of parts. As we discussed in Rational Physics, the synthesis of apprehension produces an intuition by ‘marking’ its representation by a moment in time. Apprehension does not cease at this moment but continues on, and one intuition ‘evolves into’ or ‘becomes’ a succeeding intuition when the synthesis of sensibility is marked again by another moment in time.

Kant called this on-going synthesis the ‘successive addition’ of one representation after another. Here we should not think that Kant was speaking of ‘addition’ in an ‘arithmetic’ sense of that word (e.g., $3 + 4 = 7$). Addition in this context is rather more like the notation used in chemistry to describe reactions in which two or more reactants are compounded, e.g.,



This analogy is a particularly appropriate one inasmuch as the “summary” given by a chemical formula hides a number of details (for instance, the “breaking apart” of the two hydrogen molecules and the oxygen molecule “on the way” to the final formulation of water). Likewise, we

need not suppose that the ‘addition’ of a sensational ‘snapshot’ going into the synthesis of apprehension involves, so to speak, an operation where ‘new’ data of the senses is merely ‘glued to’ the accumulated representation to which it is ‘added.’ Instead, we must examine this idea of the synthesis of apprehension in terms of the three *modi* of time to discover the formal conditions of quantitative representation. In what manner does time’s synthesis give rise to the apprehension of appearances?

First, we must consider the *modus* of persistence in time. If there is no representation of some form of composition between an intuition at one moment in time and the intuition at the succeeding moment in time, we could have no universal and necessary ground for the combination of discrete intuitions of appearances in a representation of a phenomenal object. Looked at from the side of concepts, the possibility of an *a priori* rule for the combination of intuitions in the representation of a phenomenal object requires a scheme of thinking in which the condition of persistence in time is contained. We name this category **unity** (*Einheit*) because the concept it produces is necessarily thought as the concept of *one* object. Without a category of unity we could have no ground for the *productive* imagination since the synthesis of reproduction in imagination would be limited to the mere ‘re-play’ of intuitions given directly by the data of the senses, and no ideas of supersensible objects would be possible. **The first schema of Quantity is aggregation of distinct intuitions in one object.**

Second, we must consider the *modus* of succession in time. Intuitions at successive moments in time must *differ* from each other; if they did not, we would have no ground for marking one representation in the synthesis of apprehension as distinct in time from its predecessor. Put another way, our ability to perceive *change* necessarily presupposes the ability to compose a combination of intuitions such that a difference can be thought. Thus, along with ‘the one’ conceptualized under the category of unity, we must also be able to think ‘the many’ – e.g., many different appearances *in* one object, or ‘the many objects *in* one Nature.’ The category that conforms its composition of concepts to the condition of the *modus* of succession in time with regard to Quantity we call the category of **plurality**. **The second schema for Quantity is change of composition in extensive magnitudes.**

Finally, we must consider the *modus* of coexistence in time. The schema of this *modus* we can regard as the synthesis of the first two. To be able to think multiple phenomenal objects falls under the schema of plurality. But to be able to represent more than one *object* in one intuition necessarily requires a scheme of representation by which manifold appearances in a single intuition can be represented conceptually. Such a category is neither unity as such nor plurality as such but, rather, is a notion of **totality**. This notion is that of a scheme of aggregation of ‘many objects in *one* Object.’ Hence, **the third schema for Quantity is integration of extensive magnitudes in time.**

Collectively, **these three schemata define the synthesis of a series in time** and that character of apprehension by which an intuition is an extensive magnitude. This series refers to the series of sensibilities that “go into” the synthesis of an intuition. In *Critique of Pure Reason* Kant did not break out these three schemata explicitly, as we have done here. Rather, he simply lumped all three together under what he called “the schema of number” (*die Zahl*). To the modern reader this may seem a peculiar terminology. It is obvious Kant was not trying to imply that “numbers” make up in any way some store of innate ideas; after all, young children must be taught how to count, just as they must be taught how to tell objective time. What, then, was Kant saying? Number, he tells us, is

a representation integrating the successive addition of unit to (homogeneous) unit. Thus number is nothing other than the unity of the synthesis of a homogeneous intuition in general, because I beget time itself in the apprehension of the intuition⁶ [KANT1a: 274 (B: 182)].

Thus this idea of ‘number’ is the idea of a construct springing from the synthesis of extensive magnitude in sensibility; it is *not* the idea of, say, cardinal numbers. This peculiar terminology may well seem to us, after two hundred years of on-going development of the sciences, an odd and even misleading choice of words on Kant’s part. After all, do we not all know what a number is?

Interestingly, while most of us probably feel very comfortable in the belief that we know what ‘number’ means, it seems that *mathematicians* are not quite so confident. The question “what is a number?” has been frequently asked by mathematicians. It was an especially hot topic at the opening of the twentieth century, but attempts to define ‘number’ go back a very long time. The following are some snapshots of what ‘number’ meant to different great thinkers of the past.

For number is units, and the unit is something whose essence is to be one.

Aristotle (c. 330 B.C.)
Metaphysics, III. (1001^a26)

Number is limited magnitude or a combination of units or a flow of quantity made up of units; and the first division of number is even and odd.

Nichomachus of Gerasa (c. 100 A.D.)
Introduction to Arithmetic, I., vii., 1

If in the consideration of a simply infinite system N set in order by a transformation ϕ we entirely neglect the special character of the elements, simply retaining their distinguishability and taking into account only the relations to one another in which they are placed by the order-setting transformation ϕ , then are these elements called *natural numbers* or *ordinal numbers* or simply *numbers*.

Richard Dedekind (1893)
The Nature and Meaning of Numbers, §73.

⁶ Kant means we could not have any idea of objective time without this synthesis in subjective time.

Every aggregate M has a definite "power," which we will also call its "cardinal number."

Georg Cantor (1895)

"Contributions to the Founding of the Theory of Transfinite Numbers," §1.

The number of a class is the class of all those classes that are similar to it . . . A number is anything which is the number of some class.

Bertrand Russell (1919)

Introduction to Mathematical Philosophy

A natural number is an ordinal such that itself and every element of it is either 0 or a successor.

Paul Bernays (1958)

Axiomatic Set Theory

As we can see, the definition of 'number' has gotten progressively more abstract, especially in the course of the twentieth century. (We might imagine how Descartes might comment on this!). Today in mathematics we have definitions for a variety of "types" of numbers (e.g., natural numbers, integers, rational numbers, real numbers, complex numbers and transfinite numbers), but it would seem that 'number' itself – without a qualifying adjective – is explainable only as the set of all these "special kinds" of numbers (which, of course, leaves it open to amendment whenever it might serve the purposes of mathematics to define another new class of 'numbers'). Seen in this light, perhaps Kant's constructive explanation of 'number' is not so quaint and curious after all.⁷

§ 3.2 The Schemata of Quality

Kant's treatment of the schemata of Quality in *Critique of Pure Reason* is, even by normal Kantian standards, extremely vague – a shortcoming that has been bemoaned by a number of commentators (e.g. Paton in [PAT2: 17-32]). In the earlier quote, we saw that these schemata pertain to the representation of the "filling of time" found in the synthesis of perceptions. Unfortunately, Kant does not clearly draw the connection between the three *modi* of time and the schemata of Quality in the *Critique* and we must try to "reconstruct" the theory of these schemata to address this shortcoming.

While the schemata of Quantity deal with the extensive magnitude of an appearance, those of Quality pertain to the intensive magnitude of its sensation.

Perception of an object is a consciousness of the object through sensation [KANT19: 467 (29: 999)].

The sensational content in an intuition is what separates phenomenal objects from *noumena* of thinking. This aspect – composition as the coalition of sensation – is what these schemata must treat.

⁷ Piaget came to define 'number' as "a synthesis of class inclusion and relationships of order" [PIAG17: 38]. He presented this definition in 1970 on the basis of his research published in [PIAG10].

Let us remind ourselves of what the term ‘intensive magnitude’ denotes. Intensive magnitude

is understood as a magnitude whereby the parts are not recognized previously in order to determine the magnitude; rather it must be recognized as unity, and the parts drawn out from the unity [KANT19: 467 (29: 999)].

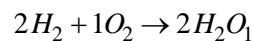
In every empirical intuition the form of the representation is given by the pure intuitions of outer and inner sense. These alone, of course, provide us with no objective ground for knowing the “real existence” (*Dasein*) of the object because the pure intuitions pertain to one’s own representation of the *Existenz* of the object. The phenomenal reality of an object is grounded in sensation, and we call **the matter of the object** that in the object which corresponds to sensation in the empirical intuition of its appearance.

Reality is either phenomenon or noumenon. Everything that is presented positively to our senses is called phenomenal reality; and everything that is presented positively to our pure understanding is noumenal reality. Phenomenal reality or reality in appearance (or seeming-reality) is that which lies only in our senses [KANT19: 324 (28: 560)].

The categories of understanding, on the intellectual side of cognition, are entitled to be called categories of Nature solely because they are the functions of judgment that pertain to thinking of an object in terms of *phenomenal* reality. Thus, it is this aspect of Reality with which the schemata of Quality must be concerned.

Ever since Aristotle made use of the term, the idea of “quality” has been a difficult and sometimes controversial idea in philosophy. Locke attempted to establish the objective validity of human knowledge of a “material” world on a distinction between “primary” and “secondary” qualities – a distinction Berkeley later demolished. To gain an understanding of what “quality” means under the Copernican hypothesis, and how this matter-of-the-matter of cognition is schematized, it might perhaps be best if we avoid Kant’s very abstract treatment at first and approach the problem by way of an illustrative analogy.

We will use our earlier chemistry analogy



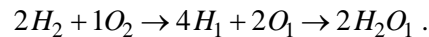
to illustrate the basic idea.¹ In this formula the letter symbols *H* and *O* stand for sensations. The numbers are analogous to the extensive form (Quantity) of the intuition – i.e. since the schema of Quantity is Kantian “number,” using cardinal numbers to illustrate extensive magnitude seems a natural choice.

¹ I have slightly modified the notation of the formula in order to better illustrate the analogy. The standard notation in chemistry omits the “1” symbols since it is understood that any letter symbol which appears in a formula occurs at least once.

We will take the left-hand and right-hand sides of this formula as intuitions at two successive moments in time and the " \rightarrow " symbol as denoting the synthesis in time going from one moment to the next. Thus, the manifold in the first intuition 'contains' three objects (the two hydrogen molecules and the one oxygen molecule); the manifold in the second 'contains' two objects (the two molecules of water). We will let the chemist (played by ourselves) be the analog of the phenomenon of cognition.

In terms of representation, the two intuitions depicted in our formula are *singular* representations. The intuition of the left-hand side is not that of three objects but rather of *one appearance*. To recognize this appearance as three objects we must *conceptualize* it by breaking it down (via judgmentation) into the three individual terms. In other words, to re-present the left-hand side as a phenomenon, we must be able to *think* "hydrogen molecule" and "oxygen molecule" and to furthermore recognize the hydrogen and oxygen molecules as *distinct* objects coexisting in time. The issue before us is how the filling of time in the schemata of Quality can make such a conceptualization possible.

To better grasp this situation, let us analytically break down the synthesis of apprehension by re-expressing our formula as



The new middle term in this formula does *not* stand for an intuition. Rather, it represents to us a snapshot of the elements of perception re-coalescing in the synthesis of apprehension from one moment to the next. Let us now take a look at the formula-as-a-whole going from left to right.

First, we can take note of the presence of "elements of sensation" (the *H* and *O* "atoms") as something that is *the persistent in time*. The intuitions on both the left-hand and right-hand sides 'contain' the "persistent sensational elements" and we view these sensational elements within the process of synthesis (between moments).² Furthermore, these 'persistents' are precisely those factors in representation that present phenomenal reality to cognition. This illustrates our **first schema of Quality: sensation persistent in time**. We call the corresponding category for this schema *reality* (*Realität*) to denote that this schema of Quality is the condition of sensibility for intuition and empirical concepts by which we think *the phenomenally real*.

Now let us examine the succession in time. Here we find a factor in the representation that does not persist in time, namely the extensive *form*, which we may call the *pure intuition of outer*

² It is understandable if you, the reader, feel a bit uncomfortable with this idea of "the time between moments." This is an idea we will discuss in much greater detail when we come to the theory of the pure intuition of time, but here it must be said that subjective time is not the mere "moments *in time*." It may perhaps be helpful to recall James' idea of "substantive" and "transitive" parts of thought in his stream of thought model; we must make the "transitive part" into a "substantive part" if we are to be able to discuss it, and this is what the middle term in the second formula is doing.

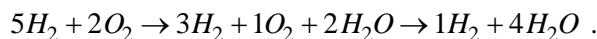
sense. In the middle term, the "4," "2," and "1" symbols, metaphorically speaking, “come from out of nowhere” since we do not find them (in the same form) in the initial intuition. They are, so to speak, the ‘**kinematical factor**’³ of intuitive representation. *They are also the factor in sensibility that is non-sensational* and, under the Copernican hypothesis and our rejection of the copy-of-reality hypothesis, are pure manifestations of the power of representation. The kinematical form is, indeed, the very hallmark of successive intuitions in time. Clearly this kinematical aspect in the filling of time must be capable of being conceptualized, and so **the second schema of Quality is the kinematic form without sensation**. Because the kinematic form contains nothing of sensation, we logically regard it as that which in intuition is the opposite of sensation. Accordingly, Kant called the corresponding category *negation*.

Finally, we may note that sensational and kinematical elements of sensibility are found coexisting in time throughout the entirety of the synthesis of apprehension. Kant seemed fond of saying that the “being” (*Sein* – which we take to mean the sensational element) and the “non-being” (which we take to mean the kinematical element) of an Object are both present in the representation of appearance. If we look at representation in terms of these two quite different ‘constituents’ of composition, what we find is that the kinematical factor (Kant’s “non-being”) is represented as *affecting* the perception of the sensational factor (the “being”). This might be better put by saying the kinematical form goes to the determination of the *Existenz* of an appearance; the *Dasein* of the object is thought from the ground of sensation. Now this relationship – represented throughout the synthesis of time – whereby the kinematic form affects the phenomenally real in sensation must be capable of being conceptualized, and the representation of the possibility of this is the schema under the *modus* of coexistence in the filling of time. The category for this schema is called *limitation* because the kinematic form *determines* (i.e., limits) the intuitive perception. **The third schema of Quality is perception as the coalition of sensation in a kinematic form.**

Strictly speaking, this is all we need for our explanation of the schemata of Quality. However, Kant persistently mixes in with these ideas another idea, namely that the sensational content of an intuition *has a degree*. Indeed, one can suppose from the emphasis Kant placed on this idea that this auxiliary idea is of profound theoretical importance. We should, therefore, examine the meaning of the idea of the ‘degree’ of Quality.

Turning again to our chemistry analogy, let’s add some more “gas molecules” to the illustration and consider the sequence of “intuitions”

³ Recall that *kinesis* in Aristotelian Greek means "change" of any kind.



This time the middle term does stand for an intuition at a determined moment in time. Keeping in mind that each of these three "intuitions" must be regarded by us as a singular representation, in this illustration the "sensation of O_2 " gradually disappears and the "sensation of H_2O " gradually appears in the sequence of intuitions. The "sensation of H_2 " persists in the sequence, but the "amount of H_2 " appears diminished from moment to moment. Even though the "elements of sensation" (H and O) persist throughout the synthesis of the succession in time, there is a *qualitative difference* in sensation among the three intuitions from moment to moment. This difference is a difference in *the content in time* brought about by changes in how the kinematical form limits the sensational elements in sensibility.

It might be objected – and rightly so – by a chemist that the difference of which we speak is not 'qualitative' but, rather, 'quantitative.' This requires an explanation. In chemistry the term 'qualitative analysis' refers to "determining the nature of a pure unknown compound or the compounds present in a mixture." 'Quantitative analysis' refers to "measuring the proportions of known compounds in a mixture."⁴ If we take the view that the individual constituents in each of the three moments in time in the example are known, then the differences among the three terms in the formulae above are 'quantitative' differences.

However, to look at this illustration from that viewpoint is to look at the contents of each of the three terms as already broken out in distinct representations. We can only call these differences 'quantitative' if we do *not* regard each term as a representation *in intuition* because an intuition is *always* a singular representation and a *unit* of representation. If we regard each of the three terms above as intuitions, we do not know the constituents in the manifolds and the difference between successive moments in time can therefore correctly be called 'philosophically qualitative.' Any analogy is a simile; consequently, an analogy can be used properly only if it is understood from the viewpoint of the similarity between it and that which it is meant to illustrate. In chemistry qualitative analysis must precede quantitative analysis; in the Critical Philosophy intuition formally precedes its empirical concept.

Now let us consider this idea of the sensations 'in' an intuition that 'fill' a moment in time. A difference is a magnitude (in the Kantian usage of that term) and, as we said earlier in this treatise, the representational state of apprehension between moments in time is not itself an appearance. In our example we have moments in time where the sensation of O_2 is present in the intuition of appearance, and we have a moment in time where it is no longer present in appearance. Furthermore, in the succession of time there are qualitative differences in how much

⁴ These definitions are taken from the *Concise Science Dictionary* published in 1984 by the Oxford University Press.

the presentation of O_2 contributes to or ‘fills’ the successive appearances in time. This idea of “how much contribution” the sensation makes to the intuition of appearance is what is meant by the idea of the degree of perception. As Kant put it, “the amount of a quality is a degree.”

There is an almost irresistible inclination to schematize this idea of ‘degree’ using numbers. We can, after all, look at the sequence $5H_2$, $3H_2$, $1H_2$ and say, “Ah! The degree decreases as 5, 3, 1.” Even Kant’s description just quoted promotes thinking about degree in this way. However, this is not quite what is meant by the idea of degree. The idea of degree is perhaps more accurately understood if we think about the experience of watching a sunset. At first, we perceive the day as “well-lighted” by the sun. As time goes on the “degree of light” we perceive gradually diminishes “with the coming of darkness.” Eventually we perceive the world as primarily “dark” with only the stars, streetlights, etc. providing points of light in the darkness. Normally we do not even attempt to put a number on the degree of lightness and the degree of darkness; we merely say “it is getting darker” or “the light is fading.” Degree is *ordinal*, not cardinal.

The idea of the degree of a quality is, before all else, an *idea* exhibited in empirical cognition. As such, it is bound up in part with a category, namely *limitation*. It is, however, also tied in to a schematism of Quantity insofar as our understanding of ‘degree’ also involves *extensive* magnitude. This is why I called the idea of degree an “auxiliary idea.” It is a very useful idea for our theory, but it is not a *primitive* notion.

§ 3.3 The Schemata of Relation

The pure intuition of subjective time is the form of inner sense. Among other things, this means that every conscious representation of every sort is represented as being (“taking place”) *in* time. Even the ideas we use to describe our cognitions are unimaginable without making use of an idea of ‘time’ as a kind of ‘substrate’ of representation. For example, we have talked about a “moment *in* time” and, likewise, of the “synthesis occurring *in* time ‘between’ moments.” The intuition of time is not an idea of “duration” for “duration” is itself an idea grounded in the pure intuition of time – i.e. duration is imagined as an “interval between moments” *in* time that we conceptualize in the idea of *objective* time as a magnitude measurable by the use of clocks.

Likewise, other ideas such as *the past*, *the present*, and *the future* are ideas that depend upon this pure intuition of an absolute time *from which* these ideas get a common ground of cognition. Such, likewise, is the case with our three *modi* of time: persistence *in* time, succession *in* time, and coexistence *in* time. This subjective time is not a *thing* apart and separate from other things but, instead, is a *pure condition of sensibility*.

Now we can describe this pure intuition to ourselves only by objectifying it because the pure intuition of time is not something we perceive but, instead, is a condition of sensibility in general. However, the possibility of objectifying time in cognition necessarily presupposes the ability to

schematize empirical intuitions in such a way that *ideas* of persistence, succession, and coexistence are possible in accordance with time as a formal condition of sensibility. The schemata of Relation are the elements of the ‘bridge’ that links the *form of the manifold of concepts* to the condition of the pure intuition of time.

We begin with the *modus* of persistence in time. If we examine this idea, we find that it always contains two attributes invariably paired together. First, it grounds the possibility of an idea of some object that exists or could exist (in the *Dasein* connotation) *in* time from one moment to the next. Second, it grounds the possibility for recognizing this object as a *phenomenon* by uniting in one object many appearances *in* time. Put another way, it grounds the recognition of the *Existenz* of this one phenomenal object as being represented in the accidents of appearance from moment to moment. Thus, we have the changeable in appearance inherent *in* some *one* object that persists from moment to moment. The category that corresponds to this schema we call the Relation of *substance and accident*.⁵ **The first schema of Relation is Object persistence in time.**

The schema of substance is the persistence of the real in time, i.e., the representation of the same as a substratum of empirical time-determination in general, which therefore remains while everything else changes. (Time itself does not elapse, but the *Dasein* of the changeable elapses in it. Therefore to time, which itself is unchangeable and lasting, corresponds in appearance that which is unchangeable in *Dasein*, i.e. substance, and only in it can the succession and coexistence of appearances be determined in regard to time.) [KANT1a: 275 (B: 183)].

We should pay some close attention to the phrase “that which is unchangeable in *Dasein*” since it is only a short step from this phrase to thinking of substance as “something that lasts forever, unchangeable” rather than as a notion of Relation in determinant judgment linked to representation in intuition by a schema of persistence in time. The pure intuition of time is not some vague knowledge of a thing in the environment, and so the idea of “a” substance that is “eternal in time” cannot in any way be what Kant is talking about. Such an idea violates the Copernican hypothesis in two ways. First, it places “time” as something “outside” the thinking Subject – transforming it from the form of inner sense to objective time. Second, it makes “substance” an idea almost synonymous with the idea of “corpuscular matter” in the atomists’ or Boyle’s usage of the word “matter.”

The *idea* of “the pure intuition of time” is the idea of a *noumenon* that *grounds* our inner sense of the representability of perceptions in terms of the *modi* of persistence, succession, and

⁵ *Substantia et accidens*. Kant also called this category *der Relation der Inhärenz und Subsistenz* - inherence and subsistence. Furthermore, he often abbreviated the name of this category to simply the "category of substance" (e.g., see [KANT2: 51 (4: 303)]). Of all these usages, he seems to have preferred "substance and accident" in his lectures, and given Kant's oft-repeated admonition that the most important yet difficult terms should be phrased "in a dead language so their meaning will not change" - as it often does in a "living" language - we will adopt the Latin phrase as our official name of this category.

coexistence. Without this rational idea and that of subjective time we could have no basis for understanding persistence, succession, and coexistence or for them having any connection with each other (i.e. we could not say they are accidents or *modi of time*).⁶ The *notion* of substance and accident is to determinant judgment what the *modus* of persistence is to the pure intuition of time – a representation that is necessary for the possibility of connecting, in the *nexus* of the manifold of representation, otherwise diverse representations in a *single* common Object and connecting this Object in Nature. We perceive neither substance itself nor time itself; what we perceive are the accidents of appearance and the evolution of perceptions. The notion of substance and the *modus* of persistence are but the pure elements of representation by which we have the capacity or *Vermögen* for *thinking* appearances to be “accidents” of the *Existenz* of objects and for *intuiting* perceptions as phenomena. The schema of persistence is the ground for what James called “the substantive part of thought.”

Now we turn to the *modus* of succession in time. Kant tersely described the schema of Relation in succession as follows:

The schema of the cause and the causality of a thing in general is the real whereupon, as soon as it is granted at one's discretion, something else always follows. It therefore subsists in the succession of the manifold insofar as it is subject to a rule [KANT1a: 275 (B: 183)].

The phrase I have rendered as “as soon as it is granted at one's discretion” is usually translated as “whenever it is posited.” There is nothing particularly incorrect with this traditional rendering, as a straightforward matter of linguistics, but Kant's original wording gets at an important aspect of the schema. He uses the phrases “*der Ursache*” and “*nach Belieben*”, both of which speak to *judging* ‘the real’ in a *discursive* determination – i.e. as a made concept of a ‘something’ of which its *Dasein* is *recognized* through an act of judgment. There is no pure *intuition* of cause and effect – which omission of “at one's discretion” in the traditional rendering could be taken to imply – and therefore the possibility of *thinking* the *Dasein* of a cause (*Ursache*) requires a transcendental schema by which such a Relation between objects can come to be represented in intuition.

It is all too easy to lose sight of how vague and non-specific the word “thing” is, and to fall into a habit of equating the word “thing” with “material thing.” This we must not do. The stock market “crashes” and people ask, “How could *such a thing* happen?”; the Nazis massacre millions of people and historians and psychologists ask, “How could *such a thing* happen?”; an

⁶ It is perhaps worth reminding ourselves that mental physics - and Kant's Critical Philosophy in general - is a *theory* and that *all* theories are the product of thinking. Hence theory is expressed in ideas by which we comprehend Nature. We do not *perceive* the pure intuition of time; we deduce it as the rational ground of that which we *do* perceive and recognize, e.g. change, and the *modi* of persistence, succession, and coexistence of appearances of objects.

apple falls from a tree and Newton asks, “How did *such a thing* happen?” In all the appearances of the phenomenon of mind, few are more profoundly strange and interesting as *the ability to ask such questions* and make an effort to answer them. A machine can be made to produce representations of “objects” from the data supplied through its sensors (and, indeed, such machines have often been built); but no machine to date asks itself *why?* or *how?* questions. The ability to do so is a mark of what we call intelligence and the *act* of doing so is a manifestation of what we call curiosity.

But before we can attempt to study these issues, we must first understand the *possibility of making* representations of causal relationships. Here we focus our attention on Kant’s phrase, “insofar as it is subject *to a rule*.” Now here by “rule” we can only mean *an empirical concept* since concepts *are* the rules for the reproduction of intuitions. The possibility of *positing* such a rule requires in determinant judgment the *a priori form* necessary for the possibility of making such a rule. This *a priori* form is a category.

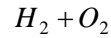
A rule that addresses the *nexus* of appearances in succession in time falls under the category we call ***causality and dependency***. It is one thing to intuit at successive moments in time; it is quite another to comprehend appearances at these moments in terms of appearances *necessarily connected* both in apprehension and in the manifold of concepts. The schema that forms the bridge between apprehension and concept in this case we name **the schema of association in time-order** (*Zeitordnung*). We can perhaps adequately describe this schema as one where, following the recognition in concept of an appearance previously intuited, there follows a synthesis of imagination which recalls to intuition a sequence of appearances that previously followed the former appearance. It is thus the schema for cognition of temporal phenomena.

That our *empirical* concepts of causes and effects are contingent does not destroy the objective validity of the notion of causality and dependency because if there was no objective validity in positing Relations of causality and dependency, the unity of Nature would be impossible to recognize in terms of divers objects. We may call the schema of succession a hypothetical schema (that is, a schema of anticipation), but nonetheless such a hypothetical schema is necessary for the possibility of experience.

Finally, we come to the schema of the *modus* of coexistence in time. Here the schema, its corresponding category, and even the idea of the *modus* of coexistence have, curiously, been little regarded, and even sometimes wrongly dismissed, by twentieth century scientists (and by some twentieth century philosophers as well). Perhaps this is due in part to a misunderstood and undue influence of Einstein’s theory (since they are frequently mis-associated with the idea of “simultaneity” in *objective* time). Be that as it may, Kant’s description of the schema was by most accounts too brief:

The schema of community (reciprocity), or of the reciprocal causality of substances with respect to their accidents, is the coexistence of the determinations of the one with those of the other, in accordance with a general rule [KANT1a: 275 (B: 183-184)].

‘Coexistence’ (*Zugleichsein*, “conjointly being”) refers to the presentation of two or more different phenomenal objects represented as present *at the same moment* in time. We see something of the flavor of this in chemical notation of our earlier analogies, e.g. in the notation



where the "+" sign is used to denote the distinction of the hydrogen and oxygen molecules as separate but coexisting objects. We may contrast this notation with that of, say, hydrogen peroxide



where such a distinction is not signified and the notation denotes a single object.

When such a distinction is recognized as being contained in an intuition – a distinction that obviously involves the Quantity of the concept as well as the Relation of substance and accident – then the general rule or concept of the intuition must, *ipso facto*, also contain the representation of how one represented object affects the representations of the others, and vice versa. The pure intuition of time is never part of any concept, but the form of concepts given in determinant judgment must reflect, in accordance with the appropriate schema, the time-determination, and this is what the categories (as schemes of judgments) do.

There would seem to be, at first glance, a fundamental contradiction between the idea of intuition as a singular representation and the idea of a singular representation also being “at the same time” an intuition of a multiplicity of phenomenal *objects*. Indeed, there would be a contradiction if the categories of understanding did not provide a pure *a priori* form of judgment of a reciprocal Relation between objects appearing in the same moment in time. Such a category we call the notion of *community*, and it pertains to the necessity of representing divers phenomena in terms of each representation simultaneously determining the other and being itself determined by the representation of the other. If we think “A is *to the left* of B”, we must *also* think “B is *to the right* of A”. **The schema that links such conceptual representations with the singular intuition is co-determination of the manifold in an intuition.**

Whether an intuition of an appearance is recognized as the appearance of one phenomenal object or more than one is the business of the schemata, categories of Quantity, schema of persistence in time, and category of substance and accident in Relation. However, if an intuition

is conceptualized as containing a multiplicity of substances, the presentation of this multiplicity in a single moment of time belongs to the schema for the Relation of community. Thus, like the schema of succession in time and the notion of causality and dependency, the schema of coexistence in time and the notion of community necessarily must presuppose a prior act of judgment that determines the representation of a phenomenon in which multiple substances are represented.

§ 3.4 The Schemata of Modality

Modality, in general, is the matter of the form of representation. In the previous three titles of representation we have seen the transcendental schemata as imaginative time-determinations conditioning conceptual representation of objects (Quantity and Quality) and the form of the manifold of concepts (Relation). These time-determinations go to, in Kant's words, **time-series** (Quantity), **time-content** (Quality), and **time-order** (Relation) in apprehension. This might seem to cover all that could be said of the relationships between the pure form of inner sense, time-determination, and the pure schemes of determinant judgments (categories). What, then, is left for Modality? Kant's answer is: **the determination of *Zeitbegriff* or 'time-quintessence'** [KANT1a: 276 (B: 185)].

This is a grandiloquent phrase, but what does it mean? To find an answer to this question, we need to remind ourselves of what the transcendental schemata bring to the organization of cognitive representation. A transcendental schema is the product of the synthesis of imagination and its role is to bring homogeneity to the otherwise non-homogeneous representations of intuitions and concepts. The word 'quintessence' means "the most perfect manifestation or embodiment of something," and for transcendental schematism this can only mean that the synthesis of imagination, which produces transcendental schemata, succeeds in achieving perfect coherence between the *nexus* of conceptual representation and the conditions of sensibility in general. However, imagination cannot be the judge of this perfect accord since it belongs to the faculty of sensibility, whereas determinant judgments belong to the conceptual faculty. The synthesis of intuitions and the making of determinant judgments are two logically distinct powers of cognition, and so if coherence is to be possible between the outcomes of these two processes the schemata of time-quintessence must provide the ground for this possibility.

The 'logical essence' of the pure intuition of time is that time is the pure *a priori* form of inner sense. But a *time-determination* of 'quintessence' must be a schema that presents in some fashion a condition under which the structure of concepts can be put together such that this structure conforms to inner sense. Hence, Kant described the schema of Modality as "time itself, as the *correlatum* of the determination of an object, whether and how it belongs to time" [KANT1a: 276 (B: 184)]. Unfortunately, he went little beyond this in elaborating on this point:

The schema of possibility is the harmonization of the synthesis of various representations with the conditions of time in general (e.g. that contraries in a thing cannot be together but can only be after one another), thus determination of the representation of a thing in any time.

The schema of actuality is *Dasein* at a definite time.

The schema of necessity is the *Dasein* of an object for all time. [KANT1a: 275 (B: 184)].

This too-brief description is not sufficient for our purposes, and so we must try to reconstruct the arguments Kant neglected to provide.

As the product of a pure *process* of intuition, time is a condition of all sensible appearances. It is utterly impossible for us to recognize an object without ‘placing’ this object ‘in’ time. A schema of time-quantessence therefore pertains in all cases to conditioning of the *Dasein* of phenomena. Recalling that for our general 2LAR of representation we have three ideas – the determinable, the determination, and the determining factor – let us examine the schemata of Modality in terms of these ideas.

The schema of the determinable will be the schema that addresses the condition of sensibility under which there can be *any* representation of a phenomenon. Here we can make use of a description Kant provided in his *Inaugural Dissertation*:

Time is a continuous *quantum*¹ and the principle of the laws of continuity in the changes in the universe. For a *quantum* is continuous if it is not composed of simples . . . and the simples which exist in time, namely *moments*, are not parts of time but limits with time in between them . . .

The metaphysical law of continuity is this: *all changes are continuous* or flow; i.e., opposite states do not succeed each other except through an intermediate series of different states. For since two opposed states are at different moments of time, and between two moments of time there is always some intervening time, and in the infinite² series of its moments the substance is not in either of the given states, nor yet in no state, it will be in different states, and so on *in infinitum* [KANT7: 134-135 (2: 399-400)].

Now, in the synthesis of the imagination there is nothing *per se* that prevents imagination from attempting to produce an intuition from concepts that are in contradictory opposition. However, such an attempt will produce a cancellation. Kant sometimes illustrated this idea by saying that if we put together two equal but oppositely-directed forces, the net effect is to have no net force at all. If a “triangle” is defined to be a closed figure composed of three straight lines, it is impossible to imagine a “four-sided triangle” since the concept of “having four sides” contradicts the concept of the definition of a triangle.

¹ A continuous *quantum* is an intensive magnitude.

² Kant always uses the term "infinite" to refer to something which is without established limits or boundaries. His use of the term should not be confused with the modern-day idea of "mathematical infinity" and, especially, "the infinite" should not be thought of as any sort of number. Before the work of Cantor, *et. al.*, the philosopher's "infinity" was, as Poincaré put it, more or less synonymous with "becoming."

Thus, the determinability of intuition is **the schema of non-contradiction: contradictory characteristics cannot exist in the same object at the same moment of time**, although an object can exhibit in its accidents opposite characteristics at different moments in time and two different objects coexisting in the same moment in time can have opposing characteristics. The category corresponding to this condition of time is called the notion of *possibility and impossibility*. If the synthesis of imagination, in combining different concepts, can produce no sensible intuition from these concepts (owing to the cancellation effect of contradictory rules), no such determinable object exists (i.e., there is no determinable *Existenz*), and the object must be judged ‘impossible.’ Otherwise, the determinable is ‘possible’ in representation.

Our second schema of Modality pertains to the idea of the determination. Here our context is that of the determination of a phenomenal object in a determinant judgment. It is one thing for the concept of an object to be imaginable; it is quite another for the appearance of that object to be given in the apprehension of the data of the senses. The appearance of an object that arises originally from the power of spontaneity, without having *at the same moment* any contribution by the power of receptivity, we call an ‘imaginary object.’ On the other hand, receptivity without spontaneity is merely the capacity for *nous* to be affected by transcendental objects, and an appearance grasped solely through the synthesis of apprehension without any contribution from concepts is merely an undetermined appearance and not the appearance of a phenomenon.

It follows that the determination of a phenomenal object requires in the synthesis of sensibility contributions from both the synthesis of apprehension and the synthesis of reproduction in imagination. **This condition laid upon sensibility we call the schema of actuality**. The corresponding category is a notion that the phenomenal object is *actual* (physically real); the absence of this condition corresponds to opposition of this notion, which Kant called *Nichtsein* (“non-being”). For an actual phenomenal object, the conditions of sensibility require that there must have been *some moment in time* in which sensation in receptivity and sensibility through spontaneity combined to produce the intuition of the object, and so the schema of actuality is the schema that represents this connection of receptivity and spontaneity at some definite moment in time. If even one such moment is presented ‘anywhen’ in time, the phenomenal object is actual (is thought to “have” *Dasein*). Only the accidents of the object can be said “to be or to have been” – i.e., can be conceptualized as having *Existenz* in combination with the object we call objective time – but the “substantial object’s *Dasein*” is without reference to past, present, or future in objective time. (Recall that substance is a notion corresponding to the *modus* of persistence in *subjective* time).

The first two schemata of Modality are more or less straightforward once we have become accustomed to adopting the Copernican perspective in our analysis. The third schema of time-

quintessence is perhaps less obvious since the principle that underlies it tends to run contrary to the sort of habits of thinking that the practice of mathematical methodology promotes. The first schema of Modality addresses that which can be determinable at a moment in subjective time; the second addresses that which is sensuously determined at such a moment. In the first schema we need only the power of imagination; in the second, we need a concordance or “harmony” between spontaneity and receptivity. In both cases, though, we are dealing with conditions at a moment in time rather than with the condition of subjective time in general.

For our third schema, we must consider subjective time in its entirety as a “continuous *quantum*” or intensive magnitude. Every moment in subjective time is merely a signification marked by reflective judgment, and every such moment must be viewed as arising from the synthesis of apprehension proceeding from those moments that prior in time-order. It is only by virtue of these ‘antecedent moments’ that concepts are constructed since the synthesis of recognition in a concept presupposes an intuition at a definite moment in time as the matter of this synthesis. Put another way, spontaneity cannot take place without concepts as the *materia ex qua* on which it operates in the synthesis of reproduction in imagination. Thus, every *cognition* that carries the Modality of the category of possibility-impossibility must ultimately be grounded in or derived from concepts that carry the Modality of the category of actuality (*Dasein - Nichtsein*).

Now, we have seen that it is impossible for two contradictory concepts to be combined in a single intuition of one object. However, this does not prevent us from presenting two different intuitions, each of which contains one of these two contradictory concepts. If the two schemata we have already discussed exhaustively covered the schemata of Modality, then it would have to follow that nothing is empirically impossible if only one adopts the proper scheme of conceptualization in a cognition. Such a conclusion is obvious nonsense.

But into these considerations we must now incorporate the idea of the determining factor in Modality. Cognition does not involve some mere haphazard or random collection of whatever concepts might lie at hand ready for incorporation into the synthesis of reproductive imagination. Rather, our principles demand unity of consciousness and, therefore, the consideration of the whole of the manifold of concepts in the synthesis of imagination in the making of an intuition through spontaneity. It is not simply a matter of conceptualizing conveniently selected concepts from some subset of “privileged” moments in time; instead, we must consider the possibility of *comprehension* in view of the sum total of *all* conceptualized moments.

Here we find two cases that must be considered in examining the possibility of making the two separate intuitions we spoke of above. Either: 1) both intuitions – each involving one of the two contradictory concepts – are comprehensible when placed in the context of the rest of the manifold of concepts (i.e., neither contradicts the rest of the manifold of concepts); or, 2) one of the two intuitions is *globally impossible* (i.e., does stand in contradiction with the manifold as a

whole). In the first case, the phenomena are called *contingent*; each rests on the ground of a concept that transforms the two aforementioned contradictory concepts into merely *contrary* concepts. In the second case, the other cognition, the one that *is possible* when comprehended under the limits imposed by the entirety of the manifold of concepts, is called *necessary*.

This, then, is the role of the third schema of Modality: that every cognition must cohere with the manifold of time schematized from *actual* experience in the synthesis of imagination. In considering this schema of time-quintessence a *possible* cognition is judged (in determining judgment) under a notion of *necessity* if and only if the opposite cognition is impossible in the *nexus* in time; otherwise, this possible cognition is judged as *contingent*. The notion of *necessity-contingency* is the third category of Modality.

The *idea* of necessity (as opposed to the *notion* of necessity) is often regarded as being somehow tied to an attribute of certainty; deductions in formal logic are usually regarded in this manner. Necessity defined in this manner should be called “absolute necessity” since it is supposed to be a necessity that somehow transcends mere human thought and attaches to “nature itself” as some sort of “natural law.” This view, however, has no objective validity under the Copernican hypothesis; rather, the notion of necessity-contingency is thoroughly bound up with the conditions of sensibility insofar as the sum-total of everything actual in every moment of time is concerned. The idea of necessity viewed in this manner is of a *hypothetical* sort. Transcendental necessity is not something that can be directly given to us in *an* experience but, instead, is a Modality of representation (arising from the third transcendental schema of Modality) in which ‘an actuality’ can be anticipated *a priori* (“before the fact”), and such a cognition *of the possible as actual* (the notion of which is seen as the synthesis of the categories of possibility and actuality) must always have a *ground in actual experience*.

The congruence of an object with the conditions of thinking is the possibility of it; actuality is absolute positing, i.e., it establishes the object with regard to itself, and not in regard to thinking.³ Actuality, insofar as it can be known *a priori*, is necessity. Now this necessity can be *hypothetical*, when the *Dasein* of a thing is known *a priori* in some respect, or absolute, when the *Dasein* of a thing is known *a priori* simply speaking. To know something *a priori* in some respect is: when I know something from concepts without experience but know the ground from experience. I can never know the *Dasein* of things fully *a priori*, from mere notions, for it cannot be derived from mere notions, but rather from the very beginning through experience. A ground must be given that still can be known through experience . . . Therefore I can never conclude to actuality from possibility, but perhaps to possibility from actuality . . . Perception is the representation of the actual. Thus cognizance of the *Dasein* of a thing is never possible without experience; either I know things wholly from experience, or I know the grounds of experience. It is thus wholly impossible to know absolute necessity . . . The cognition of necessity is therefore a hypothetical cognition. All things have derivative necessity [KANT19: 322 (28: 556-557)].

³ In other words, the cognition of the object is not a product of spontaneity alone but must also involve receptivity in its synthesis. The *Dasein* of an object must be connected with some actual in experience.

The idea of the determining factor in the schemata of Modality is, in short, an idea arising from the “quintessential property” of the synthesis in the pure intuition of time as a “continuous quantum” or “flow” in which the moments in time are merely marks (“points” or “limits of time intervals”) and must be regarded as “state-determined” in the sense that a moment “grows out of” the previous moment. We cannot throw out prior experience of the actual in judging a possible cognition as knowledge “which follows” from other knowledge. Thus, the sum-total of the actual in the synthesis of experience in time is the *condition* of the notion of necessity-contingency.

This view of necessity does, as I said above, seem to run counter to how we are accustomed to thinking of “necessity” in mathematics and in formal logic. Indeed, I would not be very surprised if some mathematicians, logicians, or other scientists found the idea of “hypothetical necessity” somewhat uncomfortable at first encounter. But calling something “necessary” is not the same as calling it “apodictic”. Furthermore, Kant’s theory of the schema and category concerning “necessity” finds support from empirical psychology. Piaget found:

The principal results of the present research can be summarized in the following three points: (1) Necessity pertains to the compositions carried out by the subject and is not an observable datum inherent in objects; (2) it is not an isolated and definitive state, but the result of a process (necessitation); and (3) it is directly related to the constituting of possibilities that generate differentiations, whereas necessity is related to integration - hence, the two formations are in equilibrium.

In considering whether necessity has an exogenous or endogenous origin, one might think of the necessity of a slope causing a marble to roll (chapter 3) as a good example of that *real necessity*, the one situated in things, that Aristotle believed existed. If we stick to observable facts, we see only that a marble placed on a slope will “always” roll down and “never” up. But this is merely extensional generality, and therefore a law that becomes necessary only when there is a deductive model furnishing explanations. With the advent of Newtonian gravity, necessity was based on a model that proposed an explanation: “universal attraction.” But this is still only disguised description, and it is only with Einstein and Misner and with Wheeler’s geometrodynamics that explanations are proposed that are more basic because they are related to the geometric operations of the subject (including possible future elaborations). True, one might say that any general fact - such as the descent of marbles on a slope - appears intuitively necessary because the subject knows that there is an explanation even if she does not know what it is. However, this argument leaves the door open for pseudonecessities as well as for valid justifications.

In short, necessity does not emanate from objective facts, which are by their nature merely real and of variable generality and therefore subject to necessary laws to a greater or lesser extent. They only become necessary when integrated within deductive models constructed by the subject. The necessity of p can thus not be characterized only as the impossibility of not- p , since new possibilities can always emerge, but must be described in Leibniz’s manner as the contradiction of not- p , and this relative to a specific, limited model.

But why do there have to be necessities? It is because without them thinking would constantly contradict itself, if it retained all prior assertions, or would get lost in Heraclitean flux, if it forgot or neglected them. And since thinking is always in development it cannot do otherwise, if it is to avoid these two problems, than to integrate the past within the current state. Such integration, once

complete, is the source of necessity.

But this only moves the problem one step back: where, in turn, does this need for integration come from? Two objects or events may be similar to each other, and this relation of similarity, once established, is one condition for integration. On the other hand, they may be dissimilar. Unlike similarities, however, which tend to be absolute (as in identity), dissimilarities are never complete: no matter how different two real or conceptual entities are, they still have certain analogies as empirical or cognitive objects. Inasmuch as similarities lead to assimilation and dissimilarities to accommodations, the latter relations are subordinated to the former as accommodations are to assimilations. The fact that there has to be mutual assimilation of schemes at all possible levels of their interaction then imposes a permanent need for integration, from which necessitations proceed.

Stated more simply, the assimilatory schemes cannot function in isolation. Their constant need to find new inputs must lead to coordinations, which we characterize in terms of their mutual assimilation. These compositions, and not the initial individual constitutions, ensure the integrative process.

We thus define as necessary those processes the composition *C* of which cannot be negated without leading to a contradiction.

What is to be learned from these situations is rather obvious: there exists no more an absolute beginning in the development of possibilities than one can determine an absolute end to necessity. Any necessity remains conditional and will need to be transcended. Thus, there do not exist any apodictic judgments that are intrinsically necessary [PIAG14: 135-143].

Thus, what Piaget and his collaborators have learned in their studies of cognitive development in children agrees at the most fundamental level with Kant's conclusions regarding the notion of necessity and contingency and its schema, i.e. **the third schema of Modality is possibility coherent in the sum-total of the actual in time**. This is what is called the schema of necessity.

§ 4. The Categories of Understanding

Looked at from the empirical perspective, the pure notions of understanding are the categories (and logically, they are fundamental rules of 'predications') by which mental representations are able to be given *objective* meaning and by which such representations can become objective *knowledge*. Both 'meaning' and 'knowledge' are empty terms if they are considered without regard to the thinking Subject who is said to understand the meaning of a representation and to have knowledge of the object deemed to be the object of the representation.

Epistemology is logically prior to ontology in the Critical Philosophy. Since the categories of understanding *comprise* the primitive core ontology of determinant judgments and *construct* the ontology of Nature, any elucidation of the categories must begin with and at all points bear in mind this logical priority. As the ontological primitives of determinant judgments, the categories cannot be defined in terms of more primitive concepts. At the same time, however, it is evident that we dare not simply assume these categories are so obvious that no explanation of them is needed. (History has amply demonstrated this). How then shall we proceed?

To begin to appreciate the doctrine of method by which we proceed in order to understand the categories, it is essential for us to understand what is entailed in making a *Realdefinition*. Kant explained this term in a footnote in the first edition of *Critique of Pure Reason* as follows:

I understand here the real definition [*Realdefinition*], which does not merely attach other and more intelligible words for the name of an object-matter¹, but rather contains in itself a clear *mark* by means of which the *object* (*definitum*²) can always be securely recognized and makes the concept to be explained usable in application. A real explanation [*Realerklärung*³] would therefore be that which does not make merely a concept but at the same time its *objective reality* distinct. Mathematical definitions which present the object in accordance with the concept in intuition are of the latter sort [KANT1a: 342 (A: 241-242)].

We find here two special kinds of definitions distinguished by the terms *Realdefinition* and *Realerklärung*. It is the latter which today we are accustomed to using in the mathematical sciences, and when we employ the term ‘primitive’ in such a science we are referring to some idea for which no more fundamental definition is to be given. It is only because such primitives are deemed sufficiently clear to be *usable in application* that science can make use of ‘primitive terms’ in its theories. Ordinarily modern science relies on this clarity by default and pursues no inquiry into the philosophical *Realdefinition* of its primitives.

The categories admit to no *Realerklärung* because they *are* the primitives that are the basis of the *Realerklärung* of all other ideas and theoretical constructs. Hence, we must seek the *Realdefinition* for each category of understanding, and **these definitions can only be essentially practical** and not *theoretical*. We must understand how the categories are “usable in application,” and since their only use is in the determinant judgments of objects, what we must look into is *what they do*. To ask the question *what is a category?* is to ask *what does it mean to know an object?*

There is something strange and even nonsensical that there should be a notion that must have some meaning but is not liable to explanation. Only in the case of the categories is there this special circumstance, that they can have a determinate meaning and reference to any object only by means of the general *sensible condition* but that this condition is omitted from the pure category, since this can contain nothing but the logical function for bringing the manifold under a concept . . . Hence the categories need, beyond the pure notion of understanding, determinations of their application to sensibility in general (schema), and without these are not notions through which an object can be recognized and distinguished from others, but only so many ways to think of an object in possible intuitions and to give it its meaning in accordance with some function of understanding (under the

¹ *Sache*. This word denotes a "thing" or an "object" in the sense of "the matter of what something is about." Its connotation is not exactly "object" (*Gegenstand*) nor "thing" (*Ding*) but rather a "thing-like object." We will later make use of this term in the empirical perspective of the categories of Relation.

² having limits, finite, limited; clearly defined, precise.

³ In the doctrine of method given in his *Logik*, Kant discussed a variety of different distinctions and divisions that are applied to the idea of a "definition." The *Realdefinition* discussed here falls under the general division of what Kant called an *analytic* definition of a "concept given *a priori*." *Realerklärung*, on the other hand, falls under the general division called a *synthetic* definition of a "made concept" [KANT8: 141-145 (9: 140-144)], [KANT8a: 490-493 (24: 757-760)].

requisite conditions), i.e., *of defining it*: they themselves therefore cannot be defined⁴ . . . The pure categories are nothing other than the representation of things in general insofar as the manifold of their intuition must be thought through one or another of these logical functions . . . thus without the condition of sensible intuition, the synthesis of which they contain, the categories have no reference at all to any determinate Object, thus they cannot define one, and consequently have in themselves no objectively valid concepts.

Now from this it follows irrefutably that the pure notions of understanding can *never* be of *transcendental*, but always only of *empirical* use, and that the first principles of pure understanding can be related to objects of the senses only in reference to the general conditions of a possible experience, but never to things in general (without taking regard of the way in which we might intuit them).

The Transcendental Analytic accordingly has this important result: That understanding can never accomplish *a priori* anything more than to anticipate the form of a possible experience in general, and, since that which is not appearance cannot be an object of experience, it can never overstep the boundaries of sensibility, within which alone objects are given to us. Its first principles are merely principles of the exposition of appearances, and the proud name of ontology, which presumes to offer synthetic *a priori* knowledge of things in general in a systematic doctrine, must give way to the modest one of a mere analytic of pure understanding [KANT1a: 344-345 (A: 244-247)].

The categories themselves are not concepts of objects but they nevertheless ground the *making* of cognitions of objects as phenomena. Thus there enters in to our exposition of the categories the considerations of metaphysics proper, i.e. the branch of metaphysics applied to objects. Metaphysics *as a whole* is the system of pure rational knowledge [KANT19: 427 (29: 956)]. Ontology (the “system of metaphysical knowledge” as “a mere analytic of pure understanding”) must be considered concurrently with the application of Critical metaphysics proper to objects because *the categories establish ontology*. This is Kant’s ‘Copernican’ turn.

That we must adopt such a methodology in our examination seems quite clear from Kant’s own description of the categories quoted above. This does, however, present us with the need for organizing this methodology along the lines of some plan. Otherwise, if we simply juxtapose the elements of metaphysics proper with the categories, what we will have is not a organized doctrine of method but, rather, a hodgepodge approach. As Kant put it,

. . . pure speculative reason is, in respect of principles of knowledge [*Erkenntnisprinzipien*], an entirely separate and self-subsisting unity in which, as in an organized body, every part exists for the sake of all the others as all the others exist for its sake, and no principle can be taken with certainty in *one* regard unless it has at the same time been investigated in its *thoroughgoing* reference to the entire use of pure reason [KANT1a: 113-114 (B: xxiii)].

Palmquist calls this idea of the reference (*Beziehung*) of a principle to one part of the body of Kant’s theory a *reflective perspective*. We have given names to four such reflective perspectives – the logical, the transcendental, the hypothetical, and the empirical – along with one other ‘top level’ perspective (namely the Copernican Perspective, which Palmquist also calls the

⁴ That is, the categories admit of no *Realerklärung* of their objects divorced from the conditions of sensibility. We cannot, for instance, follow Leibniz and say that “substance” *is* a monad. The only definition possible for a category is an *operational* definition, i.e. *Realdefinition* through analysis of the role or function the category fills in the theory of knowledge.

Transcendental Perspective, and which he capitalizes to distinguish it from the transcendental reflective perspective). He writes:

. . . the full significance of the "perspectival" approach [to Kant's theory] is rarely appreciated. To counteract this neglect, I will argue that the general transcendental assumption which guides the Critical method implies most fundamentally a thoroughgoing "perspectival revolution" in philosophy . . . For the Transcendental Perspective in general includes within it several levels of subordinate perspectives, which are equally important in guiding the development of the various systems and subsystems which compose Kant's System. Thus, what I shall call the "*principle of perspective*" (i.e., the general rule that the truth is always relative to some perspective) can be seen functioning throughout the System [PALM1: 28].

We can make use of this principle of perspective in aligning our four reflective perspectives with metaphysics proper. Doing so gives us:

logical reflective perspective	⇔	Rational Physics
transcendental reflective perspective	⇔	Rational Psychology
hypothetical reflective perspective	⇔	Rational Cosmology
empirical reflective perspective	⇔	Rational Theology.

Rational Physics and Rational Psychology are the two branches of metaphysics proper concerned with sensible objects. They speak to the *composition* of Nature and hence to the representation of the *Existenz* in appearance of the sensible objects in Nature. Rational Cosmology and Rational Theology, on the other hand, take Nature as a whole – that is, the entire manifold of conscious representation – for their object and therefore pertain to the *Existenz* of this *manifold* in consciousness. Rational Cosmology speaks to the form of this manifold – the interrelations that connect concepts in the unity of apperception. Rational Theology deals with the matter of this manifold – namely, with the meaning in *Reality* of this representation.⁵

Since no one branch of metaphysics proper can be said to be generally superior to the others, our examination of the categories of understanding must give equal weight to each of them. As we shall see, each reflective perspective provides us with a particular viewpoint of these ontological primitives. The only use we can make of the categories of understanding is to regard them as the rules under which the process of determining judgment makes concepts of objects

⁵ It may seem strange indeed that the branch of metaphysics proper which bears the word "theology" in its title should be paired up with the *empirical* perspective. However, we should not let a word - "theology" in this case - get too much in the way. Let us bear in mind two points. First, Rational Theology in Kant's terminology is not *religious* theology - a point Kant makes rather clear in the Transcendental Dialectic section of *Critique of Pure Reason*. Second, considerations of what it means for something to be "real" rather naturally match up with the empirical rather than the hypothetical perspective - a convention which we might with some justification lay at the feet of the Neo-Platonist Aristotle, "who" viewed *Theos* (God) as the "first cause" and "prime mover" and therefore "the sum of reality, the Entity whose possibilities are all real" [MARI: 73]. Kant does use Rational Theology as the jumping off point for the development of a theology of religion, e.g. [KANT22], but this is distinct from Rational Theology proper.

and endows these concepts with *practical* real significance in empirical consciousness. Thus the *Realdefinition* of a category is given in no other way than in the exposition of the form of objective understanding, and this is nothing else than the explanation of the organizing of the *faculty* of concepts as seen *in toto* from each of the four reflective perspectives.

We have previously given English names to the categories when we discussed the transcendental schemata in the previous section. In preparation for our perspectival discussions of the categories, it is worthwhile for us to summarize these names and set them beside the German names given to them by Kant in *Critique of Pure Reason*. By doing so we can enter into these discussions with something of the flavor of each category in mind, and this will help us to establish the significance of each category from each reflective perspective. The categories are:

Quantity

Unity = *Einheit* ('one-ness')
Plurality = *Vielheit* ('many-ness')
Totality = *Allheit* ('all-ness')

Quality

Reality = *Realität* (this denotes 'reality' rather than 'Reality' in our terminology)
Negation = *Negation*
Limitation = *Limitation*

Relation

Substance and Accident = *Inhärenz und Subsistenz* ('inherence and subsistence')
Causality and Dependency = *Causalität und Dependenz* ('causality & dependence')
Community = *Gemeinschaft* ('mutual participation')

Modality

Possibility and Impossibility = *Möglichkeit und Unmöglichkeit*
Actuality⁶ and Nonexistence = *Dasein und Nichtsein* ('existence and non-being')
Necessity and Contingency = *Notwendigkeit und Zufälligkeit*

Where no parenthetical translation of the German term is given, the English name is the literal translation.

In making our exposition of the categories, we will follow the maxim of maintaining the tie with those phenomena that are "clearer to us" since these experiences are the empirical starting points of our science – which, after all, seeks to comprehend these experiences in a general

⁶ *Dasein* is literally 'being-there' or 'there be . . . ' and is 'existence' in this sense. It denotes 'real existence', but this connotation is what the English word "actuality" *implicates*. We might paraphrase this Modality function as, "There actually *is* & There actually *is-not*."

framework. The principles of metaphysics proper serve in this regard to exhibit the theory of the categories with respect to: 1) external experience; 2) internal experience; 3) Nature; and, 4) Reality. We deal with (1) and (2) in this chapter; (3) and (4) are the topics of Chapter 9.

§ 5. The Categories from the Logical Perspective

We have already seen that the word ‘logic’ is employed in a variety of different ways, including Aristotle’s “logic”, traditional Scholastic logic, mathematical (symbolic) logic, Piaget’s “logic of meanings” and so on. In what sense, then, do we take ‘logic’ when we speak of viewing the categories from the logical perspective? Considering what we have said above concerning the categories, and bearing in mind that we seek a *Realdefinition* for them, the answer to this question can be found in Kant’s introduction to his transcendental Logic:

And here I make a remark, the influence of which extends to all of the following considerations and that one must keep well in view, namely: that not all knowledge *a priori* must be called transcendental, but only that by means of which we know that and how certain representations (intuitions or concepts) are applied entirely *a priori* or are possible (i.e., the possibility of cognition or its use *a priori*) . . .

Accordingly, in the expectation that perhaps it can give notions that might refer *a priori* to objects - not as pure or sensible intuitions but rather merely as acts of pure thinking, that are therefore notions but of neither empirical nor aesthetic origins - we then make to ourselves beforehand the Idea of a science of pure understanding and ideas of reason, by which we think objects fully *a priori*. Such a science, which determines the origin, scope, and objective validity of such knowledge, would have to be called *transcendental logic* because it merely has to do with the laws of understanding and reason, but exclusively so far as it is relative to objects *a priori* and not, like general logic, to the empirical as well as pure ideas of reason without distinction [KANT1a: 196-197 (B: 80-82)].

Now, the knowledge *a priori* with which we are presently concerned is the ‘know-how’ we say is ‘provided by’ the pure notions of understanding, by means of which we do not merely make mental representations but, further, *think* these as representations *of objects*. Put another way, the *consciousness* of these representations takes the form of what we call our consciousness of objects. The mental phenomenon is not “I think this representation,” but “I think this *object*.” Our logical perspective is the perspective from which we *analyze* “the origin, scope, and objective validity” of the pure notions of understanding insofar as these categories make possible *objective* thinking – i.e., cognition through (empirical) concepts.

An empirical concept is a rule for the reproduction of representations in intuition. However, it is clear that something beyond what is contained “merely” in intuition must “go into” the making of a concept, for empirical intuitions all by themselves are merely representations of appearances in which the object of appearance is still undetermined as a phenomenon. Furthermore, concepts are representations of a more general nature than a sensibly-given intuition

for it is by means of concepts that divers intuitions are combined in thinking an object of phenomenon. There is, however, nothing contained in our idea of a singular intuition that necessitates or even validates the combining of multiple intuitions into one representation. Thus, if the combining of intuitions is to produce a representation having objective validity, something must be added to representation in the making of a concept that provides this objective validity.

There are only two possible cases in which synthetic representation and its objects can come together, necessarily relate to each other, and, as it were, meet each other: Either when the object alone makes the representation possible, or the representation alone makes the object possible. If it is the first, then this reference is only empirical, and the representation is never possible *a priori*. And this is the case with appearance in respect to that in it which belongs to sensation. But if it is the second, then, since representation regarded as it is in itself . . . does not produce its object as far as its *Dasein* is concerned, the representation is thus still determining in consideration of the object *a priori* if it is possible through it alone to *know something as an object*. But there are two conditions under which alone the cognition of an object is possible: first, *intuition*, through which it is given, but only as appearance; second, *concept*, through which an object is thought that corresponds to this intuition . . . The question now is whether *a priori* notions do not also precede as conditions under which alone something can be, if not intuited, nevertheless thought as an object *in general*, for then all empirical cognition of objects is necessarily in accord with such notions, since without their presupposition nothing is possible as an Object of experience. Now, however, all experience contains outside of the intuition of the senses, through which something is given, a *notion* of an object that is given in intuition, or appears; hence notions of objects in general lie at the ground of all experiential cognition as *a priori* conditions; consequently the objective validity of the categories as *a priori* notions is due to: that through them alone is experience possible (as regards the form of thinking). For they then refer in a necessary manner and *a priori* to objects of experience, since only by means of them generally can any object of experience be thought at all [KANT1a: 224 (B: 124-126)].

Let us take careful note of Kant's phrase "object *in general*" [*Gegenstand überhaupt*]. What he refers to in this phrase is not this object or that object but, rather, *the very notion of an object*. The ability to combine divers appearances under the title of some one *object* necessarily presupposes 'objects' as the logical foundation for this union. However, since this *a priori* 'know-how' cannot predict in advance of actual appearances (which provide the *materia ex qua* of an object) the specific form that the *Existenz* of a specific object of experience will take, this 'know-how' can only be regarded as the rule of a 'mark' or attribute that must be *placed*, by determining judgment, in the concept of any object. Put another way, the judgment must follow some *scheme for representing* a representation of an object. Thus from the logical perspective the categories are the *a priori* rules for the making of a determinant judgment – the *materia circa quam* – that define a representation to be the representation of an object. We do not say that a category has objective validity but rather we say that *categories define what it means to be an objectively valid form of conceptual object representation*.

A category is therefore something that, on the one hand, imputes objective significance to a conceptual representation and, on the other hand, serves as a rule for the making of empirical

concepts. In both cases we are speaking of the category in terms of something a determinant judgment *accomplishes* in the act of making a conceptual representation. It is worthwhile to compare this with Piaget’s general description of a “scheme”:

A scheme is the structure or organization of actions as they are transferred or generalized by repetition in similar or analogous circumstances [PIAG15: 4, fn 2].

The action in this case is the making a determinant judgment and so, in Piagetian terminology, a category is viewed from the logical perspective as a scheme of determinant judgment since it provides the structure or organization of thinking as a mental action. This, of course, stretches the Piagetian terminology somewhat because Piaget and his co-workers are always quite careful to tie their interpretations to the observable activities of their subjects and the mental act of determinant judgment is not amenable to direct observation. Still, the idea of a scheme provides us with a useful picture of the logical role of the categories overall. What we must next undertake is to examine this idea in greater detail and in the framework of our idea of representation in general.

If a scheme is a “structure or organization of an action,” in what manner do we regard a category as a scheme from the logical perspective? Let us look first at the action for which the category is a scheme. The action is none other than the making of a determinant judgment and this, in its turn, involves the performance of an act of understanding. It is “cognition through concepts,” i.e. “thinking.” Now thinking, taken in our technical usage of that term, is that act of the power of spontaneity which makes representations objective. In the context of transcendental logic, the act draws its objective validity from the principle of the objective unity of the apperception of concepts.

But if I investigate more closely the reference⁷ of given knowledge in every judgment, and distinguish it, as belonging to understanding, from relationship⁸ according to laws of reproductive imagination (which has only subjective validity), I find that a judgment is nothing but the manner⁹ of bringing given knowledge to the *objective* unity of apperception. This is plain from our use of the copula *is* in the aforesaid, in order to distinguish the objective unity of given representations from the subjective. For this indicates the reference of these representations to original apperception and their *necessary unity*, even though the judgment is empirical, therefore contingent . . . I do not say by this that these representations *necessarily* belong *to each other* in empirical intuition, but rather they belong to each other by virtue *of the necessary unity* of apperception in the synthesis of intuition, i.e. according to principles of the objective determination of all our representations, so far as knowledge can arise from them, these principles being all derived from the first principle of the transcendental unity of apperception. In this way alone can there arise from this relationship *a judgment*, that is, a relationship that is *objectively valid* and is perfectly distinct from the relationship

⁷ *Beziehung*. Palmquist calls the *Beziehung* of principles a ‘perspective’ [PALM1: 27-47].

⁸ *Verhältnisse*.

⁹ *Art*.

of the very same representation which has only subjective validity according to the laws of association [KANT1: 104-105 (B: 141-142)].

Kant defines the word ‘function’ to mean the unity of the act of ordering different representations under a common one [KANT1a: 205 (B: 93)]. Viewed as a *scheme*, a category is, formally, precisely such a “unity of the act” and, therefore, is a **logical function of determinant judgment**. Breaking down this idea of a logical function one step farther, we may speak of such a logical function in terms of its form and its matter. Kant bluntly tells us, “The logical form of all judgments subsists in the objective unity of the apperception of the concepts contained therein” [KANT1a: 251 (B: 140)]. The logical function of the categories provides this form in virtue of their role as the *conditions* under which all sensible *intuitions* can, despite their “manifoldness”, be united *in* a common representation, i.e., “come together in one consciousness” [KANT1a: 252 (B: 143)].

As for the ‘matter’ of this logical function, we take note that the unity of apperception is, in this case, an *objective* unity. The representation brought forth by thinking is the representation of an *Object*. The matter of the category, as a scheme, lies in the *objective signification* of the representation it produces. To establish the ‘nature’ of this, we require some *principle of signification* by which we can establish the objective meaning contained in the scheme of the category. Thus, a category has not only logical significance but, in addition, metaphysical signification. In transcendental logic – that is, from the logical reflective perspective – we obtain this metaphysical *objective* signification from Rational Physics (which provides us with our metaphysics proper of objects of outer sense).

§ 5.1 The Categories of Quantity from the Logical Perspective

We begin with the form of the matter, or Quantity, of *conceptual* knowledge (the product of determining judgment) from the logical perspective. Here we have for our principle of signification the principle of Quantity in Rational Physics, namely the principle of the Axioms of Intuition. The objective signification of these primitives of judgments – i.e. the categories of Quantity – are drawn from how we must view the form of the matter of knowledge under this principle.

The Axioms of Intuition states: All appearances are (as regards their intuition) extensive magnitudes. This principle makes a statement about the ‘nature’ of appearances, and the principle itself is deduced from Kant’s analysis of the synthesis of apprehension which produces intuitions. It is therefore quite proper to re-state this principle, as Kant did in the second edition of *Critique of Pure Reason*, as: All intuitions are extensive magnitudes. However, we can and should examine the Kant’s deletion of the word “appearances” in this second statement.

The most obvious difference in how Kant stated the account of the Axioms of Intuition in the first and second editions is this: The first statement seems to predicate something about the appearance, while the second seems to predicate this same thing of intuitions only. Paton, among others, is quick to notice this distinction.

It is difficult to say why in the second edition Kant should have altered this formula, especially as the conclusion of the argument in the second edition conforms more closely to the formula in A than to that in B. Mellin believes that in the formula of B Kant really intended to assert that all *appearances* (not all *intuitions*) are extensive quantities. This is supported by the conclusion of the argument in B, and Kant is not speaking of intuition *quâ* intuition, but of intuition *quâ* appearance of an object [PAT2: 111, fn. 1].

Paton's observation regarding "intuition *quâ* appearance" is really the central point here. The intuition is the outcome produced by the synthesis of apprehension. The appearance is the transcendental object of this representation and, at the level of sensibility alone, this object is determinable but not yet determined (appearance but not yet phenomenon). Since the intuition is built up in the form of an extensive magnitude, the appearance must likewise be formally regarded as an extensive magnitude. This is the account from a *theoretical Standpoint*.

When we pass to the determinant judgment that conceptualizes this intuition, the object of this conceptual re-presentation of the intuitive knowledge, as the conceptual knowledge of a phenomenon, must retain this character of extensive magnitude because the concept must formally be such that from it the intuition can be reproduced by imagination. Only then can we have recognition of the object as phenomenon. It follows from this that the Axioms of Intuition is likewise consequential for determining judgment (the account from a *judicial Standpoint*) and, therefore, for the primitives of Quantity in determinant judgments. The categories of Quantity must impute to conceptual representations the objective signification of this extensive magnitude.

Concepts

To carry our discussion forward, we must at this point pause to discuss in more detail our idea of a 'concept.' As a representation and viewed from the logical perspective, a concept is a 'general' representation. This is quite different from an intuition, which, as a representation, is always a singular representation. Kant explained this distinction in the following way.

Intuition is a singular representation (*repraesentatio singularis*), the concept is a general (*repraesentatio per notas communes*) or reflected representation (*repraesentatio discursiva*).

Note 1: Concept is contrary to intuition, for it is a general representation or a representation of what is common to various Objects, therefore a representation *so far as it can be contained in divers ones*.

Note 2: It is mere tautology to speak of general or common concepts - a mistake based on an incorrect division of concepts into *universal*, *particular*, and *singular*. Not concepts themselves, only their *use* can be divided this way [KANT8: 96 (9: 91)].

Elsewhere he tells us:

A *conceptus* is a *repraesentatio communis*, which is common to many things. He who wished to have a representation of the color red must first see the color red. When he compared red in the red of cinnabar, *carmoisin*, and *ponceau* thus would he know that there is something general in the color red, that is contained in other representations of the color red and thought by red, which was common to many objects, and that was a concept. A concept is then a representation that is common to many things. With *intuitus*, I consider individual things. E.g., the sun, the earth. If I think of a certain genus of planets, however, then this is a *repraesentatio communis*, i.e., a *conceptus*. - Concept differs from intuition by virtue that all intuition is singular. He who first sees a tree does not know what it is, what he sees. If he becomes aware that these objects have something common, then he omits everything they have that is different, and takes together what they have in common, and thus he has a *repraesentationem communem*, i.e., a *conceptus* [KANT8a: 348-349 (24: 904-905)].

We can easily put together from these descriptions the link connecting the logical origin of concepts with the *Verstandes Actus* of comparison and abstraction and with the process of the synthesis of re-cognition in a concept.

How do concepts arise? I.e., how do representations become concepts? Logic does not trouble itself with how *data* for a cognition must be provided, but rather merely with what understanding does with them [with the data], then gives heed to form and not to the object. How does it happen, then, that a *repraesentatio singularis* becomes *communes*? *Resp. per comparationem, separationem, seu abstractionem*.¹ I compare things and attend to that which they have in common, and I abstract from all other things . . .

. . . Abstraction thus brings forth nothing in addition to, but rather cuts off all that does not belong to, the concept and notes merely what it has in common with other representations. The differentiation of a concept thus requires *comparatio* [KANT8a: 350-351 (24: 907-908)].

The form of a concept subsists in common validity. *Repraesentatio, quae pluribus est communis*² . . . For if a representation is not a *repraesentatio communis*, then it is not a concept at all . . . We do not divide concepts into *universales, particulares, singulares*, then, but instead [we so divide] judgments, as we shall soon hear. In my judgment I can compare the thing with all, some, or an individual thing . . . Understanding has the capacity for concepts, and one can also define it thus. We said above that understanding is the faculty of rules. But this is the same thing, for when I give a concept, I always give a foundation for rules.

Concepts arise *per comparationem, reflexionem, et abstractionem*.³ I grasp in one consciousness many representations, in which I compare what is only a reiteration of the rest. From reflexion, then, one recognizes that which many things have in common; afterward one's abstraction takes away that wherein they do not come to terms, and then a common representation remains. No concept comes to be, then, without comparison, without perception of a mental preparation, and without

¹ "through comparison, separation, or abstraction".

² "a representation that is common to many".

³ "by means of comparison, reflexion, and abstraction," our three *Verstandes Actus*.

abstraction. [KANT8a: 352-353 (24: 908-909)].

From the explanations just quoted, we see Kant's theory holds that the *origin* of concepts does not lie with determining judgment. Rather, the 'defining moment' of a concept lies in an act of reflective judgment in which we achieve the perception of an agreement among the comparates involved in the *Verstandes-Actus* of Kant's three-fold synthesis. This is in accord with our earlier distinction between determining judgment (which proceeds from the general to the particular) and reflective judgment (which proceeds from particulars to the general). Determining judgment does not *create* concepts; it *uses* concepts by combining them to produce concepts *in a cognitive structure* – a process which needs the re-presentation of these combinations in a *spontaneous* intuition and therefore involves also the synthesis of productive imagination. We have earlier described a concept as a rule for the re-production of intuitions; from the logical perspective we can also look at concepts in another way, namely in terms of their logical employment in determinant judgments. Seen in this way, a concept is called a *mark*.

Concepts and Their Use in Determinant Judgments

Kant's view of concepts as *repraesentatio per notas communes* ("representation by means of a common mark") far pre-dates the publication of *Critique of Pure Reason* in 1781. In his 1762 essay on syllogisms we found his statement,

To compare [*vergleichen*] something as a mark with a thing is called *judging*. The thing itself is the subject, the mark is the predicate. The comparison [*Vergleichung*] is expressed through the sign of combination **is** or **are** [KANT21: 89 (2: 47)].

Kant had not yet, at the time of this essay, come to the Copernican view of judgments that he would pronounce in his later Critical works but, with the advantage of hindsight, we can see how this then-widely-accepted idea of a 'mark' could later evolve to become Kant's view of how concepts are logically employed in determinant judgments.

A mark is that in a thing which constitutes a part of the cognition of it, or - what is the same - a partial representation, so far as it is taken as ground for cognition of the whole representation. All our concepts are marks, accordingly, and all thinking is nothing other than a representing through marks.

Every mark may be considered from two sides:

First, as a representation regarded as it is in itself;

Second, as belonging, as a partial concept, to the whole representation of a thing, and thereby as ground of cognition of this thing itself [KANT8a: 564 (9: 58)].

Before passing on to look at the Copernican idea of judgments, it is worthwhile for us to

briefly examine the idea of “comparison” and Kant’s usages of this idea. In the 1762 essay and elsewhere throughout the Kantian corpus, we encounter the term *Vergleichung*, which is, literally, “comparison.” However, in discussing the *Verstandes-Actus* Kant uses the term *Comparison*, which is a Germanized form of the Latin term *comparationem*. Thus, we have here another instance of Kant making a fine technical distinction, in this case between comparison in general (*Vergleichung*) and comparison in the context of the *Verstandes-Actus*. In the text of the Jäsche *Logik* we find ‘*Comparison*’ introduced as

1) *Comparison*, i.e. the *Vergleichung* of representations among one another in relationship to the unity of the state of consciousness [AK9: 94].

Thus *Comparison* is comparison in the context of a relationship between representations in sensibility and the unity of consciousness. Even though this context is *subjective*, nonetheless it is here in this subtle limitation of the general idea of ‘comparison’ where we find the connection between the *Verstandes-Actus* and determinant judgments. Earlier we noted Kant’s comment that a judgment “is nothing other than the way to bring knowledge to the *objective* unity of apperception.” Now let us take a look at his logical explanation (*Erklärung*) of representative judgment (*Urtheil*).

A judgment is the representation of the unity of consciousness of diverse representations or the representation of their relationship insofar as they constitute a concept [KANT8a: 597 (9: 101)].

We have in this two distinct types of representative judgment. The first, representation of the unity of various representations in the state of consciousness, is reflective judgment. It ‘determines’ nothing but the state of the thinking Subject with regard to its representations. The second type, representation of the relationship among representations “insofar as they constitute a concept,” is determinant judgment. This second type of judgment is *objective* rather than subjective. But, from the logical perspective, what does it mean for a judgment to be objective under the Copernican hypothesis?

The manifold of representations can be given in an intuition that is merely sensuous, i.e. is nothing but receptiveness, and the form of this intuition can lie *a priori* in our capacity of representation without being anything other than the way in which the subject is affected. Yet the *combination* (*conjunctio*) of a manifold in general can never come to us through the senses, and therefore cannot already be contained in the pure form of sensuous intuition; for it is an act of spontaneity of the power of representation, and, since one must call this understanding, in distinction from sensibility, all combination is an act of understanding, whether we are conscious of it or not, whether it is a combination of the manifold of intuition or of several concepts (and in the first case be either of sensuous or non-sensuous intuition), which we would designate with the general title *synthesis* in order at the same time to draw attention to the fact that we can represent nothing as joined in an Object without having previously combined it ourselves, and that among all representations *combination* is the only one that is not given through Objects but can be executed only by the

subject itself, since it is an act of its self-activity [KANT1a: 245 (B: 129-130)].

Let us dissect this run-on sentence (so typical of Kant's writing) and extract the several points being made within it.

The first point is that representation as sensibility is entirely subjective – “the way in which the Subject is affected.” There is no judgment in an intuition. However, consciousness of the relationship between sensible representation and the *state* of consciousness, while subjective, still requires a synthesis, namely that of the reflective judgment. Recalling that reflective judgment is concerned with the *Zweckmäßigkeit* or ‘formal expedience’ of representations for a purpose of pure practical Reason, this judgment bears directly on what mental *acts* may follow in regard to the synthesis of the representation of an object. If the sensible representation is harmonious with – or, at least, not in conflict with – the purpose, then the sensible representation is expedient and no further act of thinking need be required. On the other hand, if the relationship between representation and the state of consciousness is one of purposive discord, pure practical Reason must ‘take an interest’ in resolving this discord. Its agent for acting in this matter is the power of understanding through determinant judgments.

Which brings us to the second point. The combination (*conjunctio*) of divers representations into *one* representation – that of the Object – is an act of spontaneity. It is, in other words, an act of *determination* and, logically, that which is so determined is what we call the object. This act of determination is precisely what we mean by the phrase *making a determinant judgment*. “We can represent nothing as joined in the Object without having previously combined it ourselves, and that among all representations *combination* is the only one that is not given through Objects but can be executed only by the Subject itself, since it is an act of its self-activity.” Without the schematism of the representation of combination, there is no thought of an object and *this* is what it means for a judgment to be objective.

The link between the *Verstandes-Actus* and determining judgment is now clear. It proceeds beginning with *Comparison* to a reflective judgment of expedience (hence to the connection with a purpose of pure practical reason), to intuition, to concept, and finally to the exercise of determining judgment in making combinations in the manifold of concepts. But so far as understanding is concerned, these combinations are Object structures, and these structures set the rules of cognition by which this representational ‘loop’ is closed by the re-presenting of the represented phenomenon in an intuition via the synthesis of imagination.

Concept is the indirect illustration or presentation of an object by virtue of a common mark of various objects as a ground of cognition. Thus it always has for a ground a representation which is common to several objects, and through recognition of this, as a mark, belongs merely to the thinking of an object . . . Thus the concept has reality only insofar as an object can be imputed to it . . . But concepts belong merely to the thinking intellect, and sensibility can be mixed in with them

only insofar as they rest on intuition; otherwise in and for themselves they can have no sensibility through which they are perceived [KANT19: 442 (29: 971)].

Combination, Aggregation and the Scope of a Concept

For a representation to be a mark it *must* originate in the three-fold synthesis of the *Verstandes-Actus*. As a representation of what is common in two or more individual representations, the concept can be viewed in relationship to its ‘parent’ representations as illustrated in Figure 8.5.1 below. In this figure, *M* denotes the mark while *A* and *B* denote the representations ‘in’ which *M* is common. *M* is said to be ‘contained in’ *A* and *B*, *A* and *B* to be ‘contained under’ *M*.

Now, the representation shown in Figure 8.5.1 goes beyond we are justified in depicting strictly from the *Verstandes-Actus*. In this figure, we show *M* as a ‘conjunction’ of *A* and *B* when we draw the two lines that link the parent representations to the concept *M*. Such a combination (a *conjunctio* in Kant’s terminology) is implicit in the description of the *Verstandes-Actus*, but the making of a representation *as* combination is not part of the description of the *Verstandes-Actus*. The representation of *M in intuition* always involves abstraction as the final step in the three-fold process; the representation of *M as a concept* belongs to the synthesis of re-cognition in a concept. Consequently, in the logical sequence of the conceptualization of *M*, by the time we have a representation of the *concept M*, neither *A* nor *B* is explicitly to be found in the outcome of this representative act. *To represent M in combination with A or B is an act of determining judgment.*

By virtue of this act of representing the combination of *M* with *A* we say “*M* understands *A*” and “*A* stands under *M*.” The illustration depicted in Figure 8.5.1 gives us a visualization of the idea of what ‘understanding’ means from the logical perspective. When combined in a determinant judgment, *M* is said to be the ‘higher’ concept in relationship to representations *A* and *B*; reciprocally, *A* and *B* are ‘lower’ concepts in relationship to *M*. These ideas of higher and lower concepts are, therefore, relative ideas and are meaningful only in the context of the form of the manifold of concepts in determinant judgments.

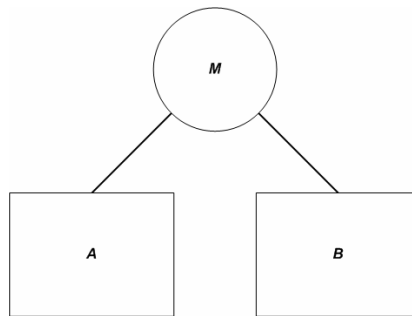


Figure 8.5.1: Illustration of a Common Mark

By the Axioms of Intuition, the appearance of A has extensive magnitude and, consequently, we must regard the concept M as a partial concept of the multiplicity ‘contained in’ the extensive magnitude of A . Thus in the terminology of the Critical Philosophy we say, “ M determines A .” However, this does not mean that M *completely* determines A since this would be the case only if M and A were identical representations (in which case we would have to say that A was a mark of B , contrary to Figure 8.5.1). Further *Comparison* of A with other representations will typically lead to additional marks of A – a situation we illustrate using Figure 8.5.2 below. Each mark in this figure gives a partial determination of A . The combination of all these marks to form the whole determination of A (insofar as these marks compose the whole conceptualization of A put together by determining judgment up to this point) is called an **aggregate** and the acts of determining judgment in forming this composition is called the **aggregation** in the determination of A . The marks in this aggregate are said to be **coordinate** because each is represented as an *immediate* mark in the determination of A . The marks are said to be ‘contained in’ A .

Now each empirical mark is itself a concept capable of being exhibited in an intuition. Consequently, in the three-fold synthesis of the *Verstandes-Actus* it is possible for the marks of different representations A and B to be made comparates of *Comparison* and, therefore, it is possible that reflexion may find in different marks a partial representation common to each. Such a partial representation would be *the mark of two marks* and would itself then be conceptualized in the synthesis of re-cognition and combined with its parent concepts by determining judgment. Such a situation is illustrated in Figure 8.5.3.

In this figure mark M_7 is a concept of which we say both marks M_3 and M_4 ‘stand under’ M_7 . Furthermore, we can follow the combinations of judgment leading from M_7 through M_3 and finally back to A . Thus we also say that A ‘stands under’ M_7 and that M_7 is a *remote* or **subordinate** mark of A . This combination through subordinate marks (for we may have marks of M_7 itself, which would be even ‘more remote’ subordinate marks of A) is called a **series**. We say

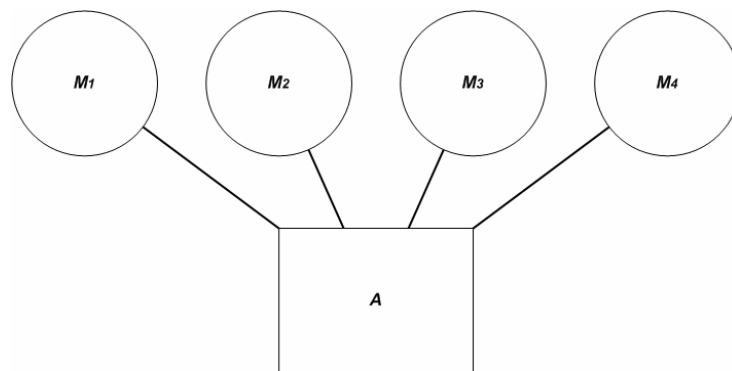


Figure 8.5.2: The Aggregate Determination of A. More is thought ‘under’ a mark than ‘under’ A ; more is thought ‘in’ A than ‘in’ one of its marks.

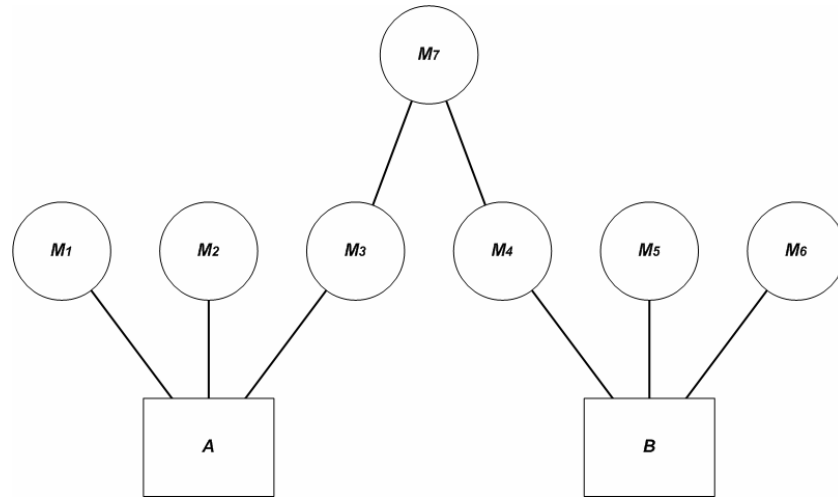


Figure 8.5.3: Illustration of Combination in a Series

the subordinate mark of *A* is a *condition* of *A* and, reciprocally, that *A* is *conditioned by* its subordinate (as well as by its coordinate) marks. The same holds between M_7 and *B*.

If we consider the series descending from M_7 down to *A*, this series of combinations is called an *episylogism*. The opposite direction, ascending from *A* up to M_7 , is called a *prosylogism*. Determining judgment proceeds from given higher marks of the concepts being judged and acts in the direction of an episylogism. (The making of a prosylogism also involves reflective judgment and speculative Reason, and the overall process is called *judgmentation*). In Figure 8.5.3 concepts M_3 , M_4 , representation *A*, and representation *B* all ‘stand under’ concept M_7 . We therefore say these representations are ‘contained under’ M_7 . The net aggregate of all *objects* of representations contained under a concept is called the *Umfang* or **scope** of that concept. The aggregate of all *representations* contained under a concept is called its **sphere**.

Every concept, as *partial concept*, is contained in the representation of things; as a *ground of cognition*, i.e., *as mark*, these things are contained *under* it. In the first respect, every concept has a *content*, in the other a *scope* . . . The more things that stand under it and can be thought through it, the greater is its scope or *sphere* [KANT8a: 593: (9: 95-96)].

This quote may seem to say that the terms ‘scope’ and ‘sphere’ were used synonymously by Kant. However, this is incorrect. What we find in examining the rest of Kant’s logic lectures is that ‘scope’ is used in reference to ‘things’ (*Dinge*) whereas ‘sphere’ is employed in discussing in the abstract the logical structure of the manifold of concepts. Scope, in other words, has a *transcendental* implication, whereas sphere has only a formal and logical connotation. This is evident from Kant’s description of sphere as “the magnitude (*Größe*) of the scope of a concept” [KANT8a: 593 (9: 96)] because ‘magnitude’ is an idea we understand only in terms of representations. Such is the distinction between these terms. With these explanations in mind, we will now turn to the logical *momenta* of Quantity in determinant judgments.

Momenta and Functions of Quantity

As we begin this subsection, it is important for us to deal straight-away with an idea – or, more properly, a set of closely-related ideas – that has historically been the breeding ground of not a small amount of puzzlement and controversy among Kant scholars. At the center of the issue is the question of what meaning or interpretation we are to give to the word ‘function’ and the question of how this term differs (if at all) from another Kantian technical term, namely the idea of ‘*momentum*.’

We have previously given what will remain the basic explanation of the term function: the unity of the act of ordering different representations under a common one. We get a glimpse of the problems that have beset this term if we go on to speak of such things as “the logical functions of judgment” – where now we would seem to be speaking as if there could be more than one kind of ‘unity of the act’ even within one act of judging. Kant himself often phrases things in this manner, which, of course, provides all the kindling one needs to ignite the controversies that have historically trailed in the wake of *Critique of Pure Reason*.

Paton, who is on the whole generally sympathetic to Kant, nonetheless clearly outlined the problem with Kant’s ‘function’ terminology in his discussion of Kant’s deduction of the categories in *Critique of Pure Reason*:

The obscurity of the first section is due partly to the fact that Kant gives us no help as to the stages into which his argument is divided, and partly due to his use of the word ‘function’ in different senses. With very little trouble Kant could have made this section much easier.

Thus he asserts (1) that functions of unity are to be found *in* judgments; (2) that judgments *are* functions of unity; and (3) that a function *is* the unity of an act which (I believe) is judgment. Any attempt to interpret these statements mechanically is bound to produce as pretty a mess as one could wish to see [PAT1: 245-246].

Paton goes on to point out the various manners in which Kant uses ‘function’ in his argument and offers some possible clarifications as to the different connotations he thinks Kant is employing in his use of the word *Function* [PAT1: 246-248].

The “function question” in *Critique of Pure Reason* is tamed somewhat by Kant’s introduction and use of the technical term *Momente* (‘moments’ or ‘*momenta*’). In philosophical contexts momentum is often used in the Hegelian sense of “any of the constituent elements of a complex entity.” The way Kant employs this term is illustrated in his *Prolegomenon*:

The logical momenta of all judgments are so many possible ways to join representations in one state of consciousness [KANT13: 48 (4: 305)].

In other words, there *is* more than one way to give a *form* to the ‘unity of the act’ in the act of

making a determinant judgment. It is in this sense that we use the phrase ‘functions of judgment’ in this treatise, i.e., as unities established by the form of momenta in Kant’s sense. The unity in an *act* of determining judgment subsists in the manner of the combination by which concepts are welded into a union of one determined representation. Nothing in this prevents us from having more than one way to do this welding or from having different ‘spot welds’ within it.

All combination (*conjunctio*) is either *composition* (*compositio*) or *connection* (*nexus*). The first is the synthesis of a manifold, the parts of which *do not* belong *necessarily to each other*, as, e.g., the two triangles into which a square is divided by the diagonal, for these do not necessarily belong to each other; and of such a sort is the synthesis of the *homogeneous* in everything that can be considered *mathematically* (which synthesis can be further divided into that of *aggregation* and of *coalition* - of which the first is directed to *extensive* and the second to *intensive magnitudes*). The second combination (*nexus*) is the synthesis of manifolds in so far as they *necessarily belong to one another*, as, e.g., an accident belongs to some substance, or the effect to the cause - thus also represented as *unhomogeneous* but yet as combined *a priori* - which combination, since it is not arbitrary, I call *dynamical*, since it concerns combination of the *Dasein* of manifolds (which can again be divided into the *physical* combination of the appearances with one another and the *metaphysical*, their combination *a priori* in the faculty of knowledge¹) [KANT1a: 285-286 (B: 201-202 fn)].

This brings us to what Kant called “the logical function of understanding in judgments”:

If we abstract from all contents of a judgment in general, and give attention only to the mere form of understanding in it, we find that the *function of thinking*² in this can be brought under four titles, each of which contains under itself three momenta. They can suitably be represented in the following table [KANT1a: 206 (B: 95)].

Kant then lists the four “titles” and their accompanying twelve logical *momenta* as follows:

<u>Quantity</u>	<u>Quality</u>	<u>Relation</u>	<u>Modality</u>
Universal	Affirmative	Categorical	Problematic
Particular	Negative	Hypothetical	Assertoric
Singular	Infinite	Disjunctive	Apodictic.

The adjectives used here were (and are) quite familiar to logicians, and not a few commentators have concluded from this table that Kant claimed “to have built his transcendental logic on a framework provided by formal logic” [KANT8a: xvi], [GUY: 101-110]. We are about to see that this interpretation is a misunderstanding of Kant’s theory and that it is in fact the opposite: formal logic must get *its* framework from the Critical ontology of judgment.

As a representation, every determinant judgment requires one category from *each* of the four ‘titles.’ We will discuss Quality, Relation, and Modality in the following sections. Here our

¹ *Erkenntnißvermögen*.

² These italics do not appear in *Critique of Pure Reason*. I have added them here to emphasize what “function” Kant is talking about.

concern is with the logical momenta of Quantity and their relationship to the categories of Quantity. To understand this relationship we must first understand the ideas of the universal, particular, and singular momenta. These ideas we might term functional ideas in the sense that they produce a single representation (a composition) from a multiplicity of given concepts.³

Kant explained the logical momenta of Quantity using the language of formal logic. However, in examining the logical momenta of Quantity using this language we must bear three things in mind: 1) the terms we are concerned with are concepts, not objects; 2) the logical momenta of Quantity must be viewed in the context of determining judgment, which means that in all cases we are examining the formal structure of combination *a parte posteriori* (i.e., in the direction of an episyllogism); 3) the characteristic feature in the explanation of the logical momenta of Quantity is centered on the idea of the *sphere* of a concept, not its *scope*. The third of these considerations has the consequence that the explanation of the logical momenta is restricted to ideas pertaining to concepts without consideration of the objects these concepts represent. The scope of the explanation is therefore restricted to the formal structure of the manifold of concepts; in contrast, the categories *do* pertain to objects and therefore are *real* momenta – i.e. they are notions for judging concepts *qua* objects.

In every determinant judgment there is always a given determinable concept and a given determining concept. We call the former the ‘subject concept’ or, more loosely when the logical context is clear enough, ‘the subject⁴.’ The latter concept we call the ‘predicate concept’ or, more loosely when the logical context is clear enough, ‘the predicate.’ What the judgment adds to combine these we will call the ‘copula’ and the outcome is the aggregate concept. Let us now proceed with the explanations of singular, particular, and universal judgments.

All logical compositions of Quantity are defined in terms of the sphere of the subject concept and its relationship to the sphere of the predicate concept. If the subject concept has no sphere (i.e., has no other concepts under it), its composition of Quantity with the predicate concept is *singular* [KANT8a: 598-599 (9: 102)]. Thus, combination in the singular logical momentum *terminates an episyllogism* at the subject concept.

When the subject concept has a sphere (i.e. the concept does have other concepts under it), we have two possible relationships between the spheres of the subject and predicate that must be considered. The logical momentum of the composition of Quantity is *particular* if part of the

³ My choice of the term "functional" is based on an analogy with the mathematical idea of "functionals." A "functional" in mathematics is a mathematical function having a domain that is a set of functions and a range belonging to another set of functions. One example is a functional that attributes a scalar number to every function defined over a vector space. I liken the idea of a scalar to the idea of a combination and that of the vector space to the multiplicity of concepts upon which judgments act. However, we will not press this analogy too far.

⁴ The subject *of* a concept is the object represented by the concept.

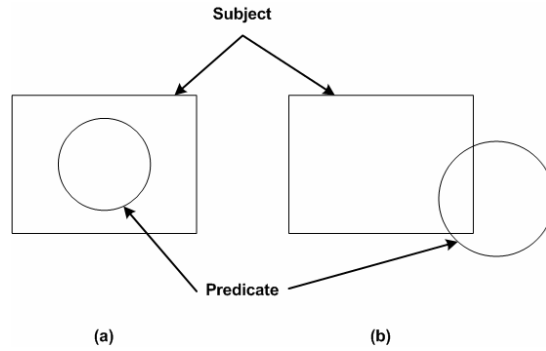


Figure 8.5.4: Illustration of the Particular Logical Momentum

sphere of the subject falls inside the sphere of the predicate *and* part of the sphere of the subject is *not* within the sphere of the predicate. Kant used Euler diagrams (which differ from today’s Venn diagrams⁵) to illustrate the logical momenta of Quantity. Figure 8.5.4 illustrates two cases of the particular logical momentum. In Figure 8.5.4(a) the composition is particular because the sphere of the subject is greater than that of the predicate and completely contains the predicate sphere. The case illustrated in (a) is one in which the subject concept contains more under it than does the predicate. Because it completely contains the predicate sphere under it the subject is the *higher* concept. For the case illustrated in Figure 8.5.4(b), the composition is particular because some *but not all* of the sphere of the subject is included in the sphere of the predicate. In this case, the Euler diagram tells us we cannot regard one concept as the higher with respect to the other. This form Kant called *contingently particular* [KANT8: 109 (9: 103)].

The *universal* logical momentum of Quantity is the logical composition of Quantity in which either: (1) the sphere of the subject concept is entirely contained within the sphere of the predicate, or (2) the sphere of the subject is entirely *excluded* from that of the predicate concept. These cases are illustrated in Figure 8.5.5. For the case illustrated in Figure 8.5.5(a), the predicate

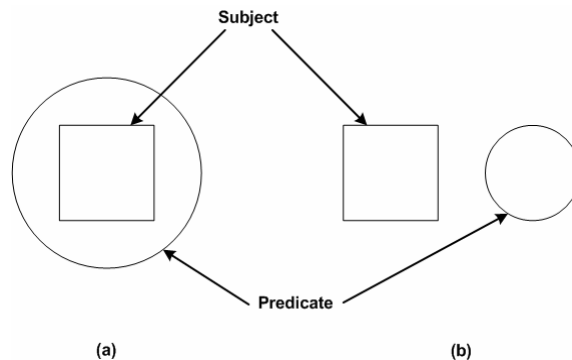


Figure 8.5.5: Illustration of the Universal Logical Momentum

⁵ Venn diagrams were introduced by the English mathematician John Venn in 1881 as a topological model of the Boolean algebra of logic. Euler diagrams represent concepts' *spheres* (extension); Venn diagrams represent their *contents* (intension) [KNEA: 349-350, 420-421].

concept is the higher concept with respect to the subject concept because the subject concept is contained under it. The subject representation must be a *concept* because it must have a sphere; an intuition, for example, has no sphere because it contains no representations under it. For the case of Figure 8.5.5(b), it is meaningless to call one concept the higher and the other the lower. It is nothing other than the fact that the judgment has entirely excluded the sphere of the one from that of the other that determines the composition of Quantity as universal.

The Momenta of the Categories of Quantity

If we compare the definitions of the logical momenta of Quantity given above with our previous discussion of the origin of concepts, we quickly discover that these definitions of the logical momenta do not appear to ‘fit’ with the idea of the origin and nature of concepts *if* we try to regard these momenta as ontological primitives. One obvious example arises in the case of the particular logical momentum. The definition permits the subject concept to be the higher concept. But: 1) a determinant judgment always proceeds *a parte posteriori*; 2) when a higher concept is logically combined with a lower concept, and no more consideration than this – i.e. that the two concepts are combined – is brought to bear, then the lower concept *and all of its sphere* analytically comes into the sphere of the higher concept. Therefore in Figure 8.5.4(a) is it not the case that the subject concept is what we have called a *mark* of the predicate concept (possibly a remote mark), and does this not contradict the earlier quote of Kant’s statement in the 1762 essay, i.e. that it is the predicate which is the mark of the subject?

Here is where the distinction between a ‘subject concept’ and a ‘subject’ becomes crucial. The strict *subject* of a concept is its *object* and, more to the point, the object regarded as a *thing*. Referring back to the earlier quote from the 1762 essay, the predicate *object* is the ‘thing’ that we must regard as the ‘mark’ of the subject *object*. The logical momenta of judgment *carry no reference to ‘object’ or ‘thing’ but only to combination in the manifold of concepts*. The subject concept is so called only because it is the concept of the determinable object; the predicate concept provides the determination *once the concepts are combined*. But this relationship – that of determinable and determining – does not implicate a ‘direction’ (epi- vs. pro-syllogism) in the synthesis of the manifold of concepts because here we speak of Modality in judgment.

Another problem with regarding the logical momenta of Quantity as ‘fundamental’ or ‘primitive’ is found with the universal judgment of Figure 8.5.5(b). This judgment is universal but it is characterized not by the inclusion of the subject under the predicate but, rather, by its exclusion. Thus, we seem to encounter a mixing of Quality and Quantity because affirmation and negation belong to Quality. An ontological primitive cannot admit such an admixture.

A third issue that prevents the momenta of Quantity from being regarded as ontologically

primitive is this: *we have given a clear and complete definition, a Realerklärung, of these momenta. However, as we discussed earlier, the primitive notions of understanding admit to no definition of this type.* The idea of the logical momenta of Quantity is not a pure and *a priori* concept of understanding, but rather a *made* concept with a *complete* definition easily constructed in a sensible representation. A concept of this type was called a *mathematical concept* by Kant.

Thus, while the logical momenta of Quantity are wholly adequate to the task of representing the *logical* composition of Quantity in a clear and sensible formal manner, they are, by themselves, inadequate as explanations of the *real* composition of Quantity in a determinant judgment. Something more is required – something more fundamental and necessary for the possibility of such a judgment – to explain the composition of Quantity in a determinant judgment. This need, at long last, brings us to the momenta of the *categories* of Quantity.

Combination in a determinant judgment produces a representation – namely the *concept* of the combination – that we view as a determination of the subject by the predicate. A concept, in turn, is at its root a rule for the re-presentation of the knowledge it represents in an intuition. However, we must also bear in mind that it is not the process of formulating an intuition nor is it imagination that carries out the combination of concepts in a determinant judgment. This act belongs exclusively to the process of determining judgment. It is an act of spontaneity and insofar as combination is concerned sensibility plays no immediate role.

And yet, in the simple logical *compositio* of aggregation, the subject and predicate *concepts* could contain in their spheres *contradictory representations* that are incapable of being exhibited together in the same intuition. In such a case, the sphere of the combined representation cannot include *either* of such contradictory elements and so *the sphere of the combination is a synthetic* rather than a mathematical union. We must permit this possibility or else deny objective validity to particular judgments. To permit this possibility *in a judgment* requires a *notion* that such a construction *is objectively valid*. In other words, we must have *a rule for the scope of the construction of the rule* represented in a particular determination of the subject by the predicate. This rule is the category of plurality ('many-ness') seen from the logical reflective perspective. ***Plurality is the notion of the scheme for representing extensive magnitude in a particular judgment.***⁶

Now let us consider the universal judgment. For cases such as the one illustrated in Figure 8.5.5(a) there would seem to be no issue or paradox with the simple definition of the universal

⁶ The formal description of the category of plurality in the logical perspective given above might appear to have little to do directly with the idea of "many-ness." The connection will be more apparent in later perspectives. Kant seems to have had a habit or preference for naming many of his technical terms for the "end result" in the "big picture" of philosophical tradition rather than, as is the custom today, naming them in a manner more descriptive of their direct use or application. (I call this Kant's tendency to name his terms "teleologically.")

logical judgment. If the sphere of the subject is contained entirely within the sphere of the predicate, the judgment is obviously unhindered logically and the need for some special rule for the construction of this type of judgment does not obviously present itself. If we permit combination in judgment at all, surely must we not permit universal determinant judgments of this type?

But now let us ask ourselves: what does it mean for us to say we “permit” some combination in determinant judgment? Clearly combination in determinant judgment is not something over which we “have a say in the matter”; combination is what the acts of determining judgment *do*. To say we “permit” some combination in judgment is really to say that our theory must establish the ground of the real possibility of such judgments. Now every determinant judgment of Quantity is a determination involving the sphere of the subject concept. In the case of the particular judgment we establish this sphere as being contained only in part within the sphere of the predicate. In the universal judgment the sphere of the subject concept is determined as either wholly contained in *or* wholly excluded from the sphere of the predicate. Figure 8.5.5(b) is just as much a combination as is Figure 8.5.5(a). Judgments of Quantity do not ‘place’ the spheres of the concepts with respect to each other; *they establish what these spheres are* by determining which representations are thought as ‘within’ and which are thought as ‘without’ the determined sphere of the aggregate concept.⁷

The sphere of a concept is not something ‘contained in’ the concept. A determination of the sphere by a determinant judgment expresses the extensive magnitude of the concept insofar as this magnitude can be represented in intuition in accordance with the Axioms of Intuition principle, and it determines this concept with regard to the form of the composition of its object. Nothing contained *in* a concept dictates the sphere of that concept, but this sphere is necessary *a priori* if the appearance of the object is to be represented in intuition as an extensive magnitude. And *this* requirement is not contained in the ideas of the logical momenta of Quantity. To these ideas we must add an *a priori* rule, namely, the rule of an *objectively valid scope*. The momenta of the categories of Quantity merely refine the rule of magnitude in terms of three *modi* of Quantity, expressed in terms of the determination of the concept’s sphere but ultimately grounded in the *scope* of the concept. It is this ‘link’ between the idea of the scope and the idea of the sphere which takes the judgment of Quantity from being merely a logical structuring of concepts *and turns this representation into the representation of an object*.

Thus, even the ‘logically obvious’ idea of the momentum of a universal judgment requires its *a priori* rule for the construction of the combination of concepts so far as this construction concerns the extensive magnitude of the appearance of the object. The notion of this rule is the

⁷ A concept is *any* rule for re-production in intuition. A determinant judgment combines concepts, and this combination *also* can serve as such a rule and thereby be called a concept.

category of totality.⁸ ***Totality is the notion of the scheme for representing extensive magnitude in a universal judgment.***

Finally we come to the category of unity ('one-ness') and its corresponding logical momentum of the singular judgment. Looked at 'topologically' in terms of a Kant-Euler diagram, the singular judgment is no different from the universal judgment in its logical form. There is, though, one great difference between the two. In the singular judgment, *the subject concept has no sphere*. Its *scope* is restricted to the raw appearance that is the object of the intuition for which it is the rule of reproduction. Considered in terms of a sphere, the subject concept is a kind of 'point' because it has no concepts *under* it. We might say "the subject concept in a singular judgment stands-under but does not under-stand."

However, while the subject concept in a singular judgment has no sphere, *it still has a scope*. It is the mediate representation of an appearance and this appearance constitutes the scope of the concept. Therefore, while in the singular judgment the subject concept has no sphere to be determined, the combination in a determinant judgment has phenomenal scope and therefore such a judgment requires its own *a priori* rule legislating extensive magnitude. The singular judgment terminates the series of combinations *a parte posteriori*.⁹ But this *series* must nonetheless be comprehended in intuition to complete the cognition. ***The category of unity is the notion of the scheme for representing extensive magnitude in a singular judgment.***

This completes our exposition of the categories of Quantity from the logical perspective. Quantity in a determinant judgment is, in this perspective, the scheme for the composition of judgments insofar as this composition is concerned with the extensive magnitude of the appearance of the object of cognition. The momenta of the categories of Quantity – unity, plurality, and totality – are the notions of schemes for composing aggregate concepts in regard to the extensive magnitude of a phenomenal object and with respect to the *scope* of the concept.

§ 5.2 The Categories of Quality from the Logical Perspective

Our rather lengthy discussion of concepts in the previous section allows our discussion of the categories of Quality in this section to be relatively more brief. In terms of the combinations (*conjunctio*) of determinant judgments, composition (*compositio*) is the matter of combination and Quality is the matter of composition. (Quantity, of course, is the form of composition). For the matter of composition our guiding principle from Rational Physics is the principle of the Anticipations of Perception.

⁸ Kant's tables of the logical momenta and the categories in *Critique of Pure Reason* and the *Prolegomenon* do not clearly show the link between totality and the universal momentum. However, he does clearly show this in [KANT19: 454-456 (29: 985-987)].

⁹ Loosely speaking, we might say it "gets to the point."

This principle, we recall, states: In all appearances sensation, and that which corresponds to it in the object (*realitas phaenomenon*), has intensive magnitude, i.e., has a degree. Intensive magnitude, in turn, is that magnitude which can only be apprehended as a unity, and in which multiplicity can only be represented through approximation to negation [KANT1a: 291 (B: 210)]. Intensive magnitude is the idea of the ‘real content in time’ (a ‘what-it-is’ that ‘fills time’) in any representation of the composition of an object. Its *logical* counterpart is the idea of the content (*Inhalt*) of a concept [KANT8a: 593 (9: 95)].

Compared to the time and effort Kant put into describing the sphere of a concept, his description of its content is rather sparse and more than a little vague. Generally speaking, ‘logical content’ means “that which is common to various representations of several things” [KANT8a: 353 (24: 910)] and it is represented as ‘contained in’ sensibility when these various representations undergo comparison, reflexion, and abstraction in the *Verstandes-Actus*. To ‘draw out’ and recognize the content of a concept is a synthesis *a parte priori*. Venn’s diagrams (which hadn’t been invented in Kant’s day) rather than Euler’s would be used to diagram logical content.

The Logical Momenta of Quality

From the viewpoint of formal logic, the *content* of a concept does not enter in to the logical description of a determination. Rather, the content of a concept is a logical presupposition for the determination of the relationship of concepts in terms of their spheres under the principle of contradiction and identity. The logical momenta make abstraction from any sensible matter in the concepts they combine (since we say a concept has sensible content only insofar as the concept rests on intuition in its original formation). Consequently, while the logical momenta of Quality are descriptive of the organization of combinations – in particular, the composition of the combination – they are entirely devoid of any real meaning beyond this context. Kant explained the logical momenta of Quality as [KANT8a: 220 (24: 274), 600 (9: 103-104)]:

The affirmative momentum is the logical momentum of judgment by which the subject concept is thought in connection with the sphere of the predicate.

The negative momentum is the logical momentum of judgment by which the subject concept is thought in opposition to and set outside the sphere of the predicate.

The infinite momentum is the logical momentum of judgment by which the subject concept is set in the sphere of a concept that lies outside the sphere of the predicate.

The first two of these momenta are not difficult to comprehend and in the language of traditional logic they are expressed as “*S is P*” and “*S is-not P*”, respectively. The distinction is

entirely in the copula of the judgment, which tells us that the predicate term either is or is not predicable of the subject term. In this way these momenta place restrictions on the sphere of the *subject* concept and its grounding marks. This is why the illustrations in Figure 8.5.5 can both be universal judgments; the distinction between them lies in the judgment of the logical domain of the sphere of the subject concept. In both cases *S* and *P* are combined but the compositions of these combinations differ *qualitatively*. It might be helpful to think of the *S* and *P* spheres in Figure 8.5.5(b) by picturing them as having a dotted line linking them to illustrate explicitly that the concepts are combined.

The case of the infinite momentum requires more discussion. In traditional logic no distinction is drawn between the negative momentum and the infinite momentum. Kant, however, does draw a distinction between them which is illustrated by phrasing the infinite momentum as “*S* is not-*P*”. In other words, the copula is that of the affirmative momentum but the infinite momentum restricts the sphere of the *predicate* and bans it from having any part of its sphere containing or contained within the sphere of the subject.¹ In such a composition, we do not make any further positive determination of the composition of the subject, nor do we do so for the predicate. What we determine is a rule that affects both.² Explicitly, we judge the subject as necessarily contained in the sphere of some third *undetermined* concept while, ‘at the same time,’ we *implicitly* make a judgment pertaining to the relationship of the sphere of the predicate with respect to this undetermined third concept. The infinite logical momentum is a very busy momentum indeed!

Kant’s use of the word ‘infinite’ in naming this momentum also deserves a brief discussion. The first thing we must realize is that this term ‘infinite’ does not carry any implication of ‘mathematical infinity.’ In the first place, the infinite momentum belongs to the title of Quality whereas the idea of mathematical infinity is something we associate with the idea of ‘number’ – which is the transcendental schema of Quantity. Indeed, for most of the history of Western thought, “the infinite” did not have the connotation of ‘number’ at all; instead, it was an idea associated with that of a *process* for which the termination of that process was indefinite. The idea of ‘mathematical infinity’ as a kind of ‘number’ is of relatively recent origin. This introduction was the work of Cantor and is today regarded as an element in the very foundations of modern mathematics. However, this introduction was not achieved without arousing its

¹ A *mark* of the predicate can also be a mark of the subject because a mark is not “in” the sphere of either. They are in *its* sphere.

² If I say “Fred is not-German” I only establish that whatever “Fred” may be, he does not come under whatever the concept of a “German” means. By this judgment I do not establish that Fred is French or even that Fred is a man (rather than, say, a beagle). Nor do I make any clearer what a “German” is other than whatever this may be, the concept is not a ground or partial concept of “Fred.” But if I say “Fred is-not German,” I establish that “being German” *contradicts* “being Fred.” Then if a mark *G* is a *sufficient* ground for recognizing “German” it cannot also be a mark of “Fred.”

detractors, some of whom were among the most eminent mathematicians of their day. Poincaré, for instance, wrote:

The notion of infinity had long since been introduced into mathematics, but this infinity was what philosophers call a *becoming*. Mathematical infinity was only a quantity susceptible of growing beyond all limit; it was a variable quantity of which it could not be said that it *had passed*, but only that it *would pass*, all limits.

Cantor undertook to introduce into mathematics an *actual infinity* - that is to say, a quantity which is not only susceptible of passing all limits, but which is regarded as having already done so. He set himself to such questions as these: Are there more points in space than there are whole numbers? Are there more points in space than there are points in a plane? etc. . . .

Many mathematicians have followed in his tracks, and have set themselves a series of questions of the same kind. They have become so familiar with transfinite numbers that they have reached the point of making the theory of finite numbers depend on that of Cantor's cardinal numbers. In their opinion, if we wish to teach arithmetic in a truly logical way, we ought to begin by establishing the general properties of transfinite cardinal numbers, and then distinguish from among them quite a small class, that of the ordinary whole numbers . . . This method is evidently contrary to all healthy psychology [POIN2: 143-145].

We will not jump into the quarrel about the views on this point at present (although it will come up again in Chapter 23, where we will find we must side with Poincaré). For our present purposes it is sufficient to say that Kant could not have meant by his use of the word 'infinite' to refer to mathematical ideas that did not come into being until many decades after his death. It is in the 'quality of a becoming' that we should look for the context of the term 'infinite.'

The Momenta of the Categories of Quality

If the principle of the logical momenta of Quality is the principle of contradiction and identity, what does 'contradiction' in *real* judgment mean? Mathematical logic more or less equates contradiction with falsity and, in doing so, sometimes seems to make the *principle* take the *logical momenta* for its ground. In their symbolic logic textbook Ambrose and Lazerowitz wrote:

It is to be noted that whenever an *inference* is made, not only is an implication asserted to hold between the premises and conclusion, but both premises and conclusion are asserted to be true. Both these facts are relevant to a consideration of the means of escaping commitment to the truth of an inferred conclusion. There are, in general, two ways of doing this. One way is to deny that the implication holds. This amounts to pointing out that the argument is formally invalid. The second way is to take exception to the material truth of the asserted premises, i.e., either to refuse to agree to the initial assumptions or to point out their actual falsity. The relevance of denying the truth of the premises depends upon a logical fact about the relation between the antecedent and consequent of any implication . . . when the antecedent is false . . . In general, if one denies the material truth of the premises or refuses to assent to it, there is no logical necessity of assenting to the truth of the conclusion [AMBR: 124-125].

In other words, to "escape commitment to the truth" of a logical argument, we must either show that the argument has somewhere violated the rules of formal logic or we must "refuse to assent"

to the “material truth” of one or more of the premises. The latter tactic is one that modern formal logic simply banishes from the domain of its science; consideration of the matter of premises is not in the job description of mathematical logic. The former is based on whether or not the argument follows both the rules set down in the definitions of the logical operations and the rules for combining propositions in an inference. And in both cases formal logic simply holds that there is merely no “logical necessity of assenting to the truth of the conclusion.”

If we make abstraction from all material content of concepts, as the logical momenta do, then, to put it bluntly, the only way contradiction can arise from logical functions of understanding is if the process of determining judgment doesn’t follow its own rules. This, of course, is a manifest absurdity in mental physics for it would be tantamount to saying that there actually is no *a priori* process of determining judgment at all but merely some sort of mechanistic or probabilistic combination of concepts. Unless ‘concepts’ are mechanical, such an explanation is occult. Even if we were to argue that mental illness is an example of such a thing, the argument would be fallacious because while we can assume that a defining trait of mental illness is symptomatic behavior that indicates an individual ‘follows a different set of rules’ in his thinking, no one holds that the mentally ill person does not “have his own set of such rules.” Nor do cases of mental illness prove for a fact that it is a breakdown within an *a priori* organization of determining judgment rather than in the person’s own *empirical* maxims of thinking or in the sensorimotor process of ‘information gathering’ that is at the root cause of his illness.

Thus, while the definition of the logical momenta of Quality is clear and concise in terms of the outcomes in the organization of the *spheres* of concepts, there is found nothing in the definition of these momenta that grounds the act of judgment *in a rule*. The problem is that the logical momenta of Quality address the outcome (the determination by judgment) but not the means or criterion needed to constitute the *action* that produces the outcome. They tell us “what judgment produces” but not “how judgment comes to do it.”

We should also find it troublesome that the definitions of the logical momenta of Quality are so tightly bound up with the idea of the spheres of concepts, ‘sphere’ being an idea of Quantity. Note well that these momenta do not speak to what is contained *in the scope of the aggregate concept* produced by a determinant judgment, but only to what is determined of the spheres by such a judgment. Of course, all determinations as to the *compositio* of a judgment involve both Quantity and Quality; that is not the problem. The problem is that the definition of the logical momenta of Quality seem to rely implicitly in some way on the momenta of Quantity and vice versa. This alone is enough to demonstrate that these momenta are mathematical concepts in our theory and are not ontological primitives. They *necessarily require* primitive rules that ground their possibility, their *Realerklärung*, and the manner of their deployment in determinant judgment. (These primitive rules are, of course, none other than the categories of Quality). For

these reasons, we cannot consider these logical momenta to be ‘laws of thinking’ but merely to be schemata of structuring the manifold of concepts in determinant judgments.

Every determinant judgment makes a determination of an aggregate concept, and this concept must be capable of exhibition in intuition. With this in mind, let us look at what it means for the principle of contradiction and identity to have a *real implication* for a judgment and what role the material content of concepts plays in this.

Sensation – the matter of an intuition – is perceived as an intensive magnitude since the intuition itself is a singular representation perceived at a single moment of subjective time. The concept is a rule for the reproduction of its object intuition. It follows that the content of the concept is a representation from which the sensational matter of the intuition can be exhibited, although this exhibition is not necessarily the same as sensation apprehended from receptivity. The exhibition of the concept in an intuition takes place by means of the synthesis of imagination.

Power of imagination is the substitute of the senses, the capacity for intuitions in the absence of objects . . . Consciousness is the capacity for grasping representations so that we can reproduce them; the skill for that is called the capacity of remembrance, memory [KANT19: 375 (28: 673-674)].

The sensibility of concepts via imagination, of course, necessarily presumes that there is something ‘in’ the concept which, as data, provides imagination with the information required for its synthesis.

Empirical intuitions are representations of an object, how our senses are affected by it. Empirical intuition has two parts, matter and form, and empirical concepts likewise. - The matter of everything empirical - for empirical intuition, is sensation, the form is the *Gestalt*³. The concept has matter, i.e. content, representations, data, which are given; the form is the reflexion of understanding, by which it brings the sensations together that thereby it thinks something general. The concept is a sum of sensations fashioned by understanding . . . In every empirical concept is matter, i.e. sensation, and form, which is required for understanding because it is logical [KANT19: 150 (29: 795-796)].

We cannot take this to imply that the sensational content of a concept is a copy of the intuitive sensation, i.e. a copy-of-reality, an idea we discussed and dispensed with earlier in this treatise. Every concept arises as a representation of that which is common among two or more representations in sensibility. (Concepts, we recall, are *common* representations). All that we require of the sensational part of the content in a concept is the representation of information sufficient to make it possible for imagination to reproduce a sensibility of that content.

Now let us take a look at the composition of the combination of two or more concepts in the determination of the aggregate concept in a judgment. This composition must take place in such a manner that the concept can be exhibited in intuition. So far as the sensational content of this concept is concerned, the sensational information must, in accord with the Anticipations of

³ This word has no exact English equivalent. It means roughly the whole "figure" or "configuration."

Perceptions, be able to be composed *by degrees*. There are only three *modi* in which this is possible.

First, the sensational content of each concept may be such that these *subjective* marks⁴ of sensational representation in the predicate can coexist with those of the subject concept. This is the case, for example, with auditory and visual marks of sensation. In this case there is no contradiction between the diverse sensational contents, and the representation of one such mark in no way precludes the possibility of representing the other. In this case the sphere of the aggregate can contain conceptual marks of the predicate without itself being in the sphere of the predicate.⁵

Second, there may be identity in the sensational marks of the subject and predicate concepts. For example, the representation of the same color sensation in both subject and predicate presents no contradiction to the representation of this color sensation in the aggregate concept because imagination is not thereby being called upon to both represent and not represent this color sensation in its synthesis of the intuition. The aggregate concept may, therefore, be represented in the spheres of both subject and predicate where the conceptual mark is a ground.

Finally, the sensational marks of the concepts may oppose each other. In this case, imagination is called upon to both represent and, simultaneously, negate the sensational content in the aggregate. This *is* a contradiction and, indeed, a *real* contradiction. Those parts of the spheres of the subject and predicate concepts that stand in such a contradiction cannot be represented in the same intuition. In such a case, those parts of the spheres that are grounded in these marks *must* be excluded from the sphere of the aggregate.

In regard to the first *modus*, intuition also has a form and, in particular, an element of this form is the pure intuition of outer sense. All that we have just said regarding the sensational content of intuitions and their concepts also holds for the representation of the form of outer sense. Concept *materia* whose re-presentation by imagination would contradict each other if they appeared in the same form of outer sense do not contradict each other if the schemata of their re-presentation differ in the form of intuition. Kant called the form of outer sense the ‘space’ of intuitive representation (admittedly an ambiguous term at this point, but one which we will clear up later in this treatise).⁶ Sensational contents that would be contradictory if given the same spatial representation are not contradictory if each is represented in a different spatial form.

⁴ I introduce the term ‘subjective mark’ to mean a sensational characteristic (mark) of a perception. See the glossary for the distinctions between mark, subjective mark, and conceptual mark.

⁵ E.g., if the mark is a partial concept but not a sufficient ground of the predicate.

⁶ In science and in mathematics, "space" is yet another of those ideas that differ widely in appearance from one application to the next. The word "space" takes on many meanings depending on the object of the science. Thus we speak of "physical" space, "state" space, "Hilbert" space, "geometric" space, "metric" space, "representation" space, "solution" space, etc. The idea of "space in general" is not unlike the idea of "number in general" or "time in general" or "object in general" or many other very abstract ideas. We will later (Chapter 17) devote considerable "discussion space" to the pure intuition of space.

Consequently, the real meaning of ‘non-contradiction’ and ‘contradiction’ pertains to the possibility or impossibility of representing an aggregate concept in *one* intuition through the synthesis of imagination. I can say, “A and B are the same, only different,” without contradiction because my intuition of how they are the same and my intuition of how they are different are conceptually separable in my representations of the appearances of A and B. I can think of myself as a boy and myself as a grown man as appearances of “the same me” without contradiction because “myself as a boy” and “myself as a man” are different *limitations* in concept and in intuition, joined in the unity of apperception by an aggregate concept of “me” which, without “qualifications”, I cannot represent (in terms of *Existenz*) by any sensuous intuition at all.

The act of determining judgment in representing the aggregate necessarily presupposes schemes or rules by which this representation is possible in intuition in the various *modi* just described. Thus from the logical perspective we have the categories of Quality as:

***Reality*: the notion of the scheme of determining the intensive magnitude in an affirmative judgment;**

***Negation*: the notion of the scheme of determining the intensive magnitude in a negative judgment;**

***Limitation*: the notion of the scheme of determining the intensive magnitude in an infinite judgment.**

§ 5.3 The Categories of Relation from the Logical Perspective

Taken collectively, the categories of Quantity and Quality constitute what Kant called the *mathematical* categories. The reason for this name is that these categories ground the composition of determinant judgments, i.e., the construction of aggregate concepts of objects. Mathematics, as Kant uses that word, is knowledge through the construction of concepts (rather than knowledge *through* concepts, a fine distinction to be sure; knowledge through concepts is a task he assigns to philosophy). We now turn our attention from the *compositio* of combinations in determinant judgments to the *nexus* of combination in judgment. Here the categories are those of Relation and Modality which, taken collectively, Kant called the *dynamical* categories.

While the previous categories have to do with the composition of specific objects as phenomena, the dynamical categories are concerned with the overall unity in consciousness of the whole of the manifold of concepts, i.e. with the *connection* of object concepts to each other in the whole of conceptual representation. These categories are those which are necessary for the possibility of the *Dasein* of such a manifold. It is not out of place to remind ourselves at this point that in the Critical Philosophy ‘being’ is not a real predicate. We hold nothing to be a real something unless we have not only a representation of the object *but also* a representation of this

object in connection to other objects in the manifold of concepts. All objects in the *scope* of Nature constitute the magnitude *of* Nature.

As before, we will proceed by first examining the logical momenta to understand the appearance of the forms of connection, i.e. *nexus*. As was the case for the mathematical momenta, we will find that the dynamical logical momenta “co-mingle” the ideas of the logical momenta of Relation and Modality and, therefore, that these logical momenta are not primitives in the ontology of determinant judgments. They will each require a primitive notion that grounds their possibility, and these notions will be none other than the dynamical categories.

Also, as before, we must call upon the metaphysics proper of Rational Physics to provide us with the metaphysical principles that take us from mere relationships of a formal logic to real relationships in cognition. For the categories of Relation we have the principles of the Analogies of Experience: As regards their *Dasein*, all appearances stand *a priori* under rules of the determination of their relationship to each other in *one* time.

Propositions and the Categorical Logical Momentum

In our description of the momenta of composition the idea of ‘concepts’ played a central role. Composition in combination, as the matter of combination, is concerned with the formal representation of the organization or ‘faculty’ of understanding insofar as this pertains to the material unity of representations (the matter of representation) in consciousness. *Nexus* in combination, as the form of combination, pertains to the formal unity of *all conceptual representation* in consciousness. When we turn to the consideration of this formal element of combination, we find something new needs to be added to our exhibition, namely the idea of ‘propositions.’

In their *Principia Mathematica* Whitehead and Russell regarded the idea of a proposition as a primitive, i.e. as something we just accept without further reduction in terms of other ideas and without definition. Russell later described it this way:

When, in the preceding chapter, we were discussing propositions, we did not attempt to give a definition of the word "proposition." But although the word cannot be formally defined, it is necessary to say something as to its meaning, in order to avoid the very common confusion with "propositional functions" [RUSS1: 155].

Russell then proceeded to give a somewhat vague description, primarily through examples, of “what ‘proposition’ means.” This meaning he confined to something involving “symbols” that “give expression to truth and falsehood.” This “truth” or “falsehood” he also confined to be limited to particular instances, and he held that no proposition could be said to be always true or always false.

When one insists on stripping out all material content from one's ideas, as Whitehead and Russell do in the *Principia*, perhaps this vague description of the idea of a 'proposition' works well enough for the purposes of logical positivism. Fortunately, we will be able to do rather better with this idea here. What we will find is that the idea of 'propositions' is *not* primitive and that a *Realerklärung* of this idea is easily obtained from the logical momenta of Relation. Since these logical momenta are not themselves primitive, the idea of 'propositions' will be rather greatly demoted from the lofty Platonic standing Russell assigned to it.

It will prove useful to develop the idea of propositions in two steps. In the first, we will consider what we may regard as the primary or most basic idea of a proposition. This is the idea that most closely corresponds to what is usually meant in traditional logic by the word 'proposition' and we will call a proposition of this sort a *categorical proposition*. We will then consider derivative propositions, which will take their definitions in terms of combinations made from categorical propositions. The categorical proposition takes its *Realerklärung* from the categorical logical momentum of Relation.

All the logical momenta of Relation pertain to the representation of the representation given in a judgment in relationship to the unity of the state of consciousness [KANT8a: 601 (9: 104)]. The 'representation given in a judgment' is the composition of the judgment by the momenta of Quantity and Quality. As an example, let us use the venerable predication '*S* is *P*'. So far as composition is concerned, this is merely the determination of an aggregate concept in regard to the spheres of *S* and *P*. The form of this composition is that of aggregation, the matter is that of coalition. To this, however, we must *necessarily* add *connection* between what is represented in the composition of the judgment and *how this composition is to be regarded*, in terms of the *Existenz* of this compositional matter, in empirical consciousness. In our consideration of the composition, the focus of our attention was given to the *terms*, *S* and *P*. When we consider the *nexus*, the focus of our attention shifts to the copula (which in this example is represented by the word "is") *as the manner of conscious unity*.

This representation of the form of connection, this copula *x*, is a representation that was not presented in the initial representation of the concepts *S* and *P*. It is a spontaneous representation of determining judgment by which we make the manifold of concepts and through which we have a conscious connection, *SxP*, in that which we call *the judgment*.

In categorical judgments, subject and predicate make up their matter: the form, through which the relationship (of agreement or of opposition) between subject and predicate is determined and expressed, is called the *copula* [KANT8a: 601 (9: 105)].

This *nexus* of representation in the form *SxP* is what we call a *categorical proposition*.

The categorical proposition is obviously a judgment. Are we also to regard it as being a

concept? If so, the formal similarity between this theory and Locke's "simple ideas" (*S, P*) and "complex ideas" (*SP*) is striking if we ignore Locke's metaphysical views on the origin of simple and complex ideas. On the other hand, viewing the categorical proposition as being a concept would seem at first glance to contradict what we said earlier about the origin of concepts, namely that concepts originate via the *Verstandes-Actus* of comparison, reflexion, and abstraction through reflective judgment. Yet Kant himself, if we may judge from his logic lectures, seems to promote the idea that the categorical proposition is to be viewed as being a concept when he says,

The matter of all categorical propositions consists of a concept, in which the concept of the subject belongs to the concept of the predicate [KANT8a: 372 (24: 932)].

Here he speaks of three distinct concepts, the third of which we perhaps could call 'the copulated concept' for want of a better name. Next to this we set Kant's general *Erklärung* of a judgment, i.e. a judgment is the representation of the unity of the state of consciousness of various representations, or the representation of their relationship *insofar as they constitute a concept*.

The answer to our question becomes evident if we understand that Kant's writings employ the word *Begriff* (which is invariably translated as "concept" in most English translations of Kant's work) in several different senses.¹ That this is the case is evident from Kant's famous plea in *Critique of Pure Reason* that we preserve unsullied the "original meaning" of the word *Idea* (*Idee*):

A perception that solely refers to the subject as a modification of its state is a *sensation* (*sensatio*); an objective perception is *cognition* (*cognitio*). This is either *intuition* or *concept* (*intuitus vel conceptus*). The former refers immediately to the object and is singular; the latter refers mediately, by means of a mark which can be common to several things. A concept is either an *empirical* or *pure concept*, and the pure concept, so far as it has its origin solely in understanding (not in the pure image of sensibility), is called *notio* ['notion']. A concept out of notions which passes beyond the possibility of experience is the *Idea* or *idea* of reason [*Vernunftbegriff*] [KANT1a: 399 (B: 376-377)].²

Now this taxonomy of terms clearly is not exhaustive, particularly in regard to 'concept.' What, for instance, should we call a representation made up of both empirical concepts and notions that can be *exhibited* within "the possibility of experience"? This should, if we are to follow Lavoisier's dictum, be made to depend on the object of the representation, and we give it the name 'idea.' For the situation we are presently considering, 'proposition' seems as good a name as we can devise for the representation *SxP*. Since this is most certainly an objective perception it is entitled to be called a cognition when its representation of the manifold of

¹ We can no more fault Kant for the fact that the German language has several senses of *Begriff* than we can fault ourselves for the same thing in the English word "point" (which has 35 definitions).

² In this treatise "idea" is one translation of *Begriff* but "Idea" is always *Idee*. Depending on context, I also translate *Begriff* as "concept" or as "notion."

concepts can be also exhibited in intuition. When we restrict our view to just the faculty (organization) of the manifold of concepts, it is likewise entitled to be called a concept.

But how does it become a concept? Here we must remind ourselves that every determinant judgment *participates in thinking* through the three-fold synthesis of apprehension, imagination, and re-cognition in a concept. The *Verstandes-Actus* are by no means excluded from the overall act of *making* a determinant judgment, and *SxP* is a representation that must, like all other outcomes of the power of spontaneity in the faculty of empirical consciousness, undergo this synthesis and be subjected to the *Verstandes-Actus*. Relative to *S* and *P* alone, the representation *SxP* contains more than the sum of *S* and *P*. It is, in other words, an aggregate concept of a phenomenon. The copula *x* is itself a mark but differs from *sensuous* marks³ by virtue of its source in the power of spontaneity. The categorical logical momentum is the logical *nexus* of Relation in a determinant judgment that is displayed in intuition as an individual representation, *SxP*, whereby the unity of consciousness is given by the subordination of the predicate to the subject. This we take as our explanation or *Realerklärung* in the logical perspective of the possibility of cognition through concepts, i.e., *thinking SxP*. The categorical proposition is a copulated concept but not every concept is a categorical proposition.

The Category of Substance and Accident from the Logical Perspective

While the formal similarity between Lockean representationalism and the theory discussed above is evident, the similarity between Locke and Kant ends abruptly at this merely formal level. We must now consider the ground of the categorical logical momentum and its product, the categorical proposition. In doing so, we call upon the First Analogy of Experience: *All appearances contain the persistent (substance) as the object itself, and the changeable as its mere determination, i.e., the way in which the object exists.*

We said above that the categorical logical momentum supplies the connection to the state of consciousness in the combination of judgment by means of the copula. We must now inquire into the “nature” of this copula, i.e., *its* representation. This, of course, leads us to consider both the matter of the copula as well as its form. Here, in discussing Relation, we limit ourselves to the form of the copula in categorical judgments and will return to discuss its matter when we take up the momenta of Modality.

Before it can take form as a copulated concept, the categorical proposition – or, more accurately, the representations from which it is formed – must pass through the three-fold

³ By the term ‘sensuous mark’ I mean a mark in which the matter of the mark originates through the power of receptivity. A mark in which the matter in the mark originates through spontaneity can be called an *intellectual* mark in some cases or a *practical* mark in others.

synthesis, be exhibited in an intuition, be marked by reflective judgment at moment in time, and return to conceptual form through the synthesis of re-cognition in a concept. Unlike concepts determined strictly through sensuous marks (which originate by means of receptivity), it is the process of determining judgment which acts, by the power of spontaneity, as the agent to sensibility in this process of intuitive representation. Now, the intellectual mark (the copula) of the categorical proposition can originate nowhere else than in the spontaneity of thinking and therefore must take its origination *a priori* in the *nous* of Organized Being. Consequently, the judgment of the copula can contain nothing empirical; rather, it is the power of spontaneity to *produce* this copula which is the ground of the possibility of all empirical categorical propositions. As to its form, then, our idea of a copula is grounded in *a pure notion of a form of connection* in the manifold of concepts that the representation of the copula instantiates.

If all that was “fed into” the synthesis of reproduction in imagination were the terms represented in the concepts *S* and *P*, nothing in the process of imagination could give to sensibility the representation of a categorical connection between them. There is no judgment in apprehension and such a connection – which is *never* presented through receptivity – necessarily requires judgment. (“*S* is *P*” is not a representation that can be given to us merely through the data of the senses). The power of receptivity is a ‘passive’ power (in that it requires stimulation) and if the concept we call the categorical proposition is to be possible, then there must necessarily be ‘something’ *in* the scheme of the categorical logical momentum “fed into” imagination that makes possible the representation *SxP* in intuition.

This ‘something’ can be nothing other than an *a priori* rule under which this making of the representation is constituted. Furthermore, it must be “in the nature” of such a rule that the representation in intuition and the representation in concept can be bridged by a transcendental schema. Since the cognition of this representation is an *objective* perception, the Copernican hypothesis requires of this rule no less than it be a rule by which we come to our cognition of objects *as phenomena*. The principle of this rule, insofar as the categorical Relation is concerned, is the First Analogy of Experience which states *the objective character* of the appearance of an object as a phenomenon.

Now it is ‘the nature’ of a categorical proposition that it unites in consciousness two terms, namely *S* and *P*. As ‘that which is determined,’ *S* pertains to the object but can represent this object only as an appearance and this only ‘through’ *P* and the copula *x*. As concepts, *S* and *P* are already rules for the re-production of their respective intuitions, but the combining of these representations in a *new* intuition *SxP* requires for the representation *x* two aspects. In the first place, we must have the notion of an object and this is the notion of substance which, under the transcendental schema, takes the *modus* of persistence in subjective time. In the second place, we must have the notion of an *appearance* at a moment in time, and this is the notion of accident.

These two are *inseparable* in thinking because ‘accident’ without the implication of ‘substance’ (i.e., without an object) is meaningless, and ‘substance’ without ‘accident’ is an empty notion. Substance regarded-as-it-is-in-itself is never an object of perception and therefore *is not a possible appearance*.

The rule of the copula in a categorical judgment is therefore the rule that connection of the subject and predicate is always represented in the form of an appearance of an object. In this form, the subject *S* is represented *as the object* and its appearance at a moment in time is determined by the representation of the connection of the predicate concept to the subject concept. If we examine this determination *SxP*, we quickly realize the ‘nature’ of the momentum of the category of substance & accident subsists in the positing of *S* as the subject term, for nothing in the concept *S* necessarily announces “this is the concept of a subject term.” The concept *S* is the subject term only because an *a priori* rule governing the categorical logical momentum *marks S* as the subject and, likewise, marks *P* in the role of the predicate concept. This rule therefore is a rule for the form of the cognition of the object in the state of empirical consciousness. From the logical perspective, therefore, **the category of substance and accident is the notion of the scheme of determining the objective form of a categorical judgment.**

This notion takes us beyond formal logic in a fundamental way. It is no longer the case where we simply view the relationship between *S* and *P* as a relationship between two concepts. Rather, this category imputes objective *knowledge* to our representation; the *real* connection is not a connection of concepts but a connection of *the cognition of an object* with the state of empirical consciousness *in the form* of the manifold of concepts. It is not the object that determines the cognition, but the cognition which determines the object. From the *Dasein* of the manifold of concepts comes the empirical consciousness of the *Existenz* of objects. The dynamical categories make possible the *Dasein* of this manifold and, thereby make possible objective empirical consciousness – objective knowledge.

Hypothetical Propositions and the Hypothetical Logical Momentum

We now turn to the first of our ‘secondary’ propositions, which Kant called the hypothetical proposition⁴. The hypothetical proposition differs from the categorical both in terms of its matter and its form. Kant described the hypothetical proposition in the following way:

⁴ *Propositio hypothetica* [KANT8a: 372 (24: 932)]. Usually Kant referred to these as “hypothetical judgments” (*hypothetische Urtheile*). The German *Urtheil* in one sense is ‘judgment’ taken as a decision, opinion, view, finding, etc. Kant tells us, “Judgment and proposition [*Satz*] according to word-usage are actually distinct . . . We prefer to say that a judgment considers the relationship of two concepts so far as it is problematic, while by proposition we understand an assertoric judgment . . . and in just the assertion subsists the proposition [KANT8a: 374 (24: 934)].

The matter of hypothetical judgments subsists of two judgments that are connected with one another as ground and consequence⁵. One of these judgments, which contains the ground, is the antecedent (*antecedens, prius*), the other, which is related to the former as a consequence [*Folge*], is the after-proposition⁶ (*consequens, posterius*), and the representation of this manner of connection of both judgments under one another for the unity of the state of consciousness is called the *Consequenz*⁷, which constitutes the *form* of hypothetical judgments.

Note 1. What the copula is for categorical judgments, so the *Consequenz* is for hypotheticals – their form [KANT8a: 601-602 (9: 105)].

Like the categorical proposition – and for the same reason – the hypothetical proposition is a determined concept and it contains three parts – antecedent, "after-proposition" or *consequent proposition*, and *Consequenz*. We will use the notation *AyC* to denote this type of proposition.

However, the hypothetical proposition differs fundamentally from the categorical both as to its matter and as to the rule that brings it to the transcendental schema of imagination. The concepts *A* and *C* (antecedent and consequent) that make up the matter of the hypothetical proposition *must be categorical propositions*. No concept falls within the domain of the hypothetical logical momentum unless it is a determined concept that has resulted from the application in a determinant judgment of the categorical logical momentum under the rule of the category of substance and accident. If we say that a categorical proposition *SxP* is a rule, then the hypothetical proposition *AyC* is an empirical *rule about rules*.

Another difference that distinguishes a hypothetical proposition from a categorical one shows up in how the concept terms 'in' the proposition are regarded as being consciously determined by the act of judgment. In the categorical proposition the subject concept is regarded as a determinable and the predicate concept is regarded as a determination. For the hypothetical proposition the consequent term *C* is regarded as conditioned, the antecedent term *A* is regarded as the condition, and the proposition *as a whole* is regarded as a determination.

Similar to the copula in the categorical proposition, the determination in a hypothetical proposition is presented through the *Consequenz* *y* and is the product of the notion for the scheme of the hypothetical logical momentum. In this case, however, the form of this connection is not directed at the object of the judgment (as a substance) but rather to establishment of the truth of a relationship between the antecedent term and the consequent. Truth, we recall, means the congruence of a cognition with its object; while the categorical Relation corresponds to the idea

⁵ *Grund und Folge*. Another possible translation here is "ground and result." I translate *Folge* here as "consequence" because of the logico-dynamical context of Kant's discussion, but the connotation of *Folge* here is objective, referring to the object of the consequent proposition rather than the proposition itself. The primary connotation of *Folge* is succession in time. Consult the glossary for more discussion of this.

⁶ *Nachsatz*, the proposition which "follows after" the antecedent. Note well that Kant is making a number of very fine distinctions here. One does not normally find this level of distinction in formal logic, but Kant's logic must take concepts and objects into account. We will call this "the consequent" proposition.

⁷ *Consequenz* is a cognate of the Latin *consequentia* in the context of "a necessary succession."

of internal Relation in our general 2LAR, the hypothetical Relation corresponds to the idea of external Relation in this 2LAR. This external connection between the propositions in a hypothetical proposition has a two-fold character with regard to the *Consequenz*.

The first of these is called the “positing connection” or *modus ponens*. The second is called the “rescinding connection” or *modus tollens* [KANT8: 112 (9: 106)]. The connection being made in the hypothetical logical momentum is a connection to the state of consciousness regarding the *criterion*, under which the connection of judgment is held to be true, through the power of judgment to judge agreement or disagreement in its concepts according to *a priori* norms. For the *modus ponens* the norm is: if the antecedent is true, then the consequent is also true. For the *modus tollens* the norm is: if the consequent is false then the antecedent is also false. Thus, the hypothetical logical momentum finds its nearest correspondent in traditional logic in the logical construct we call the “implication”, which can be represented by the usual form “If *A* then *C*” or, alternatively, “*A* implies *C*.”

Having said this, however, we must not neglect an important detail that constitutes a subtle distinction between the hypothetical logical momentum (and its hypothetical proposition) and traditional logic. The positive act of the judgment is in the positing of the connection according to *modus ponens*. If, in the overall structure of a judgment, we have a hypothetical proposition in which the momentum of Quality in its composition is the negative logical momentum, this corresponds to a statement in traditional logic of “*A* does-not-imply *C*.” *The real ground for making such a judgment* is best described as “discovering a mistake” – i.e. encountering in experience an appearance where an *anticipation* of *C* from an actual perception of *A*, based on a prior judgment *AyC*, is *thwarted* by actual events *as they turn out this time*. But this amounts *overall* to a violation of the criterion of truth expressed in *modus tollens*, and so we can see that the idea of the hypothetical logical momentum can not be the same as the ideas of *modus ponens* or *modus tollens*. These latter two ideas pertain to truth criteria, not products of judgments. It is this distinction that leads us to say, “the form of connection in hypothetical propositions is twofold” [KANT8: 112 (9: 106)] rather than (as it has sometimes been translated), “the form of connection in hypothetical propositions is of two kinds.” We call connection according to the criterion of the *modus tollens* a ‘rescinding’ connection because this criterion is the ground for discovering an error of Relation in an empirical judgment *and adding conditions to* (‘rescinding’) this prior judgment *AyC* by altering *y* (“changing one’s mind”).

The Category of Causality and Dependency from the Logical Perspective

As was the case for categorical judgments, the hypothetical judgment requires an *a priori* rule that governs its construction and ensures the possibility of its representation in an intuition. The

Consequenz is not given in the antecedent or consequent propositions and must therefore be supplied for the hypothetical logical momentum. The principle of Rational Physics by which we understand the possibility for the hypothetical momentum is the Second Analogy of Experience: *Everything that happens (begins to be) presupposes something which it follows in accordance with a rule.*

The data of the senses obtained through receptivity does not even present us with the immediate knowledge of determined objects, much less with any mark or indication that a cause and effect relationship necessarily exists between perceptions at one moment of time and those at another. Yet how else are we to view the idea of antecedent and consequent in a hypothetical proposition? Phrasing this question another way, what is the ground from which one categorical proposition comes to be thought as “antecedent”, another as “consequent”, and that there should be a hypothetical proposition of connection made between them?

We have discussed this question before when we introduced the idea of ‘magnitude’ and the pure intuition of time. In the synthesis of apprehension, the perception of an appearance at one moment in time evolves into another perception in a subsequent intuition marked at a subsequent moment in time, and we are unable to draw a crisp boundary that delimits when the first appearance ‘ends’ and the second ‘begins.’ It is in this that we found the correspondent in the Critical Philosophy of James’ idea of ‘transitive parts’ and ‘substantive parts’ in the ‘stream of thought’ – with, of course, the crucial difference that in the Critical Philosophy we have what we should call “the stream of apprehension” rather than “the stream of thought.”

It is, however, one thing to talk about a ‘stream of apprehension’ and something else altogether to conceptualize the manifold of concepts as being connected in the form of hypothetical propositions. The latter is objectively valid only under the presupposition of an *a priori* rule for the construction of the manifold where such a connection is mandated as a *law of thinking*. **The category of causality and dependency is the notion of the scheme of determining the objective connection as antecedent and consequent in a hypothetical proposition.**

Various philosophers, and occasionally some scientists, have taken issue with Kant’s answer to the question of how we come to view Nature in cause-and-effect terms. If the category of causality and dependency is the counterpart in determining judgment to the stream of apprehension in intuition, would it not be the case that such a law would promote the famous “C followed A, therefore A caused C” fallacy to such an uncompromising extent that rational thinking as we know it would be impossible? It is true enough that human beings fall prey to this fallacy, and the thought processes of children illustrate this quite vividly. Piaget has reported numerous conversations with young children such as the following example:

GAVA (8½): "What is the sky made of?" - *It's a sort of cloud that comes.* - How? - *The steam from the boats goes up to the sky and then it makes a great blue streak* [PIAG24: 289].

In time we come to correct at least the most egregious of these fallacies and to be less prone to make judgments of this sort. Thus, it is claimed, there can be no *a priori* ‘law’ of cause and effect because facts such as these flatly contradict this theory. Piaget seems to be of this view:

Above all . . . the idea of reality undergoes with age a progressive transformation. This means that the categories of child thought are capable of evolution. Now, apriorism presupposes fixity, whereas every change in the actual structure of thought seems to show that this structure is plastic to the action of external things, whether this action be immediate or remote [PIAG8: 257-258].

If “apriorism” really did have to assume “fixity,” there would be merit to arguments such as this. What, though, is meant by ‘fixity’? Does it mean that a proposition, once made, is not subject to later modification? We have seen above, in the twofold character of the hypothetical logical momentum, that this sort of ‘fixity’ is not only not presumed but is actually denied by the *modus tollens*. The category can be both *a priori* and ‘fixed’ *without requiring that the manifold of concepts be fixed*. The evolution of this manifold with experience is, indeed, not only permitted but *required* by the categories of understanding. We have already alluded to the evolution of determined object concepts (categorical propositions) when we discussed aggregation (the combination of predicate concepts with the subject concept). Now we also see that the category of causality and dependency does indeed legislate precisely this ‘plastic’ character of cognition that empirical psychology has so capably demonstrated as a fact. In relationship to objects, the categories of understanding ground the *Dasein* of the manifold of concepts but this does not ‘fix for all time’ the *Existenz* of the phenomena in appearance.

As for the notion of the scheme for determining the hypothetical proposition, the category of causality and dependency is wholly unconcerned with objects as things-in-themselves and poses no rule for the construction of hypothetical propositions that is based on ‘what’ the objects’ appearances might be. The ordering principle – i.e. the determination of which categorical proposition is to be the antecedent and which is to be the consequent – is the condition of representing the concept *in intuition*, and the sole requirement here is its possibility of being represented in subjective time through the second transcendental schema of Relation (the schema of the *modus* of succession in time). It does not guarantee that cause-and-effect relationships will turn out to be objectively correct throughout the march of experience. It does not need to offer such a guarantee because, through the connection to the Modality of logical perfection in determining judgment by the *modus tollens*, such a guarantee is unnecessary. Truth is the congruence of the cognition with the object, and the causal connection of the category need not stand in the face of empirical falsification. The category proposes no object as ‘cause’ but merely that an objective connection of succession in time exists. It rules for the *Dasein* of a ‘cause’ but does not determine its *Existenz* as an Object.

Disjunctive Propositions and the Disjunctive Logical Momentum

In some ways the disjunctive judgment is the most peculiar of the logical momenta of Relation. In the other two momenta, the form of the judgment always involves the determination of one concept or proposition in terms of another one, this other one being thought either in terms of a characteristic (for a categorical proposition) or of a condition (for a hypothetical proposition). The disjunctive proposition, on the other hand, is a proposition concerning division of the sphere of a concept rather than a Relation coordinating higher concepts in one which stands immediately under them.

A judgment is disjunctive if the parts of the sphere of a given concept determine one another in the whole or to a whole as complements (*complementa*) [KANT8a: 602 (9: 106)].

In both the categorical and the hypothetical proposition, the connection of *nexus* is “additive”, so to speak, in the sense that what is determined enlarges or limits the sphere of a concept. In the disjunctive proposition the determination is transitive in the sense that we are dealing with a reciprocal *nexus* of concepts that all lie within the sphere of the same higher concept.

Propositiones disjunctivae, as to their matter, are two or more judgments¹ which are always regarded in opposition, and of which I represent to myself that when they are taken together all these judgments make up all that can be said of the judgment². If one of them is true, then all the others are false. If all are false except one, then the one must be true . . . The matter of disjunctive judgments are thus various judgments, these however being regarded in opposition, so that all the judgments taken all together make up the whole judgment³. One no doubt sees that they merely make up a logical division [KANT8a: 372-373 (24: 932-933)].

We must clearly distinguish *two* activities that are carried out in the making of a disjunctive proposition. The first of these is the logical division of the sphere of a concept H into mutually exclusive subspheres. A concept P brings into its sphere that of another concept S in a categorical proposition, e.g., S_1xP , S_2xP , etc. In a hypothetical proposition, the antecedent A brings the consequent C into its sphere, e.g., AyC_1 , AyC_2 , etc. The consequents, in turn, can be used as antecedents of still lower consequents (thus making A a remote mark of these still lower concepts), and of these concepts some or all of their spheres are contained in the sphere of A . The composition of this sphere may, of course, have for its matter a mixture of different relational types of propositions (including disjunctive propositions). The first act in the disjunctive logical momentum is the division of the sphere of a concept H , which we can symbolically represent as $\sigma_H = \{\sigma_1, \sigma_2, \dots, \sigma_n\}$ for a sphere divided into n parts. This logical division is mutually exclusive

¹ i.e., two or more judgments regarded as propositions.

² i.e., of the higher concept undergoing disjunction in determining judgment.

³ i.e., they constitute the entire sphere of the higher concept.

(no subsphere contains part of another subsphere in the disjunction), and it is complete (there is nothing in the sphere of H that is not also in exactly one of the subspheres).

But this is not all. The division of the sphere of H is not simply a matter of, so to speak, drawing arbitrary boundaries that partition the sphere σ_H . In making the logical division, the disjunctive proposition does so in such a way that if some concept, say that of σ_1 , is asserted in thinking as being true at a particular moment in time, then the propositions of the *other* subspheres are held-to-be-not-true *at this same moment*. Now, what is the real significance of this? Let us recall that “true” means a cognition is held-to-be-congruent with the object of the cognition. Therefore, when we say the concept of σ_1 is true, we can mean nothing other than that σ_1 is *being used* to form some part of an intuition via the synthesis of imaginative reproduction and provides *materia ex qua* for the acts of understanding (the *Verstandes-Actus*). Since σ_1 is matter in a disjunctive proposition, this also means that *at the same moment in subjective time*, the other subspheres in the disjunction are *excluded* from taking part in the representation of the cognition. (They are “not true” of the object of the cognition).

Thus, while the matters in a disjunctive judgment are all ‘given’ (i.e., we must have a representation of the sphere of H before this sphere can be divided), something new is added to the *nexus* of the manifold of concepts by the disjunctive logical momentum. This new something is *reciprocal determination* among the subspheres of the sphere of H . When we determine that one subsphere is ‘true’ in a cognition, we simultaneously determine that the other subspheres are not-true (i.e., false) *for this same cognition*. As a higher concept, H is a mark (either immediate or remote) of every concept in its sphere. Consequently, concept H is a concept ‘in’ the cognition but not everything that stands under H is made part of this cognition.

If we take away all references to objects and cognitions, it is easy to see in the disjunctive logical momentum the disjunction function of formal logic.⁴ However, in transcendental logic we are not permitted to throw away the references to objects and cognitions because if we do so then the statement “the concept of σ_1 is true” becomes an empty statement, devoid of any real meaning, and the definition of the disjunctive logical momentum therefore also loses all meaning.

The effect of the disjunctive judgment is to coordinate *under the higher concept H* all the concepts in its sphere insofar as these concepts are organized into distinct subspheres of H . It is important that we do not confuse this idea of the coordination of the subspheres with the idea of the coordinate marks in the composition of aggregation of a concept S . In the composition of aggregation, the coordinate concepts are all higher concepts predicated upon the lower concept S which, taken collectively, determine S . S in this sense is a *conjunction* of the higher coordinate marks. In a disjunctive proposition, the representations of the subspheres of H can all lay equal

⁴ i.e., “if p then not- q else if not- p then q .” In formal logic, $p \vee q$ implies $p \wedge q$ and $\bar{p} \wedge \bar{q}$ both have a “truth value” of “false.” In Kantian terms, H is the concept of the entire quoted sentence.

claim to being *possible representations* grounded under *H*. This is not to say that one and only one of these subspheres is ‘the true representation of which *H* is a mark’ because different subspheres will be true for different appearances. A concept is the representation of a general mark of *many different* representations. The disjunctive proposition merely allows us to sort out and distinguish different appearances as ‘being different’ despite the fact that *H* is a common mark of cognition for all of them.

The Category of Community from the Logical Perspective

The definition of the disjunctive proposition and our description of the disjunctive logical momentum given above poses a major question: Under what criterion or rule are we to perform the logical division of the sphere of a concept *H*? The resulting division must be one for which the representation of each of the subspheres, one at a time, is possible in intuition, and the division must be carried out in such a manner that these representations are each *objectively valid* (i.e., are predicable of a substantial concept of an appearance after the synthesis of re-cognition in a concept). For this we require, first, a principle of objective validity and, second, an *a priori* rule which stands under this principle and in which the disjunctive proposition is grounded.

The principle of Rational Physics for this judgment is the Third Analogy of Experience: *All substances, insofar as they are coexistent, stand in thorough-going community (i.e., interaction with each other)*. Substances, i.e. notions of objects, are said to coexist when the singular representation in intuition at a particular moment in time contains the conceptual *materia* of these objects in the intuition. Now, we cannot rationally think of an intuition in this way unless we regard these objects as being ‘separate’ or ‘different’ objects. In *Critique of Pure Reason* we find:

The congruence of a single category, namely that of community . . . , with the form of a disjunctive judgment which corresponds to it in the table of logical functions is not so conspicuous as in the other cases.

In order to be assured of this congruence one must note: that in all disjunctive judgments the sphere (the multitude of everything that is contained under it) is represented as a whole divided into parts (the subordinated concepts), and, since one can not be contained under the others, they are thought as *coordinated*, not *subordinated*, with one another so that they do not determine one another *unilaterally*, as in a series, but rather *reciprocally*, as in an *aggregate* (if one member of the division shall have been supposed, all the rest would be excluded, and vice versa).

Now a similar connection is thought in *an entirety of things*, in that one is not *subordinated*, as effect of another as cause of its *Dasein*, but is rather conjointly and reciprocally *coordinated* with the other as cause in regard to its determination . . . which is an entirely different manner of connection from that which is to be found in the mere relationship of cause to effect (ground to consequence), in which the consequence does not in return reciprocally determine the ground and therefore does not make up a whole with the latter . . . The same procedure of understanding is adhered to when it represents the sphere of a divided concept as well as when it thinks of a thing as divisible; and just as in the first case the members of the division exclude each other and yet are combined in one sphere, so stand likewise the parts of the latter as ones to which *Existenz* (as substances) pertains to each exclusively of the others, ere yet are combined in one whole [KANT1a:

215-216 (B: 111-113)].

Thus, the real ground for the division of the sphere of a concept lies with the representation of *coordinated substances* and, therefore, with the categorical propositions contained in the sphere under the category of substance and accident. The concepts in the sphere of every concept are connected in propositions, and every proposition either is or contains at least one categorical proposition.⁵ Every categorical proposition is a concept that results from a judgment made under the rule of the category of substance and accident. It follows that the division of the whole sphere into disjoint subspheres nucleates around categorical propositions and, in particular, around the subject concept in such a proposition as that concept is subsumed under a notion of substance by the category of substance and accident.

There is a further consideration that enters into this description. Not all of the propositions in the sphere are necessarily categorical and, in particular, some may be hypothetical. Now a hypothetical proposition contains *two* categorical propositions, and each of these contains the representation of a substance. Clearly these two substances cannot be divided into distinct subspheres because of the *nexus* between them; the two categorical propositions are connected *as a series* with the consequent necessarily *subordinated to* the antecedent. But the division in the disjunctive proposition is a *coordinating*, not a subordinating, division. Therefore the division of the sphere cannot “draw a boundary” *between* the antecedent and consequent in a hypothetical proposition. Which, then, of the two substances is the *basis* of a possible division of the sphere?

To see the answer to this question, we need merely look at the statement of the Third Analogy. The representations of substances in the intuition coexist *at the same moment in time*. Now, the antecedent in a hypothetical proposition is, by definition, a representation from a *prior* moment in time.⁶ It is the object of the consequent that is present ‘at this moment’ in the intuition. *It is therefore the substance of the subject term in the consequent around which a division of the sphere can nucleate and not that of the subject term in the antecedent.*

There are additional complications to this picture we can, without much effort, put forth. The consequent of every hypothetical judgment contains a categorical proposition. However, this does not mean that in the march of experience it is impossible for the understanding of the consequent to undergo further refinement. Suppose, for instance, that the subject concept in the consequent *C* of a hypothetical proposition comes to be represented as a mark predicated of other (‘new’)

⁵ If a proposition contained in the sphere is a disjunctive proposition, it *must* have more than one categorical proposition under it.

⁶ The category of causality and dependency is not a future-directed rule of construction. It is a ‘backward-looking’ notion. This is a consequence of the statement of the Second Analogy. We perceive the *effect* and necessarily presume the *Dasein* of a cause. But ‘the cause’ is not what is represented by the antecedent in a hypothetical proposition. Rather, we say the antecedent ‘contains the cause’ *in* its concept. If we were to equate the antecedent to the cause itself, we would be saying we represent the *Existenz*, not the *Dasein*, of the cause, and to do so is an error under Rational Physics.

subject concepts. Here we have the possibility for a division of the sphere with *C* as a mark of other categorical propositions. (We can likewise suppose that the original consequent *C* becomes the antecedent for other lower consequents). Suppose *C* is a member of a disjunction of a concept *H*. Can the higher concept *H* have *its* sphere further subdivided on the basis of a disjunction of *C*? No. In order to so divide the sphere of the *H* concept, we would have to split off some member of *C* into one of these new sub-spheres of *H*, and this is impossible because *C* is a mark of *all* its lower propositions. Put another way, the concept *C* cannot be split off and placed in a sphere of *H* that excludes some of *C*'s own subspheres. We can make a disjunction on the consequent *C*, but the entire sphere of *C* must remain in *one* subsphere of *H*. We conclude from this that the rule for the making of disjunctive propositions speaks not only to what is permissible in a disjunctive judgment, but also to what is *not* permissible. Not every 'topologically possible' disjunction – that is, a disjunction that is symbolically possible from a geometrical view – is *objectively* possible.

This example illustrates the potential for complexity in the determined manifold of concepts. Yet for all the enormous complexity latent in the faculty of understanding (represented in the combination of concepts by determining judgment), the act of making a disjunctive judgment must still fall under a *single a priori* rule governing the scheme of such judgments.⁷ ***The category of community is the notion of the scheme of determining the objective form of a disjunctive proposition.*** It is the notion of a rule under which are subsumed all acts of disjunctive judgment, insofar as the form of disjunctive propositions is concerned, and of the reciprocity that must necessarily exist among the elements of this disjunction. Like all the categories, it is the notion necessary for the possibility of its corresponding logical momentum in the making of a determinant judgment. Further discussion of this I leave to a treatise on the Logic of Meanings.

§ 5.4 The Categories of Modality from the Logical Perspective

The idea of 'modality' or 'modal judgments' in logic has a long and often controversial history. The idea was first introduced with rigor by Aristotle, although there was a kind of modal logic present in pre-Aristotelian dialectics, both from Plato and from the Megarian dialectical school (which later evolved into the Stoic philosophy).

We may, as is customary, look at Aristotle's modal logic in two parts: modal propositions and modal deductions. The theory of the former is found in *On Interpretation* [ARIS2] and, while Aristotle had a great deal to say about it, the flavor of his idea can be illustrated by its use in categorical propositions. For the so-called 'universal affirmative' form of proposition, the three modalities can be expressed in the following way:

Problematic: *S* may be *P*;

⁷ There is only one disjunctive logical momentum, not several.

Assertoric: S is P ;
 Apodictic: S must be P .

The labels problematic, assertoric, and apodictic were added later as logic was developed by other contributors. Aristotle's theory, of course, contains much more than the three simple modifications of the copula shown above. He developed modalities for all of the various forms of propositions, e.g. the particular negative form, etc.

What is important for us to keep in mind in regard to Aristotle's theory, and in regard to the tradition of logic that developed from it, is that these ideas of modality are ideas that are held to be statements concerning "things themselves" and "reality itself." For instance, the problematic proposition could be phrased as "it is not necessary that S is not- P ." For a general proposition p , the modality 'possibly p ' is defined by Aristotle as meaning " p is neither impossible nor necessary."

Aristotle also applied the idea of modality to deductions – the so-called 'modal syllogisms' – in his *Prior Analytics* [ARIS3]. This attempt, it is generally held, was not very successful and there are many who regard it as the least splendid example of Aristotle's thought. There are even those who speculate that Aristotle did little more than to pose the problem to his students and that what we find in *Prior Analytics* on modal deductions was actually added to the work, after it was substantially completed, from the results of his students' labors. The theory of modal syllogisms is generally regarded as having severe problems, and we do know that Aristotle's immediate successor at his school lost no time in amending this part of Aristotle's theory.

Modal logic remained part of the "corpus of logic" in both the Stoic system and in the "Aristotelian system" as it passed through the hands of the Neo-Platonists to the medieval Scholastics. "Scholastic logic" (also called "the logic of the schools") developed from the 12th through the 15th centuries in the hands of such notables as Peter Abelard (1079-1142), Albert the Great (1193-1280), William of Shyreswood (*ob.* 1249), Peter of Spain (*ob.* 1277), Robert Kilwardby (*ob.* 1279), Albert of Saxony (1316-1390), Walter Burleigh (*ob.* 1343), William of Ockham (*ob.* 1349 to 1350), Ralph Strode (*c.* 1370), Paul of Venice (*ob.* 1429), and Peter Tarteret (*ob.* 1480-1490) [BOCH: 149]. By the end of the 15th century the Scholastics' development of logic was essentially completed and no important developments in Scholastic logic occurred after this point.

The Renaissance gave birth to many changes in Western thought and society, and logic was not immune to its influence. Beginning in the 16th century, under pressure from the humanists (particularly Peter Ramus, 1515-1572) and from the first stirrings of modern science, logic underwent what we might loosely describe as a "change in its philosophy" from how it had been viewed by Scholasticism. The end result of this movement was a "new way of logic" that held the

stage for the next four hundred years until the development of mathematical logic in the 19th and 20th centuries. This logic is the logic most people today mean by the name “traditional logic” (or, as some moderns prefer to call it, “transitional logic,” since it lies between the Scholastics and mathematical logic). In this logic, the modal proposition disappeared altogether, not to re-emerge until around 1918.

We can identify two main branches in the development of this transitional logic. The first is the British tradition from Bacon (*Novum Organum*, 1620) to Hobbes (*De Corpore*, also known as *Computatio sive Logica*, 1655) and from there to Locke and Hume (who did not actually make any new contributions to logic *per se* but who were highly influential in casting down Scholastic logic). The line along the British tradition is largely responsible for the element of ‘psychologism’ that many moderns regard as one of the most distasteful features of “the traditional way of looking at logic.”

As important as British thought was during this time, of far greater importance, in terms of the development of a rigorous and *formal* logic, was the so-called “logic of Port Royal.” The Port Royal logic is primarily due to the work of Antoine Arnauld and Pierre Nicole in 1662. We are nearly correct in most essentials if we view the Port Royal logic and transitional logic as one and the same. The Port Royal logic borrowed a great many things from Scholastic logic, but its most significant departure comes from what it dropped altogether from Scholastic logic. The most important doctrines that were expunged by the Port Royal logic were: the doctrine of suppositions, the doctrine of consequences, the doctrine of antinomies, and, lastly, the doctrine of modal logic. Oftentimes when Kant refers to and criticizes “the logicians,” he seems to be referring to those who adhere to the logic of Port Royal.

It should be clear enough by now that the idea of Modality plays a central role in the Critical Philosophy. Those moderns who insist that Kant’s system was constructed on the foundations of an “impoverished traditional logic” (whether by this they mean the logic of Port Royal, Scholastic logic, or some other arguably “Aristotelian logic”) can, and apparently do, reconcile the absence of modal logic in the Port Royal logic with its presence in the Critical Philosophy by regarding the idea of modality as “belonging to metaphysics” rather than “logic proper.” Kant himself, in his writings, can be so interpreted if one wishes. But this does raise the question of what view of logic Kant actually did hold.

It is here in this context that we must mention Leibniz. Leibniz is a pivotal figure in many ways, but here we will confine ourselves to his contributions to logic. By all modern accounts, Leibniz was a great logician – perhaps the greatest of the 17th and early 18th century (see [KNEA: 320-345]). His works on “algebraic” logic, which actually remained unpublished for two centuries, have been cited by some modern historians of logic as ground for granting Leibniz the title of “father of mathematical logic” (although I think this is too generous in view of the fact

that the development of mathematical logic was well underway by the time Leibniz' work became widely known). However one chooses to view Leibniz in relation to mathematical logic, of much more direct impact was Leibniz' philosophy on the great German *Aufklärung* or "Enlightenment" of the 18th century.

Against the tide of the logic of Port Royal, Leibniz gave a place to modal reasoning through his idea of 'contingent propositions' (see [WIEN: 94-95]). In Leibniz we find more than a little sympathy to Scholasticism, much of which was systematized in what can be described as a 'mathematical or mathematico-syllogistic' formalism by Christian Wolff [SCHW: 256-261]. Wolff is a central figure in 18th century German philosophy, principally due to his systematic organization of Leibniz' work into a formal if dogmatic philosophy. It is in this Leibniz-Wolff tradition that the idea of modal logic survives in 18th century philosophy, albeit in altered form from where the Scholastics had left it. The textbook required for Kant's course in logic, Meier's *Auszug aus der Vernunftlehre*, was a traditional Wolffian text. Kant himself was a product of the 'Wolffian school' of philosophy and was, in his pre-Critical period, a member in good standing of this rationalist corps of philosophers who dominated German philosophy in the *Aufklärung*.

Thus from the figure of Leibniz we see two diverging lines that logic would take. We will come back to the idea of Modality in the Critical Philosophy in a moment. Before doing so, let us briefly take a look at the standing modal logic has in modern mathematical logic.

If it is possible to tag the beginning of mathematical logic with a date, there is ample reason to fix this date at 1847 in the work of De Morgan and the work of Boole. Certainly this marks the beginning of the end for the 'transitional logic' that was dominant for two centuries. From the logic of Port Royal through most of mathematical logic, modal logic is given no place at the table. Still, it *is* possible to make a place, still within the confining prejudices of logical positivism, for a modal logic. This was done briefly in 1877-1878 by Hugh McColl and again later, in 1918, by C.I. Lewis. Let us take a brief look at Lewis' modal logic. To the usual valid 'truth values' of 'true' and 'false' recognized by all mathematical logic, Lewis adds a third 'value,' namely 'impossible.' By doing so, and combining this trivalent range of truth-values with logical negation, Lewis comes up with a total of five possible 'truth implications.' Letting p stand for any proposition, these are:

- 1) p ("p is true")
- 2) $\neg p$ ("p is false")
- 3) $\sim p$ ("p is impossible")
- 4) $\sim\sim p$ ("it is false that p is impossible" = "p is possible")
- 5) $\sim\sim\sim p$ ("it is impossible that p is false" = "p is necessary").

Lewis wished to use his modal logic to solve certain problems involving what is known as "strict implication," but his efforts were less than successful. It has been forcefully argued that

“alternative logics” (i.e. modal logic) can add *nothing* to logic that two-valued logic does not already contain [KNEA: 548-575].

What does one do with a mathematical modal logic if we have one? It would seem, judging from the main-stream of mathematical logic literature, that the answer is “not much.” Fuzzy logic is multi-valued (that is, does not use bivalent truth values), but it is not a modal logic. Computer science has recently begun to make some use of various modal logics, particularly to address some important issues involving computer and network security, but as of yet it is difficult to call the logic used here “well-developed.”

The Logical Momenta of Modality

We now turn to the idea of Modality in the logical perspective of the categories, beginning with the logical momenta of Modality. Curiously, given the importance this idea has in Kant’s system, Kant spent very little effort in explaining it in either his lectures on logic or in his lectures on metaphysics. Most of what Kant has to say on the subject he says in *Critique of Pure Reason*, and even here his treatment could not be called “in depth.”

We could amuse ourselves for hours trying to guess why this topic should have received so little depth of coverage by Kant. One supposition is that perhaps Kant felt modality was not a proper part of logic; modern logicians often look upon this supposition favorably, although it does not explain the lack of in-depth treatment in Kant’s metaphysics lectures. On the other hand, it might be that Kant simply regarded the topic as too easy to warrant spending very much time upon it. If this is so, it is a judgment that at least some of Kant’s readers do not share.

Be that as it may, *our* first consideration must be to contrast the ‘traditional’ views of modal logic against how we must view it under the Copernican hypothesis. Recall that throughout almost all the history of logic and philosophy in general – with the exception of Kant – modal propositions have been propositions made regarding *things*. Examples include such pronouncements as: it is impossible that God does not exist; it is necessary that *everything* that comes to be has a cause of its existence; of all possible worlds, God necessarily made *this* world the best that is possible; it is necessary that the laws of quantum mechanics, when applied to large-scale phenomena, must produce classical physics as an approximation; the infinity of the set of real numbers is necessarily greater than the infinity of the set of integers; etc.

Now let us look at the idea of modal propositions from the viewpoint of the Copernican hypothesis. When applied to the idea of combinations in judgment, Modality is the matter of the form of judgments of the *nexus* of the manifold of concepts. What consequences does this hold for the representations of judgment?

In the first place, determinant judgments with respect to *nexus* are connections of concepts insofar as these connections are regarded as being required for unity of consciousness in the manifold of concepts. In such an act of judgment the logical momenta are *a priori* momenta and, as such, provide for the cognitive structure for experience in general. But to call experience a “structure” every “experience” in the particular, as it pertains to some specific object of experience, has its representation constrained to fit into a unity of the whole of experience. The logical momenta of Relation see to the formal connection of concepts one to another, i.e. to the form of the manifold, but the momenta of Modality pertain to the matter of the unity of the manifold in its connection to unity of consciousness. This matter ‘in’ a *connection* does not have to do with the objects represented (the matter of which is the matter of *composition*). It has to do with the connection of the manifold of concepts as an organized part of, as Kant put it, “the faculty of knowledge” (*Erkenntnißvermögen*).

To put it more plainly, the modal propositions of determinant judgments are *judgments of judgments*. They add nothing to the content of the concept of the object *qua* object but concern only *the manner in which the concept is used in thinking*.

The modality of judgments is a quite special function of them, which has the distinction that it contributes nothing to the content of the judgment (for besides magnitude, quality, and relationship there is nothing more that makes up the content of a judgment), but rather only has to do with the value of the copula in reference to thinking in general. Problematic judgments are those where one accepts the assent or denial as merely possible (arbitrary); assertoric [judgments] are those regarded as actual (true); apodictic [judgments] are those one sees as necessary [KANT1a: 209 (B: 99-100)].

To get a feel for the Modality of a proposition, let’s use the categorical proposition SxP as an example. Kant described the problematic logical momentum as forming a proposition in which the proposition is “seen as arbitrary.” Now this clearly does not mean that *S* and *P* somehow collided by chance and fused together in a random judgment. What does it mean for a judgment to be “arbitrary”? Put simply, the problematic judgment is one in which the proposition serves (so to speak) “for purposes of argument.”

Possibility, actuality, and necessity are of course logical but not metaphysical (real) predicates, i.e. determinations. We know through them not object-matters¹ but the relationship of our concepts to the capacity of the mind to posit and rescind [AK18: 125-126 (§5228)].

The problematic logical momentum underlies the ability of the power of spontaneity to exhibit *creativity* – which we can describe here as the ability to ‘put together’ through thinking the concepts of things we have never experienced, and to *know* that these concepts are not handed to us by ‘external experience’ but, rather, are our own creations. We can get the flavor of the

¹ *Sachen*.

problematic propositions by thinking of them as:

Problematic categorical: *What if SxP ?*
 Problematic hypothetical: *What if AyC ?*
 Problematic disjunctive: *What if $\sigma_H = \{\sigma_1, \sigma_2\}$?*

In using the “what if” phrase to describe these propositions, I do not mean to imply that the thinking process literally frames the problematic proposition as a linguistic question that “the mind asks itself.” Rather, I use the phrase to illustrate the tentative, uncertain and *intellective* regard with which the problematic modal connection is held in empirical consciousness.

The faculty of imagining [*facultas imaginandi*] is the capacity for intuition of objects of past time, the faculty of anticipating [*facultas praevidenti*] is the capacity for intuition of objects of future time. The capacity for intuition insofar as it is not entirely bound to time is called the fictive faculty [*facultas fingendi; Dichtungsvermögen*]. All three capacities have their laws. The first law is the law of the association of ideas [*lex associationis idearum*]. The law of the power of imagination as a capacity for seeing in advance is the law of expectation of similar occasions [*lex expectationis casuum similium*]. The law of the fictive capacity is the law of the compatibility of ideas [*lex sociabilitatis idearum*]. It [an idea] is to be conceived according to the law of compatibility, it is to be reproduced according to the law of the association of ideas [KANT19: 345 (28: 585)].

While the problematic momentum of Modality imputes no commitment to hold a problematic modal proposition as true or certain, the situation is otherwise for the other modal momenta. In experience we are each in possession of a great number of cognitions that we hold, in varying degrees, to be *factual*, i.e., true. Regardless of the degree to which we hold these ‘facts’ to be true², what all concepts of such cognitions share in common with each other is that *each is regarded as being actually true*. The judging of a such a proposition belongs to the assertoric logical momentum. We can illustrate the nature of the connection of assertoric propositions to empirical consciousness as:

Assertoric categorical: *I think SxP ;*
 Assertoric hypothetical: *I think AyC ;*
 Assertoric disjunctive: *I think $\sigma_H = \{\sigma_1, \sigma_2\}$.*

Again, the phrase “I think” is not to be taken too literally (as if the mind were “talking to itself”). The phrase is used here merely to denote that the proposition is held to be a fact.

Finally, there are some cognitions which, although not presented to us directly in sensual experience, we nonetheless hold to be true *a priori*. For instance, I hold it to be a fact that, so long

² The degree of holding-to-be-true we attach to a concept reflects the magnitude of certainty we impute to that concept. The *degree* of holding-to-be-true does not belong to determinant judgments but, rather, to logical perfection in the Modality of the state of understanding.

as both are alive, a son will never be older than his mother. I can imagine – i.e. posit problematically – that a miracle might occur tonight such that tomorrow a son will wake up and find that he has suddenly become older than his mother, but I don't think this could really happen. However, since tomorrow isn't here yet, my *conviction* of the truth that the son will never be older than his mother is a proposition I hold to be true *a priori*. Propositions such as this – propositions that carry this *a priori* conviction of truth – are called *apodictic* propositions. The making of such a proposition belongs to the apodictic logical momentum. Using the same sort of figurative manner of expression as we have used for the first two cases, we can illustrate the apodictic proposition as:

Apodictic categorical: *I am certain SxP*;
 Apodictic hypothetical: *I am certain AyC*;
 Apodictic disjunctive: *I am certain $\sigma_H = \{\sigma_1, \sigma_2\}$* .

Note that these propositions are phrased “I am certain,” not “it is certain.” The former is a proper modal judgment. The latter is a transferal to the object and, by this transferal, becomes transcendent (not objectively valid).

None of these nine types of propositions are about the object of the proposition. They are, in every case, judgments of the *manner in which* one regards the proposition as true. This manner of holding-to-be-true is the *matter of nexus* in combinations of judgment, and the ‘nature’ of this matter determines how one uses these propositions *in other judgments*. Suppose someone were to challenge me to prove my claim that the son will never be older than his mother. My reaction, in all likelihood, would be to judge *this other person* to be either an idiot or a troublemaker or an over-learned fool. After all, “isn't the proposition self evident (apodictic)?” I can likewise assert, “*Hamlet* is a great play,” even if Socrates himself were to question me on what *Hamlet qua Hamlet* ‘is’ or why ‘it’ is ‘great’ because I judge that “I know what I’m talking about.”

The Categories of Modality from the Logical Perspective

The real possibility of the three logical momenta of Modality necessarily presupposes a rational ground in *a priori* rules that govern these functions of thinking. These are, of course, what we mean by the categories of Modality. For the regulative principles of these rules we have the Postulates of Empirical Thinking in General from the metaphysics proper of Rational Physics. The categories of possibility-impossibility, *Dasein-Nichtsein*, and necessity-contingency each take for their respective principle of Rational Physics one of the general postulates:

1. What agrees with the formal conditions of experience (in accordance with intuition

- and concepts) is possible [Postulate of Possibility];
2. What coheres with the material conditions of experience (sensation) is actual [Postulate of Actuality];
 3. That whose context with the actual is determined in accordance with the general conditions of experience is necessary (exists) [Postulate of Necessity].

Now these acroams are not themselves what we mean by the categories, although they perhaps come closer to giving us a direct exposition of their respective categories than is the case in the other acroams of Rational Physics. When we first introduced the General Postulates, it was commented that these postulates are essentially definitions (in the *Realerklärung* sense of that word) of the terms possible, actual, and necessary (and, of course, their opposites). To understand the categories of Modality, however, we cannot merely stop here but, instead, must present an exhibition of the rules that illustrates the *real use* of the general postulates from the logical perspective.

A. The Category of Possibility-Impossibility: I think it is not unlikely that, of the three logical momenta of Modality, the problematic logical momentum may strike most readers with the most pronounced feeling of mystery and, therefore, that we will find its category the most demanding of explanation. There are two reasons why I think this. First, the other two logical momenta are likely to be ones we “feel the most comfortable with” if for no other reason than because we have a long tradition of logic with which these momenta obviously bear a close kinship and therefore we are somewhat habituated to a certain way of thinking about the assertoric and apodictic propositions in logic. Second, the problematic momentum, as it was just described above, is the logical momentum in which we find the logical roots of the phenomenon of creativity, and this ‘fictive faculty’ is something that has always had the reputation of being cloaked in the deepest shades of the mysterious. The category of possibility-impossibility therefore is the most demanding of a clear explanation.

The Postulate of Possibility merely requires that the representation of the concept and its intuition agree with the formal conditions of experience. This does not tell us very much, though, because *every* cognition is bound by this requirement. It is not unique to problematic propositions but can be regarded simply as a *condition* of conceptualization in general. What is it, then, implied in this principle that gives us the *real condition* that makes possible the fictive representation of concepts for objects that have not been presented to us in actual experience?

Perhaps the most unique characteristic of a problematic proposition is that such a proposition makes a representation through the power of spontaneity without the need for calling *immediately* upon the power of receptivity. What I mean by this is that the appearance of the object in intuition is one that has not been ‘presented’ to us through receptivity but, instead, has been sewed

together, so to speak, from pieces and parts of previous empirical cognitions. Furthermore, the combination of these ‘pieces and parts’ is not one that is dictated by the affections of outer sense but, rather, is produced through imagination acting in concert with the judicial processes of understanding. This is what gives the problematic proposition its ‘speculative’ character.

Because the synthesis of the conceptual matter of a problematic proposition in an intuition is carried out without the immediate contribution of receptivity, it does not seem appropriate to call this synthesis a synthesis of apprehension; a better name for it is the synthesis of *comprehension*, because while the formal process seems the same as that of apprehension, the *materia ex qua* of this synthesis has its source from the manifold of concepts rather than the affection of outer sense. Here, then, we have a glimpse of one of the properties of the category in question, namely that this category is a rule that determines what conceptual matter is to be applied to the synthesis of comprehension. In apprehension, receptivity “has a say” in what this matter is to be and thereby places a limitation on the synthesis. In comprehension, though, the synthesis is limited by the category of possibility-impossibility as an *a priori* rule of thinking in general.

Now since the synthesis of comprehension is a spontaneous mental activity, what concepts are to ‘feed’ the synthesis is undetermined prior to the act. Therefore the notion of the category is one by which the determining factor for the *materia ex qua* of comprehension is ‘presented’ – i.e. we think of this category as a *rule* for determining that which is determinable by the concepts in the manifold of concepts. But as a *function* of understanding – the unity in the act of judgment – the idea of the category as a rule of determination must include the idea that the *materia circa quam*³ available to the power of judgment in general molds the form of the synthesis of comprehension. In what do we find this *materia circa quam*?

First we remind ourselves that the cognition of any object (whether it is merely a possible cognition as yet unexperienced in actuality or a cognition of actual experience) requires a representation in intuition. But an intuition is distinguished in sensibility by an act of reflective judgment in which sensible perceptions are marked at a moment in time. Thus there is involved in every cognition an *aesthetic* factor (because reflective judgment, as the bridge between objective judgment and purposes of *pure* Reason, is a judgment of *Zweckmäßigkeit* and such a judgment is subjective rather than objective). When we examine our objective knowledge we do not find this aesthetic factor ‘contained in’ the objective perception, but among some of them at least we often find that these perceptions seem to be accompanied by or linked to ‘feelings’ of one sort or another which we say “the thought arouses.” In other cases there seems to be an ‘indefinable something’ that seems to be and at the same time not to be ‘part of’ the objective perception. Thus, for example, we speak of “the spirit of the law” and mean to express by this the intangible

³ The "matter around which".

purpose that the law is supposed to “embody.”

Let us give a name to this ‘intangible factor’ of knowledge by which we can distinguish its aesthetic character. We will call it the *aesthetic Idea*.

Spirit, in an aesthetic sense, is called the animating principle in the mind. But that through which this principle animates the soul, the stuff it employs for this, is what sets the powers of the mind into expedient swing, i.e. in such a play which maintains itself on its own and even strengthens those powers.

Now I maintain this principle is nothing other than the capacity for presentation of *aesthetic Ideas*; under an aesthetic Idea I understand that representation of imagination which calls forth much to think, but to which no determinant thought, i.e. concept, can be adequate, hence no language can make it fully attained and intelligible . . .

Imagination (as productive faculty of knowledge) is of course very powerful in creation of, as it were, another nature out of the stuff actually given us. We entertain ourselves with it when experience becomes too banal to us; we fashion this no doubt always according to analogical laws, but yet also according to principles which lie in a higher place in reason . . . thus we feel our freedom from the law of association (which attaches to the empirical employment of that ability) in accordance with which stuff indeed can be lent to us from nature, but this can be worked up by us into something entirely different, namely that which surpasses nature [KANT5c: 192 (5: 313-314)].

Reflective judgment, as the bridge between determining judgment and Reason, makes its representations through affective perceptions, among which are those we call *feelings*. In the synthesis of comprehension, where we are constructing an intuition, the expedience of this representation for a purpose is marked by feelings which *present* (not ‘represent’) the aesthetic Idea. But if such an aesthetic Idea is to “call forth much to think” without itself being a constituent concept in this process of thinking, this can only mean that what is presented aesthetically as a judgment of expedience must also affect the activity of determining judgment. The power of imagination is not a power of judgment, and so that which is affected lies in the process of thinking rather than in the representations of sensibility.

Now let us recall the role of the transcendental schemata as the ‘bridge’ between representations of sensibility and those of determining judgment. On the one side we have the presentation of sensibility; on the other we have the categories of understanding. It follows from the points raised above that if the synthesis of comprehension (or, more generally, that of apprehension) is to affect the process of thinking, the categories must be such that reflective presentations of expedience affect determining judgment through the categories. If it were otherwise, the idea of reflective judgment as the bridge between the power of Reason (which stands in a mediate rather than an immediate relationship to cognition) and the process of determining judgment (which Reason employs) would be groundless and even self-contradictory. But the categories that determine the connection of propositions to the *manner of thinking* (i.e. as problematic, assertoric, or apodictic) are the categories of Modality.

We describe something like the aesthetic Idea when we say we “have a gut feel” for

something expressed problematically in one of our ideas or when we say an idea “feels right.” When we discussed the data of the senses we introduced the idea of ‘seeming’ (Chapter 6, §7.2) as the idea of “the inducement for a judgment of experience.” Seeming was distinguished from the appearance of an object and, as an ‘inducement for judgment,’ this is an idea of something subjective in the cognitive process. We said this was the idea of something that “is not true and also not false” but this idea of a something that lies outside truth or falsity is an idea whose object stands under Kant’s description of an aesthetic Idea. We are therefore in a position where we can now describe seeming as the causality for presentations of the aesthetic Idea to affect the process of determining judgment.

The mode in which determining judgment is affected by the aesthetic Idea depends upon both the transcendental schema of Modality, by which sensibility communicates with the category, and upon the general postulate, whose regulative condition is satisfied by the *materia in qua* of the particular intuition. For the synthesis of comprehension in which all the *materia ex qua* available for possible incorporation into the intuition arise purely from the power of spontaneity, the transcendental schema is the first schema of Modality, namely the schema of possibility:

The schema of possibility is the harmonization of the synthesis of various representations with the conditions of time in general . . . thus the determination of the representation of a thing in any time [KANT1a: 275 (B: 184)].

The general postulate under these conditions is the Postulate of Possibility.

We can now see the category of possibility & impossibility from the logical reflective perspective in the following way: ***The category of possibility & impossibility is the notion of the scheme for determining a problematic proposition solely through the power of spontaneity under the inducement of an aesthetic Idea in the synthesis of comprehension.*** Standing under this notion is the idea that the scheme of this category includes the power of acting to bring to the synthesis of imagination those concepts whose inclusion in the synthesis of the intuition is both possible under the principle of contradiction and identity and also are such as to promote the reflective expedience of the intuition for a purpose of pure Reason.

Long ago, when we first discussed the *Verstandes-Actus* in Chapter 3, one of the questions we raised at that time was that of how representations came to be brought into this three-step synthetic process in the first place. We now have the answer to that question. The bringing of representations into the three-fold synthesis of the acts of understanding involves the interplay of imagination, reflective judgment, and the categories of Modality in their role as rules for the construction of rules (that is, rules for the determination of concepts of propositions). This process Kant often called ***the free play of imagination and understanding***, and by this designation he implied the factor of spontaneity in the process of thinking.

B. The Category of *Dasein-Nichtsein*: The discussion immediately above applies as well to the other categories of Modality. This is unsurprising when we consider the ‘nature’ of these categories as rules for determination of the *manner in which we think about* propositions generally. What distinguishes the category of *Dasein & Nichtsein* (a phrase Kant intended for us to understand as ‘actuality & non-being’) from the category of possibility & impossibility is that in this case cognition involves both the power of receptivity and that of spontaneity, whereas possibility-impossibility involves only spontaneity. In the determination of the ‘*I think*’ propositions, the synthesis of reproduction in imagination and the synthesis of apprehension act jointly in the construction of the intuition. The *Verstandes-Actus* still fall under the jurisdiction of reflective judgment, the verdicts of which are communicated to the category through presentation in the aesthetic Idea. In this case, though, the synthesis involves sensations of outer sense and this inclusion places the process in the domain of the transcendental schema of actuality, insofar as Modality is concerned, and under the regulation of the principle of the Postulate of Actuality.

This is, in one way, a relatively minor change in the conditions of the synthesis inasmuch as it merely introduces sensation of outer sense into the *materia ex qua* of the process. But there is in this an important feature that makes all the difference. In the synthesis of a merely problematic proposition, a concept included in the *materia ex qua* of the synthesis has no guarantee that it will still be included in the *materia in qua* of the final representation of the intuition. If the concept poses a real contradiction, or if it does not contribute to the perfection of the expedience of the representation, it will not survive the processes of *Comparison* and reflexion.

But in the synthesis of the assertoric proposition, the second General Postulate *requires* that the intuitive representation be linked up with the sensuous contents of experience and, therefore, *the intuition must include* in its *materia in qua* the matter of sensation. It need not include *all* such matter as may be available as *materia ex qua* – e.g. most of us do not attend to a sensation of hearing in recognizing the appearance of a desk – but it must include *some* of this matter. Were the synthesis to discard all sensuous matter presented through receptivity, the proposition would be problematic rather than assertoric.

In view of the discussion on the previous category, we can summarize the category of *Dasein-Nichtsein* in the following way: ***The category of Dasein-Nichtsein is the notion of the scheme for determining an assertoric proposition through the combined powers of receptivity and spontaneity under the inducement of an aesthetic Idea in the synthesis of apprehension.*** Standing under this notion are the same ideas of the power of acting in accordance with the inducement of seeming and the requirements of the principle of contradiction and identity as we had for the previous category.

Now this is not to say that assertoric propositions can only be applied to direct ‘first

encounter' with the perception of an object. After all, if this were so we could never "think things through" before taking actions or to obtain any understanding of Nature.

The postulate requires *perception* to know the actuality of things, thus sensation of which one is conscious; to be sure not immediately from the object itself, the *Dasein* of which is to be recognized, but just of its context with some actual perception according to the analogies of experience, which unfold all real connection in an experience in general.

In the *mere concept* of a thing no character of its *Dasein* can be met with at all. For even if this concept is so complete that it lacks nothing in order to think a thing with all its inner determinations, still the *Dasein* has nothing at all to do with all of this, but only with the question: whether such a thing is given to us so that the perception of it could if need be precede the concept. For that the concept precedes the perception signifies its mere possibility; but perception, which delivers the fabric for the concept, is the sole character of actuality. However, one can also know the *Dasein* of the thing prior to the perception of it, and *comparatively a priori*, if only it coheres with some perception in accordance with the first principles of their empirical connection (the analogies). For in that case the *Dasein* of the thing is still fixed to our perceptions in a possible experience, and with the guidance of the analogies we can get from our actual perceptions to the thing in a series of possible perceptions [KANT1a: 325 (B: 272-273)].

The *Dasein* of some as yet unperceived object can be first determined by, for instance, an assertoric hypothetical proposition. Such a judgment establishes the *real ground* for the *Dasein* of a cause even though the *Existenz* of this object is left wholly undetermined by this judgment. There may follow any number of problematic propositions in which one *seeks to understand* such a cause (e.g., "Lightning has struck. Is Zeus angry?"). But if we can finally determine through additional and actual experiences the *Existenz* of the causal object, the inquiry can be put to rest so far as the Modality of a determinant judgment is concerned. Such a process is at the center of experimental sciences. And this is why those things in science that are wholly objects of ideas and incapable of ever being encountered in actual experience remain problematic even if the circumstantial evidence in actual experience promotes a high degree of holding-to-be-true. A higher degree of holding-to-be-true for a problematic proposition expresses greater *confidence* in the judgment; for an assertoric proposition, it expresses greater *commitment to* the proposition. If experience should come to contradict a problematic proposition in which one has great confidence this is ground for puzzlement (an affective judgment), but if it should come to contradict an assertoric proposition this is ground for consternation.

C. The Category of Necessity-Contingency: Propositions connected in a hypothetical Relation are connected as a series such that if I should think of the consequent I must also think the antecedent. A later cognition in which the antecedent gets included in the intuition must therefore also be one in which the consequent is *anticipated* – the *facultas praevidendi* of imagination. Such anticipation is indeed one of the key maxims for experimentation, namely that given the same conditions the same outcome is expected to follow. When the expectation is met with consistently, the scientist calls this *connection* a "reproducible fact."

Anticipation appends to the apprehension of one object (in which the antecedent is contained) the need for comprehending another involving the consequent. There is in this case, in the synthesis of imagination, a situation in which the inclusion of the consequent proposition in the cognition is guaranteed if the judgment is to be in accordance with the general conditions of experience (which is, of course, the Postulate of Necessity). A like requirement is established by disjunctive propositions whereby the inclusion of the categorical proposition at the 'center' of one subsphere of a concept necessarily precludes those at the centers of the other subspheres because such an inclusion would contradict the disjunction of the sphere of the higher concept in the disjunctive proposition. If actual experience should contradict anticipation, we are *surprised*.

We should compare this condition placed upon the synthesis of comprehension with the conditions described above for the other two modal categories. Under those categories, concepts in the manifold are accorded no guarantee of inclusion (or exclusion) in intuitions produced by the synthesis. They may be either included or excluded without real contradiction arising *in the nexus of the manifold*. But in anticipation this is *not* the case when concepts are connected in a series (by hypothetical propositions) or are coordinated (by disjunctive propositions) because in these cases the concepts, once having been so connected, cannot *thereafter* be thought independently of each other. *The prior connection conditions subsequent synthesis in reproductive imagination*. The expression of this *conditioning of thinking* is the function of the rule expressed by the category of necessity & contingency. Anticipation tests the manifold.

At the core of this conditioning we find the notion of the *Dasein* of an object. Determinant judgments are ultimately judgments passed on the *Existenz* of Nature as this *Existenz* is represented for objects *in* Nature and their relationships under the principle of unity in transcendental apperception.

What finally the third postulate concerns gets to material necessity in *Dasein*, not the mere formal and logical necessity in the connection of concepts. Now since no *Existenz* of objects of the senses can be known fully *a priori*, but always only comparatively *a priori* relative to another already given *Dasein*, but after that one nevertheless can only come upon an *Existenz* that must be contained somewhere in the context of experience of which the given perception is a part, the necessity of *Existenz* can thus never be known from concepts but rather always only from connection with that which is perceived according to general laws of experience. Now there is no *Dasein* that could be known as necessary under the condition of other given appearances except the *Dasein* of effects from given causes according to laws of causality. Thus it is not the *Dasein* of things (substances) but rather their state of which alone we can know the necessity and, moreover, only from other states which are given in perception according to empirical laws of causality. From this it follows that the criterion of necessity lies solely in the law of possible experience: that everything that happens is determined *a priori* through its cause in appearance. Hence we recognize only the necessity of *effects* in Nature, the causes of which are given to us; and the mark of necessity in *Dasein* does not reach beyond the field of possible experience, and even in this it does not hold of the *Existenz* of things as substances because these can never be regarded as empirical effects or something that happens and arises. Necessity therefore concerns only the relationships of appearances according to the dynamical law of causality and the grounding possibility for inferring *a priori* from some given

Dasein (a cause) to another *Dasein* (the effect). Everything that happens is hypothetically necessary; that is a first principle that subjects alteration in the world to a law, i.e. a rule of necessary *Dasein*, without which not even Nature would happen [KANT1a: 329 (B: 279-280)].

The notion of a substance under the category of substance and accident is the notion of something merely as persistent in time. It is, so to speak, a mental coat hook upon which we hang the concepts of the accidents of appearance. Thus we impute *Dasein* to substances *contingently* (the sole exception being that of the notion of our own personal *Dasein*; the *I* of transcendental apperception, you will recall, is consciousness of one's own *Dasein* but without any *a priori* consciousness of the Nature of one's personal *Existenz*). The *Dasein* of a substance is only determined as *necessary* by way of a determination of connection by an assertoric hypothetical proposition, and such a proposition does not determine with necessity the *Existenz* of this substance. Even within a disjunctive proposition, the *Dasein* of a substance is thought necessary only by way of an assertoric hypothetical proposition contained within the disjunction, and this necessary *Dasein* attaches to the connection of a substance in the antecedent proposition with that in the consequent proposition. And if the actual consequent lies in a different subsphere from the actual antecedent (through a disjunction), then experience contradicts anticipation.

This is a lot for judgment to sort out if the unity of the manifold of concepts is to be maintained. This 'bookkeeping job' for determining judgment requires that the connection of the manifold contain *marks in the judgments* by which the connections thought as necessary can be distinguished from those which are thought as merely contingent. The idea of the placing of such marks, as well as the idea that these marks must be attended to in the making of judgments, are ideas that stand under the category of necessity-contingency. Without this *a priori* 'bookkeeping function' of determining judgment, it would be impossible to discover contradictions within the complex interrelationships of concepts in the manifold of concepts. ***The category of necessity-contingency is the notion of the scheme of determining the marks of the conditions of experience in an apodictic proposition.***

§ 5.5 A Comment on the Theory of the Categories from the Logical Perspective

Some commentators have offered the opinion that Kant's categories are somehow based on or grounded in traditional logic theory, the implication being that the categories are *derived* from the 'logical functions' (by which they mean the logical momenta we have discussed). For example, Professor Young writes:

Kant's view, more fully stated, is that the categories have their origin in "the function of thought in judgment" . . . It is the task of general logic, he holds, to give a systematic account of the various "moments" of this function . . .

Kant's claim, now, is that the categories, which are concepts fundamental to all our knowledge, have their roots in these logical functions of judgment. He claims, indeed, that the categories *are*

these functions of judgment, insofar as they are employed in the determination of the manifold of a given intuition" [B 143, my emphasis; see also B 128]. There are just as many categories as there are functions of thought, accordingly; they are represented in a second table . . . whose structure is supposed to be based on that of the first[.]

These two tables give rise to a multitude of questions and difficulties. Before turning to these, however, we should consider more closely Kant's contention that the categories *are* the functions of judgment employed in a certain way. This is plainly the central contention in the Metaphysical Deduction [GUY: 102-103].

Young goes on a bit later to write,

To a modern reader it is likely to seem that Kant's argument rests on an impoverished logical theory and perhaps a flawed conception of logic as well. Kant believes that logic is a strictly formal discipline, which "abstracts all content of cognition of the understanding . . . and deals with nothing but the mere form of thought" . . . He also believes that logic as he knows it is "a closed and completed body of doctrine" . . . which may not legitimately be altered in any substantive way. Given familiar attacks by Quine and others, Kant's view on the first point is likely to strike a modern reader as naive. Given developments in logic theory over the last century, his view on the second point is likely to seem embarrassingly shortsighted.

. . . But in any case, his logical theory is plainly impoverished. It deals, at best, with only a small fragment of propositional logic. It also provides no explicit treatment of quantification, the implicit treatment being limited to categorical propositions. Most important, his logic does not allow for the representation of multiplace predicates or of the complex quantification structures that are the engines of mathematical reasoning [GUY: 105-106].

Now, I mean no disrespect for Professor Young, for these views are by no means unique and are held by many Kant commentators, but in light of what we have seen in the previous sections, it is difficult to accept characterizations such as quoted above as accurate assessments of the Critical Philosophy. To me it seems that interpretations like the one just quoted place undue emphasis on a chapter title in *Critique of Pure Reason* which reads, "On the Guide to the Discovery of all Pure Notions of Understanding" [KANT1a: 204 (B: 91)]. Even granting the fairness of criticizing Kant for the rather "Abracadabra, presto!" impression his writing often leaves when one reads this chapter of *Critique of Pure Reason*, a "guide" is not an "origin" or a "root" any more than the phenomenon of static electricity is the origin for Maxwell's equations.

It is clear to me (and now hopefully to you) that Kant's ontology of determinant judgments is *not* "based on" traditional formal logic. Rather, the doctrine of formal logic, if it is in any way to be employed as a doctrine for reasoning, must be derived from epistemology. We can have a logic doctrine – say, mathematical logic – as a science, and if we wish to try to express another doctrine – mathematics, let us say – in terms of this logic doctrine, we are obviously free to try to do so. But in doing so we should keep in mind that this logic doctrine has no claim to being "the laws of thinking" if it is not deduced from the fundamental principles laid out in the Critical Philosophy. Paton commented:

I do not suggest that there is any necessary antagonism between Kant's point of view and that of

mathematical logic. On the contrary, it is no part of Kant's case that Formal Logic is a *sufficient* account of all reasoning. He believes it to be true so far as it goes, but he maintains, for example, that it cannot account for the special characteristics of mathematical reasoning. The modern study of relations in logic may be regarded as a development of his view that mathematical thinking depends, not on the form of the syllogism, but on specific spatial and temporal relations.

It must nevertheless be recognized that Kant's Formal Logic and modern mathematical logic are trying to do different things, and the criticism that Kant ought to have done what mathematical logic does is an unreasonable criticism [PAT1: 210-211].

Professor Young's overall attitude towards Kant actually seems to me to be a reservedly friendly one. He leaves one with the general impression that Kant did as good a job as possible given the "erroneous" first step where he "bases" his philosophy on an "impoverished theory of logic". Of a different sort are those whom I call the "unfriendly" commentators, of whom Kemp Smith is perhaps the foremost representative.

The *fons et origo* of all the confusions and obscurities of this section are thus traceable to Kant's attitude towards formal logic. He might criticize it for ignoring the interdependence of conception, judgment, and reasoning; he might reject the second, third, and fourth syllogistic figures; and he might even admit that its classifications of the forms of judgment is not as explicit as might be desired; but however many provisos he made and defects he acknowledged, they were to him merely minor matters, and he accepted its teachings as complete and final . . . The defects of the traditional logic were very clearly indicated in his own transcendental logic. He showed that synthetic thinking is fundamental; that by its distinctions the forms and activities of analytic thought are predetermined; that judgment in its various forms can be understood only by a regress upon the synthetic concepts to which these forms are due; that notions are not merely of the generic type, but that there are also categories of relation. None the less, to the very last, Kant persisted in regarding general logic as a separate discipline, and as quite adequate in its current form . . .

The resulting situation is strangely perverse. In the very act of revolutionizing the traditional logic, Kant relies upon its prestige and upon the assumed finality of its results to make good the shortcomings of the logic which is to displace it [SMTH: 184-185].

Kemp Smith's charge is a more serious one: Kant actually has overthrown traditional logic but is ignorant of his own accomplishment. This is not to say, in Kemp Smith's view, that *Kant's* system is correct; Kemp Smith seems far from this position. He tells us Kant's work has serious defects and shortcomings. Kemp Smith also ignores the systematic theme which runs throughout all of the Critical Philosophy. Rather, he says, it is a thrown-together collection of old notes compiled over a decade and merely pieced together. Consequently, he tells us,

But the *Critique* is not merely defective in clearness or popularity of exposition . . . What is much more serious is that Kant flatly contradicts himself in almost every chapter; and that there is hardly a technical term which is not employed by him in a variety of different and conflicting senses. As a writer he is the least exact of all the great thinkers [SMTH: xx].

If all this is really so, one might well wonder why Kemp Smith still calls Kant one of the "great thinkers."

The Critical Philosophy, and everything within it, is first and foremost a *system*. I find

myself in complete agreement with Palmquist on the essential points that Kant cannot be taken piecemeal and that the apparent ‘contradictions’ and other vexations of his work disappear when one pays serious attention to the architectonic structure of Kant’s philosophy. Palmquist writes:

The difference in emphasis between Kant and most of his interpreters suggests that the frequent disagreements among the latter are due not so much to the fact that, as Kant himself admits, his method of exposition is 'difficult, nay, even shrouded in obscurity' . . . as to the interpretive technique they employ. Most interpreters tend to isolate a certain aspect of Kant's System . . . and then proceed as if it 'can be disconnected from the other layers of his complicated thinking and be given a separate interpretation and critical discussion' . . . without seriously damaging the accuracy and adequacy of the interpretation. They then naturally tend to regard this aspect of Kant's System as its central or key conception. But any attempt (analytic or otherwise) to grasp the supposed unity of his philosophy merely by examining the validity of a narrow cross-section of its arguments is bound to lead to the conclusion that Kant's work 'is not the exposition of a single unified system.' . . . It is usually not even considered as a serious option that such a unity might be found where *Kant* thought it was: in the logical structure of reason's architectonic form, the organic whole which determines the content of every particular theory he propounds.

Kemp Smith is the classical example of a commentator who commits this interpretive error. His standard procedure . . . is to reject anything related to Kant's architectonic as a 'perverse' . . . form of philosophical magic . . . The resulting interpretation of Kant is so misguided that interpreters who are more sensitive to Kant's own intentions are forced to simply ignore the many errors in [SMTH], lest their own work be bogged down with refuting Kemp Smith's interpretation (an interpretation whose influence has been far wider than many scholars recognize because its obscurities are imbedded in his translation of [*Critique of Pure Reason*], which has unfortunately come to be used as the standard English edition) [PALM1: 6-8].

The primary reason it has taken us this long in the present treatise to reach a discussion of the categories of understanding has been precisely because it is necessary to see the forest before examining the trees, as the saying goes. Here in §5 we have examined the categories from the logical perspective and shown the dependence of the logical momenta on them. Undoubtedly the reader has also noticed how, even from the logical perspective, our exposition of the categories has had to reach out and bring in other elements of the system in addition to determining judgment even though, as the fundamental notions that govern the construction of concepts, the categories “belong to” – if I may use that phrase – determining judgment. This is especially apparent with the categories of Modality, which interact explicitly with the aesthetic Idea. However, we are dealing here with a *systematic doctrine* and must expect to have to deal with what goes on at the logical borders between its principal structures.

And it is precisely because we are dealing here with a system that our exposition of the categories cannot rest on just *one* of the four reflective perspectives. We have dealt with the categories from the logical perspective. We must now deal with them from the others. We will next examine them from the transcendental perspective and its associated branch of metaphysics proper, Rational Psychology.

§ 6. The Categories from the Transcendental Perspective

When we turn from the logical perspective to the transcendental perspective, we turn from considerations of objects of outer sense to those of inner sense, i.e. from objects viewed in terms of *physical Nature* to objects viewed in terms of *thinking Nature*. However, when we introduced the Ideas of Rational Psychology in Chapter 4 (§4.2), we saw that these Ideas are of a ‘negative’ character – a ‘stop sign,’ if you will, that warns us of a boundary beyond which we pass from the immanent use of Reason to a dialectical and transcendent use where objective validity is lost and the constructs of reasoning become fantastic. The question naturally arises at this point: How can the ‘negative’ Ideas of Rational Psychology tell us anything about the Nature of the categories of understanding? Can there be an immanent ‘positive’ use of the metaphysics proper of Rational Psychology and, if so, what is it?

§ 6.1 Metaphysics Proper in the Doctrine of Method

The question we have just asked is not restricted to only Rational Psychology and should be considered for all branches of metaphysics proper. In the case of Rational Physics, we have seen a detailed use of its principles in the previous section as regulative principles for deducing the role of the categories from the logical perspective. The statements of the acroams of Rational Physics (Axioms of Intuition, Anticipations of Perception, Analogies of Experience, and the Postulates) might tend to mask that these principles are fundamentally limitative, but in fact they *do* place boundaries on what can known from experience.

This is generally true of the other acroams of metaphysics proper because, used as principles in the doctrine of method, all Critical metaphysics proper is a doctrine of epistemology.

Everything grounded in the nature of our powers must be expedient and in unison with their correct use, if only we can prevent a certain misunderstanding and find out their proper direction. Thus the transcendental Ideas, too, will presumably have their good and consequently *immanent* use, even though, if their significance is mistaken and they are taken for ideas of actual things, they can be transcendent in their application and just for that reason deceptive. For considering the whole of possible experience, it is not the Idea regarded as it is in itself but merely its use that can be either overflying¹ (transcendent) or indigenous (immanent), according to whether one directs them straight on to a supposed object corresponding to them, or only to the use of understanding in general considering the objects with which it has to do . . . Thus reason properly has as objects only understanding and its expedient employment, and just as [understanding] unites the manifold in the Object through concepts, so [reason] on its side unites the manifold of concepts through Ideas by erecting a certain collective unity as the goal of acts of understanding, which are otherwise concerned only with distributive unity [KANT1a: 590-591 (B: 670-672)].

¹ Kant means, e.g., "the premature and forward haste of understanding, and its jumping or flying to generalities" - Francis Bacon [BACO2: 114].

Metaphysics is the philosophy concerned with presenting our knowledge of what Kant called “the special faculty of knowledge” in its systematic unity. One part of this presentation deals with our knowledge of Nature. Kant calls the restriction of metaphysics to this topic “metaphysics in the narrow sense” and gives it the name *metaphysics of Nature*.

The aforementioned metaphysics in the narrow sense consists of *transcendental philosophy* and the *physiology* of pure reason. The former considers only understanding and reason itself in a system of all ideas and first principles that refer to objects in general, without taking on Objects that *would be given (Ontologia)*; the latter considers *Nature*, i.e., the quintessence of *given* objects . . . and is therefore *physiology* (though only *rationalis*) . . .

Immanent physiology considers Nature as the quintessence of all objects of the senses, thus as it is given *to us*, but only according to *a priori* conditions, under which it can be given to us in general. There are, however, only two sorts of objects for this. 1. Those of outer sense, thus the quintessence of these, *corporeal nature*. 2. The object of inner sense, the soul¹, and, according to the fundamental notions of this in general, *thinking nature*. The metaphysics of corporeal nature is called *physics*, but, since it shall contain only the principles of its *a priori* knowledge, Rational Physics. The metaphysics of thinking nature is called *psychology*, and precisely for this same reason, only the *rational knowledge* of this is meant here [KANT1a: 698-699 (B: 873-874)].

Kant is telling us here that objective validity can be established only through the immanent use of Reason, never its transcendent use. Therefore all our metaphysical notions and ideas that pertain to “the object of inner sense” can be objectively valid only as principles that are necessary for the possibility of our thinking Nature.

The Ideas of Rational Psychology, as we said before, are stop signs for speculative theorizing. This does not mean, however, that Rational Psychology has no other immanent use for these Ideas. The Ideas of Rational Psychology, as I said back in Chapter 4, exterminate harmful vermin. But we still require, up to the limits these Ideas impose, principles of the unity of thinking Nature, and these regulative principles also fall within the domain of Rational Psychology. How shall we know these principles?

First, how can I expect an *a priori* knowledge, thus a metaphysics of objects so far as they are given to our senses, thus given *a posteriori*? And how is it possible to know the nature of things according to *a priori* principles and to arrive at a *rational* physiology? The answer is: We take from experience nothing more than what is necessary to *give* ourselves an Object, partly of outer and partly of inner sense. The former is accomplished through the mere concept of matter . . . the latter through the idea of a thinking being (in the empirically inner representation: I think). Otherwise, we must in the entire metaphysics of these objects abstain entirely from all empirical principles that might add any sort of experience beyond the concept in order to judge something about these objects [KANT1a: 699 (B: 875-876)].

We might expect that Kant would have laid out this system of Rational Psychology in his

¹ Bear in mind that Kant uses "soul" to refer to the "I" of transcendental apperception and this term has for him (and for us in this treatise) no religious significance.

lectures on metaphysics (since he did not do so in *Critique of Pure Reason*), but in this expectation we are disappointed. What we find instead is that he divides his lectures on psychology into two parts: empirical psychology (which he very clearly holds does not belong to pure philosophy but connects to philosophy only in the sense that it requires an *applied* metaphysics) and a discussion of Rational Psychology that is almost entirely directed at refuting the speculative metaphysics of Wolff. His lectures follow the topics in Baumgarten's textbook (a textbook of the Wolffian school) and perhaps Kant felt it was more important to make sure his students did not learn something that wasn't true (in regard to objective validity) than to profess an entirely new system of metaphysics for Rational Psychology.

Or perhaps he simply had not had time to formulate such a new system and therefore had nothing 'positive' to present. There is evidence in the *Opus Postumum* that suggests Kant was still struggling with this problem and no strong evidence that he was near to solving it. His discussions of "empirical psychology" are clearly of a metaphysical nature, although equally clearly they fall on the side of applied rather than pure metaphysics, and we do not find in his works the transition or 'bridge' from the applied to the pure metaphysics of psychology. If it is true that time ran out for Kant before he could complete this project, then what we have in the Kantian *corpus* suggests that Kant, too, was a follower of Aristotle's maxim of beginning with "that which is clearer to us" in approaching metaphysical theory.

With regard to empirical psychology, Kant asks and answers:

Where does that leave empirical psychology, which has always retained its place in metaphysics and from which in our time such great things for enlightenment has been expected, once one gives up hope of producing something useful *a priori*? I answer: It comes in where the proper (empirical) doctrine of nature must be put, namely on the side of *applied* philosophy, for which pure philosophy contains the *a priori* principles, which must therefore be allied but never confused with the former [KANT1a: 700 (B: 876)].

Judging by the contents of Kant's lectures on empirical psychology, what he seems to have in mind here is the empirical view of thinking Nature that grew out of Locke's *An Essay on Human Understanding* and underwent further analysis by Locke's successors as well as by continental philosophers (such as Leibniz and Wolff) who were influenced, or at least stimulated, by Locke's theory.

When the mind turns its view inwards towards itself, and contemplates its own actions, *thinking* is the first that occurs. In it the mind observes a great variety of modifications, and from thence receives distinct ideas. Thus the perception or thought which actually accompanies, and is annexed to, any impression on the body, made by an external object, being distinct from all other modifications of thinking, furnishes the mind with a distinct idea, which we call *sensation* [LOCK: 175 (Ch. XIX, 1)]

That the mind, in respect of its simple ideas, is wholly passive, and receives them all from the existence and operations of things, such as sensation or reflection offers them, without being able to

make any one idea, experience shows us. But if we attentively consider these ideas I call mixed modes . . . we shall find their origin quite different. The mind often exercises an *active* power in making these several combinations. For, it being once furnished with simple ideas, it can put them together in several compositions, and so make variety of complex ideas, without examining whether they exist so together in nature [LOCK: 201 (Ch. XXII, 2)].

Considerations such as these, and others as developed in the Wolffian system, are “merely” empirical and do not fit at all within pure philosophy. Even as applied philosophy they can lay no claim to forming a science *proper* so long as a *hiatus* remains that separates empirical psychology from metaphysics proper. To bridge this gap, we must deduce objectively valid *a priori* principles that provide us with a transition from Rational to empirical psychology.

Now, the transcendental role of the categories of understanding is exhibited precisely in this bridging of the gap between our knowledge of empirical Nature and epistemology under the Copernican hypothesis. Therefore, if we are to seek the ‘bridge’ between empirical psychology and our pure thinking Nature, it is to them that we must turn. In doing so, however, we must exercise great care to remain within the borders of objectively valid knowledge staked out by the Ideas of Rational Psychology.

§ 6.2 The Categories of Quantity from the Transcendental Perspective

From the logical perspective, the categories of Quantity are the notions of the schemes for determining the form of the sphere of a concept. This idea of a sphere is the idea of the aggregate of all concepts contained under the sphere’s concept. Insofar as the logical perspective is concerned, our idea of the categories of Quantity is couched wholly in terms of representations.

But we also require of these representations that they be ‘significant of objects.’ The idea of the sphere *per se* is nothing more than a representation of the magnitude of the *scope* of the concept. The scope, in turn, is the idea of *all objects* for which the concept is a mark of cognition. How, though, are we to regard this idea of the ‘scope’ of a concept? In the transcendental perspective we are concerned with the principles of our thinking Nature and thus, as far as the categories are concerned, with the possibility of unity (in thinking) in consciousness. Insofar as we consider the categories of Quantity, the principle from Rational Psychology upon which we must call is the psychological Idea of Quantity, namely the Idea of unconditioned unity in the multiplicity of time.

As we discussed in Chapter 4, this Idea is the Idea of the absolute unity of the *I* of transcendental apperception – this notion of ‘oneself’ that we know *a priori* only in terms of *Dasein* and never (*a priori*) in terms of *Existenz*. Now, in our theory we have not been the least hesitant about carving up this transcendental *I* in terms of various powers, processes, and so on. Numbered among the ideas we have been dealing with thus far we have *imagination, determining judgment, reflective judgment*, and so on. We have, however, obtained this mental anatomy from

our exposition of the faculty of pure consciousness which, we recall, deals with only *the appearance of the phenomenon of empirical consciousness*. Consequently, these elements of our theory we must regard, under the psychological Idea of Quantity, as representations of a merely logical division and never as representative of a *real* division.

The immediate consequence of this is the following: While we are permitted to posit such a logical division, in doing so we cannot make a complete separation of the elements of our theory and, in particular, we must view all these elements as standing in a relationship of complete reciprocity with one another. Just as we earlier found against the objective validity of the mind-body division of Descartes, now we find against the objective validity of any real division in the theoretical faculty of pure consciousness. Put another way, our theory is an exposition of the appearance of the phenomenon of mind and not an exposition of mind regarded as *Ding an sich*.

Moving now to the categories, from the logical perspective the categories are the constitutive rules of the process of determining judgment we must posit as the ground of our ideas of the appearance of determinant judgments and the faculty (organization) of conceptual understanding. But from the transcendental perspective the categories are the constitutive rules of cognition through concepts – *thinking* – and cognition involves representation in intuition as well as in concepts. Rational Psychology forbids to us the making of a real break between judgment and sensibility and so although we can regard the categories as ‘belonging to’ the phenomenon of judgment, metaphysics proper necessarily requires that our theory contain a transcendental ‘bridge’ between the categories (and their function as the unity in conceptual thinking) and that part of cognition that we say ‘belongs to’ the representation of sensibility.

To briefly review our exposition of the faculty of knowledge as it has been presented so far, we have identified three major components. One is imagination, which we have described in terms of three syntheses: the synthesis of apprehension in an intuition, the synthesis of reproduction in imagination, and the synthesis of re-cognition in a concept. The second is the process of reflective judgment, which is the ability to judge from the particular to the general through the principle of the *Zweckmäßigkeit* (expedience) of our cognitions for a purpose of pure Reason. In appearance this ability has been implicated in the *Verstandes-Actus* of comparison, reflexion, and abstraction. Third, we have the process of determining judgment which, from the logical perspective, we have described in terms of the logical momenta of judgment and the categories.

In addition to these, we have also discussed the transcendental schemata, which we have placed within the power of imagination and which serve as the necessary condition of experience under which we could subsume representations of intuition under the categories according to the requirement that every concept be such that its re-presentation in an intuition be possible insofar as the *form* of representation is concerned. We described these schemata as the ‘bridge’ between

intuition and concepts made possible through the pure *a priori* intuition of time. Let us now ask: What is the “mental physiology”² that unites these “anatomical” elements as an organized unit insofar as the Quantity is concerned?

The Comprehension of Extensive Scope

When we take the transcendental perspective our viewpoint shifts to the examination of what is transcendently necessary in the *Existenz* of the thinking Subject for the possibility of human knowledge *as we come to know it*. Palmquist described this perspective in the following way:

A transcendental perspective . . . presupposes the subject-object distinction: it attempts to determine what there is in the subject a priori which makes possible our knowledge of the objects we experience . . . That this knowledge arising out of this radically epistemological perspective concerns only a set of synthetic a priori forms embedded in the *subject* is spelled out explicitly by Kant when he says 'the word "transcendental" . . . never means a reference of our knowledge to things but only to the cognitive faculty' [PALM1: 124-125].

The real significance of the categories of Quantity subsists in the idea of the *scope* of the concept whose *sphere* is determined via the logical momenta of Quantity. However, as should be obvious by now, the scope of a concept is a *noumenon*. It is completely impossible for us to have an actual presentation, given in empirical experience, of any such thing as the scope of a concept *per se*. Thus, we must content ourselves with addressing not the scope *qua* scope, but only the scope *qua* appearance as this can be represented in intuition. But here we are faced with an interesting question which, as it turns out, goes directly to the heart of understanding the categories of Quantity from the transcendental perspective. There are two ways in which we might use the phrase “appearance of the scope.” The first of these ways is: the specific appearance of the scope of a concept *as that concept appears in a specific cognition*. This is the appearance of the concept’s scope *in concreto* as a specific example. The second of these ways is: the general appearance of any scope of a concept.

Only an intuition stands in immediate relationship with appearance. Hence, if we are to speak with real significance about the appearance of the scope of some concept *A*, we must do so in terms of how such an appearance is represented in intuition. An intuition, though, is *repraesentatio singularis* and is always a specific instantiation in representation. Therefore, we must look for the significance of the categories of Quantity in terms of the possibility of any appearance of the scope of any concept, *A*, *in concreto*. With respect to the idea of the general appearance of *the* scope of *A*, we must regard this as the idea of the merely *possible scope* of *A*.

² In referring to mental anatomy and mental physiology, we are using the terminology we developed in Chapter 6 for the sensorimotor idea.

The sphere of a concept *A* is the representation of the magnitude of the scope of *A*. So far as Quantity is concerned, it is the sphere as a representation of extensive magnitude with which we are presently concerned, and with the manner of partial re-production of this representation in the extensive magnitude of an intuition. The idea of extensive magnitude, in turn, is the idea of that in which the representation of the parts makes possible the representation of the whole (see Chapter 6 §3.3). So far as the intuition of the scope of a concept *A* is concerned, these ‘parts’ are the concepts contained in the sphere of *A*; they constitute the *materia ex qua* for any possible intuition of the scope of *A*. The reproduction of the intuitions of these concepts by the power of imagination furnishes the successive comparates for the *Verstandes-Actus*, and those representations which coalesce in the synthesis of comprehension to form a new intuition constitute the *materia in qua* of this outcome.

Let us suppose that some concept *A* is made the subject of a determination in judgment. To have a cognition of this judgment *A* must undergo the synthesis of reproduction in imagination. In this synthesis, the form of the extensive magnitude of the reproduced perception of *A* is determined by a transcendental schema of Quantity. *Which* schema this will be – i.e. the determination of the *modus* of time as the pure form of inner sense – is determined by the category of Quantity under which *A* is thought. At this point in the synthesis of *comprehension*, the sensibility of *A* cannot yet be called an intuition because the mere act of reproductive imagination applied to *A* is only a step in the synthesis of a comprehensive intuition, and the mere reproduction of *A* in sensibility does not mean that this reproduced sensibility is marked by reflective judgment at a moment in time. Hence, the reproduction yields merely a possible sensible perception.

Now this synthesis of comprehension through the power of imagination is said to be governed by three laws: 1) the law of association; 2) the law of anticipation; and 3) the law of compatibility (as we mentioned earlier in quoting Kant). The determination of which of these laws is applied in the synthesis of comprehension is not the concern of the categories of Quantity because the idea of these laws is a *dynamical* idea and thus does not belong to the *mathematical* categories of Quantity and Quality (which do not concern the *nexus* of the manifold of concepts). Even so, the possibility of the latter presupposes the possibility of *making* some kind of ‘association’ of concepts and it is with *this* possibility that the mathematical categories are concerned. Even the Great Skeptic accepted the necessity of positing association:

As all simple ideas may be separated by the imagination, and may be united again in what form it pleases, nothing wou'd be more unaccountable than the operations of that faculty, were it not guided by some universal principles, which render it, in some measure, uniform with itself in all times and places. Were ideas entirely loose and unconnected, chance alone wou'd join them; and 'tis impossible the same simple ideas should fall regularly into complex ones (as they commonly do) without some bond of union among them, some associating quality, by which one idea naturally

introduces another [HUME1: 10 (Bk. I, pt. 1, § IV)].

Necessary as the presupposition of association is, this presupposition nonetheless requires a rational ground if it is to be an objectively valid principle. This ground, however, is amply provided by the combination of judgment in the form of the sphere of *A*. It is not “resemblance, contiguity, and cause-and-effect,” as Hume maintained, that is the ground for the law of association; rather, it is simply the combination made among concepts by determining judgment in which we find the necessary condition that allows us to think association in terms of one concept “summoning up” another.

In all three cases (laws of association, anticipation, and compatibility), we must necessarily presuppose the possibility of extracting from the manifold of concepts the *materia ex qua* appropriate for each case. It is, in part, for the dynamical categories to determine which of these laws thinking shall follow in any specific circumstance, but in all cases it falls to the categories of Quantity to determine the extensive scope of the synthesis of comprehension by selecting its *materia ex qua* according to the form of the sphere of the concept *A*. Every intuition is a *repraesentatio singularis*. The synthesis of imagination in comprehension takes us from a representation of a manifold (the sphere of *A*) to the representation of an individual *in concreto* in the intuition. The judgment of this, in the determination of a perception as marked at a moment in time, is therefore reflective; but the possibility of making this judgment necessarily presupposes that the *materia ex qua* of this cognition be given to imagination, and this is where the categories contribute to comprehension. We can therefore speak of the scope of a concept in two ways:

- 1) As the *possible* scope of the concept; the possible scope is nothing more than the mere collection of all possible appearances that can be conditioned by the concept *A* through the sphere of *A*;³
- 2) as the specific scope of a concept in the *use* of that concept as it is employed in a specific act of thinking.

It is the second of these that makes *actual* experience and so only this second context of the idea of a scope has *a priori* objective validity, i.e., *real significance*.

The Categories of Quantity from the Transcendental Perspective

We are now ready to understand the categories of Quantity in terms of the thinking Nature of the Subject. In so doing, we will need to consider: 1) the active role of the categories in the power of spontaneity; and 2) the interrelationship of the categories with the other anatomical ideas.

³ In the language of mathematics, this is called the ‘power set of a set.’

The synthesis of the *Verstandes-Actus* is a three-fold synthesis of a succession in representation making a sensible intuition marked at a moment in time. In comprehension, thus in the interaction of the synthesis of reproduction in imagination with the synthesis of apprehension of given data of the senses, it falls to the categories of Quantity to determine the *materia ex qua* given to the *Verstandes-Actus* insofar as this matter originates from concepts. The concepts contain whatever *information* “goes into” the making (by imagination) of the reproduction of the concept as a sensible representation. But the determination of *which* concepts are to be extracted from the conceptual manifold and employed as *materia ex qua* for synthesis is a judgment *a priori* of the applicable sphere of the concept *A* insofar as this judgment is concerned only with the multiplicity thought in the extensive magnitude of a perception and not with the intensive magnitude of that perception or the dynamical determination of the law employed in imagination (i.e. the law of association, anticipation, or compatibility).

This, of course, presupposes that somehow comprehensive synthesis “gets started” by a determination of some initial concept *A*. This determination does not belong to the categories of Quantity. Rather, the *ground* for the selection of *A* must be sought in a purpose of Reason in employing the process of determining judgment in this specific instance. Such a purpose does not have immediately to do with any object, for Reason is not immediately concerned with objects but only with understanding (insofar as thinking is concerned). Therefore, the selection of some starting concept *A* can be made only through the intervention of some reflective judgment of *Zweckmäßigkeit* with regard to this purpose. It is in this where we find the aesthetic Idea coming into the process of thinking as an inducement to judgment. The mental physiology of thinking therefore has in it a *principle*, namely the *principle of the interaction of practical, reflective and determinant judgments via the Quality of seeming and the aesthetic Idea*. Put simply, a starting concept *A* becomes a starting concept because it “seems expedient” for a purpose of Reason.

Comment: In speaking of the idea of a starting concept, we should not construe this idea as implying some sort of break in the cognitive process, as if whatever apprehension that had gone on before is unrelated to that which is going on now. The pure intuition of time admits no breaks in the synthesis of apprehension because this would be tantamount to regarding apprehension and comprehension as aggregates of succession. Reflective judgment, however, marks the state of the synthesis by judging perceptions to be intuitions. The mark of an intuition *qua* intuition is that which we call a moment in subjective time. Perception so marked can then undergo the synthesis of re-cognition in a concept and it is from *this* context that we can speak of a starting concept. The similarity of this idea of a ‘stream of apprehension’ and James’ “stream of thought” is, hopefully, quite clear.

The laws of association, anticipation, and compatibility can now be seen in relationship to quantitative *modi* of the principle of interaction in the processes of judgment. Stated in terms of the 2LAR of the sensorimotor idea of Chapter 6, this principle of interaction is: the emergent property in the Relation of the ‘anatomical’ elements of reflective, determining, and practical judgments; the physiological idea of the three laws (Quantity); the Quality of seeming; and the modal idea of sensorimotor meaning. It is thus a principle belonging to the logical division of *psyche*. To apply this principle to the categories of Quantity, we note first that these categories are required to provide the extensive *materia ex qua* of cognition. It is up to apprehension to give final representation in intuition, the multiplicity in its extensive magnitude being what we call the intuition’s *materia in qua* as determined in the synthesis of the *Verstandes-Actus*.

Now it is not the case that every concept in the sphere of concept *A* is necessarily *materia ex qua* for the reproduction of perceptions via imagination. For instance, in the divided concept (that is, a concept in the form of a disjunctive proposition) the category of community “has a say” in the *materia ex qua* inasmuch as it is only a subsphere of *A* that is thought in a specific case. Similarly, anticipation means necessarily that the connected series of concepts under *A* are to be thought in a specific order in time. Considerations such as these are considerations of the *multiplicity in time* and speak to constraints placed upon the mathematical *composition of the unity* of cognition in the power of spontaneity. Such constraints are discursive rather than aesthetic and seated in the process of determining judgment.

However, limitations may also be placed upon the *materia ex qua* of the synthesis through interaction in the aesthetic Idea. If I should see a cat on my lawn, I do not automatically summon to mind everything I am able to think under the concept of ‘cat.’ The cat on my lawn may or may not “remind me” of a pet cat I owned as a boy; it may or may not remind me of the dissected cat a veterinarian friend of mine once showed me when we were students; I may or may not remember the sound of a cat’s purring, or think about the texture of a cat’s paws. Speaking now in general, the *materia ex qua* taken from the sphere of a concept is in part determined in reciprocity with the reflective judgment of the *Zweckmäßigkeit* of the state of perception. It is in this reciprocity we see the appearance of the principle of interaction as this principle is applied to the categories of Quantity. In engineering terms, we call such an interaction a ‘closed loop’ – an idea that expresses the free play of understanding and imagination in the synthesis of cognition.

But this character of the sensible representation of appearances necessarily presupposes in the categories of Quantity the responsiveness of their determinations to reflective judgments made on the state of perception. Under Rational Physics we considered the *a priori* logical function of the categories; under Rational Psychology, we now add to this the *a priori* transcendental function of the categories of Quantity, namely the ability of the category to determine concepts in accordance or *harmony* with an aesthetic Idea. From the transcendental

perspective, *the categories of Quantity are notions of association in the determination of concepts as the materia ex qua of the synthesis of reproduction concordant with an aesthetic Idea insofar as this association pertains to the extensive magnitude of the sphere of a concept* in terms of identity (unity), difference (plurality), and completion (totality).

It is worthwhile to note at this point that a similar harmony is required of the power of imagination insofar as *the category determines the transcendental schema in the synthesis of reproduction in imagination*. Thus we have the categories acting as patient to the agency of an aesthetic Idea *and* as agent to the synthesis in imagination of the transcendental schemata with regard to the synthesis of reproduction.

§ 6.3 The Categories of Quality from the Transcendental Perspective

From the logical perspective, the categories of Quality are notions of the matter of composition in the sphere of a concept. They pertain to inclusion within the sphere (reality), exclusion from the sphere (negation), or distinctions of subcontrarity by which we can represent partial inclusion and partial exclusion from the sphere (limitation), and thereby place a boundary for the sphere. When we turn to the transcendental perspective, our focus shifts from the sphere of the concept to its scope. The Idea of our thinking Nature under Rational Psychology that pertains to this consideration is the second psychological Idea, the Idea of unconditioned unity of Quality.

When we introduced this Idea in Chapter 4 we described it as the Idea by which we can regard the *I* of transcendental apperception as “logically simple” but are forbidden to regard the *I* as “materially simple,” i.e. as a simple substance.

That the *I* of apperception, consequently in any thinking, is a *singular* that cannot be resolved into a plurality of subjects, hence indicates a logically simple subject, is consequently an analytic proposition implied in the idea of thinking; but that does not mean that the thinking *I* is a simple *substance*, which would be a synthetic proposition. The notion of the substance itself always links up with intuitions . . . and hence must lie wholly outside the field of understanding and its thinking, which is all that can be properly talked about here if it is said that the *I* in thinking is simple. It would also be amazing if what otherwise requires so much care in order to distinguish what is in substance and what is unfolded in intuition, and even more whether this could be simple (as in parts of matter), were given so directly in the poorest representation of all as if by revelation [KANT1a: 446 (B: 407-408)].

Does this negatively-stated Idea say anything *implicitly* that could provide us with a positive principle for the categories of Quality from the transcendental perspective? The categories of Quality are reality, negation, and limitation, and this description of the second Idea of Rational Psychology seems at face value to have little to do with these notions. However, there *is* a tie-in between the Idea and the categories that emerges from the philosopher’s idea of ‘the simple’ when this idea is examined carefully under the Copernican hypothesis.

To get to this tie-in, we remind ourselves what is meant by ‘materially real’ in the Critical

Philosophy. The matter of a transcendental object is that which we say is ‘in’ the object that corresponds to sensation in an intuition. In other words, matter in this ‘corporeal’ sense of the word is the transcendental postulate of a correlate to sensation; it is that which we say the object “has” that “gives it the power” to have an affect on our senses. Having said this, it is important to at once call attention to the adjective “transcendental” in the phrase “transcendental object.” We are not talking about the matter of the object from the viewpoint of empirical physics here (as Aristotle, Locke, and other empiricists must do) nor from the viewpoint of rational *objective* metaphysics (as Plato, Leibniz, Wolff, and other rationalists must do). Rather, “transcendental” always pertains to rational and necessary interrelations between the object and the thinking Nature of the Subject. From the Copernican perspective, the matter of the transcendental object is a transitive Idea of the possibility that an object is capable of affecting the Subject. This Idea ties directly to our regard for the object as ‘something materially real.’

Second, reality in space¹, i.e. matter, is likewise a conditional, whose inner conditions are its parts, and the parts of those parts are the remote conditions so that here a regressive synthesis takes place, whose absolute totality reason demands, which cannot take place otherwise than through a complete division in which the reality of the matter disappears either into nothing or else into that which is no longer matter, namely the simple [KANT1a: 463 (B: 440)].

Under the Copernican hypothesis we do not say something is ‘real’ *because* it “has matter” or “produces an effect in sensibility.” Rather, we say something “has matter” or “produces an effect” *because it is real*. Reality *qua* reality is a *notion* – a category of Quality – *under* which stands the idea of “real things.” Reality *qua* reality is not an object of sense but a fundamental *Quality* of representation in determinant judgments and an *a priori* form of thinking that is necessary for the possibility of thinking “a thing.” Every conceptual representation is conditioned by the notion of reality (or another category of Quality), and this is why Kant says that an unconditioned object – i.e. ‘the simple’ – is “no longer matter” because only the categories are unconditioned in a concept. We must make our *descriptions* of the categories *in concreto* so that we may understand the appearance of the *idea* of a category and understand its role in representation. But, as a primitive in our ontology, a category is a ground for the very idea of thinking.

The Principle of Compatibility

With what we have just seen clearly in mind, we can now proceed to deduce the fundamental *positive* principle in the second Idea of Rational Psychology. This is the principle of

¹ ‘Space’ is Kant’s name for the pure intuition of outer sense. This term should *never* be equated with ‘physical’ space in reading Kant’s Critical Philosophy because Kant’s ‘space’ is a subjective space.

compatibility.

In the synthesis of reproduction in imagination, the mathematical categories are applied to the composition of the scope of a concept *A*. But, as we have seen previously, the logical momenta of propositions in the manifold of concepts includes combinations in judgments in which a concept *B* may lie outside the sphere of *A* yet still be combined with *A*. The “negative universal” form of proposition is perhaps the clearest example of this (Figure 8.5.5b). In terms of the logical momenta of judgments, the affirmative judges *B* as included (in whole or in part) in the sphere of *A*, the infinite judges *B* as merely contrary to *A*, and the negative judges *B* as contradictory to *A*.

Now let us examine the representation of the scope of *A*. In the synthesis of reproduction we understand by the categories of Quantity the bringing of concepts to this synthesis as the *materia ex qua* of a possible intuition. Now suppose concept *B* was judged as contradictory to *A*. Does this mean that *B* (which is, after all, combined with *A* even though it is not in the sphere of *A*) is to be “left out” of the synthesis of reproduction? It might seem this depends on whether this synthesis is being carried out as purely as a synthesis of spontaneity (in which case *B* has already been judged as outside the scope of the creatively produced intuition) or is being carried out as a synthesis that is both spontaneous *and* receptive, but this is not so. The synthesis of intuition must allow *materia ex qua* from the synthesis of apprehension (i.e., receptivity) as well as from the synthesis of reproduction, and in the former there is the possibility of inclusion of presentations of outer sense that can be recognized as *B* in the *materia in qua* of an intuition. *These presentations cannot be permitted to become part of the intuition* because they are contradictory to the scope of *A* (which we are here taking to be the subject of the synthesis). The only way in which the contradiction that would arise in this situation can be prevented is if the contradictory concept *B* is made part of the synthesis, but is brought “into the picture” in a “negative sense” as a compare in the act of comparison. *B* is then said to be *reproduced as a negative magnitude*, a topic we will discuss in detail later in this treatise. *B* thereby cancels the receptive presentation.

The principle we are discussing here is not that of association, for *B* is expressly ‘anti-associated’ with *A*. Rather, we are discussing here a principle of *compatibility* for agreement of the intuition with its conceptual representation. Intuition is the immediate object of a concept, and truth is the congruence of the cognition with its object. Truth is, so to speak, the ideal of understanding and the principle of compatibility is in this sense a principle regulating for the material truth of the cognition in that *unity of Quality* is a precondition for *congruence*.

This principle of compatibility is therefore the ‘positive’ principle we derive from the second Idea of Rational Psychology (unconditioned unity of Quality). It is a regulative condition for the synthesis of cognitions, which must always be oriented toward the ideal of truth in judgments. It now falls to us to apply this principle to the categories.

The Categories of Quality from the Transcendental Perspective

We begin by taking a look at the schematization of Quality by the transcendental schemata in the synthesis of imagination. The schemata of Quality from the transcendental perspective are:

for reality, being in time, i.e., presenting a representation as something that “fills” time;

for negation, non-being in time, i.e., presenting a representation as “not filling” time;

for limitation, a continuous and uniform production presenting a “being-in-time” affected by a “non-being in time” [KANT19: 466 (29: 998)] “as one descends in time from the sensation that has a certain degree to its disappearance, or gradually ascends from negation to its magnitude” [KANT1a: 274-275 (B: 182-183)].

In the synthesis of reproduction in imagination, a concept under the category of reality is imagined with a sensation that is to be posited in time. For negation, the concept is imagined in the mode of its sensation *not* being posited in time – i.e., the image of the concept serves as a kind of sensational ‘white-out’ that announces, so to speak, “this isn’t here.” This representation is *inhibiting* and is said to be presented as a negative magnitude.

The transcendental schema for limitation is perhaps the most obscure of these three schemata. Let us keep in mind that in representing the scope of a concept *A*, the reproduction in intuition has for the object of the representation the appearance of *A*. Imagine yourself conversing with another person at a table in a restaurant. As you listen to the other person, you are able to make out his words despite the hum of other conversations in the background and despite other sensations of sight, sound, smell, etc. of which you may be vaguely aware but are “paying no attention to.” You are not oblivious to these other perceptions; if someone drops his glass on the floor your attention is quickly drawn to the unexpected sensational disturbance of the sound of shattering glass. But, for all this, these perceptions mostly remain “in the background” while your companion’s words and gestures stay “in the foreground.” You are, in other words, capable of *concentrating* – making clear a specific part of the whole of your sensibility. It is this ability to divide up the whole of sensibility and give to one part of this division your “focus” that the schema of limitation describes. The schema of limitation does not *negate* the other sensations, but it does, so to speak, draw boundaries in this whole of sensibility. These boundaries may be “crisp” or they may be “fuzzy”; the degree to which the “foreground stands out” relative to the “background” may be great or it may be hard to differentiate. For some of the data of the senses, the degree to which this sensational data is “in the foreground” may be so slight that the perception is barely a conscious one; or the data of the senses may be suppressed to such a degree

that no consciousness of this data is presented.

Every intuition has, with respect to appearance, *intensive* magnitude – a unity in representation that can be thought as a multiplicity only in its approximation to negation. Now this idea of intensive magnitude is a difficult idea to grasp but an easy idea to misconstrue. When we speak of the intensive magnitude of a perception we do not speak of its degree as if this degree were like a column of mercury in a thermometer, and the categories of Quality are not notions of some kind of volume of sensation. To think of the degree of a perception in this manner is to think of it in terms of a number, but ‘number’ is the schema of Quantity as extensive magnitude, not Quality as intensive magnitude. The confusing of the latter for the former is a common error that Nobel laureate Henri Bergson emphatically pointed out.

When we assert that one number is greater than another number or one body greater than another body, we know very well what we mean. For in both cases we allude to unequal spaces . . . and we call that space the greater which contains the other. But how can a more intense sensation contain one of less intensity? Shall we say that the first implies the second, that we reach the sensation of higher intensity only on condition of having first passed through the less intense stages of the same sensation, and that in a certain sense we are concerned, here also, with the relation of container to contained? This conception of intensive magnitude seems, indeed, to be that of common sense, but we cannot advance it as a philosophical explanation without becoming involved in a vicious circle . . . The question, then, is how we succeed in forming a series of this kind with intensities, which cannot be superimposed on each other, and by what sign we recognize that the members of this series increase, for example, instead of diminishing: but this always comes back to the inquiry, why an intensity can be assimilated to a magnitude [BERG1: 1-2 (Chap. I)].

Bergson makes his case quite persuasively, and he goes on to point out several empirical examples which, upon close examination, very clearly seem to contradict all possibility of any use of ‘number’ as a valid schema of intensive magnitude. The ‘biological solution’ to this question that he proposes is not acceptable to us under the Copernican hypothesis because it attempts to use empirical results as the ground for metaphysical principles. However, this does not alter the fact that he has correctly identified that it is an error to think of an intensive magnitude as if it were a number.

We can come to this same conclusion quite readily without Bergson’s ‘biological’ substructure. Even from the simple example given above, it is easy to see that when we speak of the degree of a sensation or of a perception, this degree has nothing to do with any idea of ‘signal intensity’ (which is measurable, therefore has number as its schema, and therefore is extensive). Rather, the degree of a perception is the idea of what ‘focus’ is applied in one’s conscious attention. The distinction between ‘foreground perception’ and ‘background perception’ that we used in our illustration above describes quite well “what the appearance of this grasp of our conscious awareness looks like,” and the representation of this ‘grasp’ or ‘focus’ is what we mean by the degree of intensive magnitude in the synthesis of apprehension.

In terms of our 2LAR of the sensorimotor idea, this degree of perception is the idea of a

moving power – a Quality in a subjective agent-patient relationship in which the agency lies with either the power of receptivity or that of spontaneity to affect the conscious state. With regard to the mental physiology of *nous* in determining judgment, imagination, and reflective judgment, this moving power is the capacity of the matter of perception to affect the composition of an aesthetic Idea, and therefore to place reflective judgment in the role of patient with respect to imagination's role as agent. When we discussed Quantity from the transcendental perspective it was reflective judgment that acted as the agent; for Quality, the role is reversed and reflective judgment finds itself the patient insofar as its relationship with the aesthetic Idea is concerned.²

Now let us return our attention to the categories of Quality. The regulative principle for the categories of Quality is the principle of compatibility. Truth is the ideal of understanding, and no representation of understanding can ever be expedient for a purpose of Reason if that representation can not be cognized so as to have all of its parts compatible with each other in the *materia in qua* of intuition. The aesthetic Idea is not an objective perception, and, since all the representations of reflective judgment are subjective rather than objective, reflective judgment cannot judge objective truth. Reflective judgment can only 'reflect' – by means of its own representations – *conditions* for truth in judgments thought under the categories. Put in other words, the compatible representation 'feels right'; the incompatible representation 'feels wrong.'

But the basis for such a reflective judgment – the feeling of 'being right' or 'being wrong' – cannot lie with objects because in this case *a priori* reflective judgment could never be possible. Rather, the *materia ex qua* of sensibility must carry in its composition the information necessary for the possibility of the reflective judgment. And this is where the categories of Quality come into play. First, we have the case where the sensible representation of a concept requires a presentation in the *materia in qua* of intuition by the scope of the concept; the notion of this form of compatibility is the category of reality. Second, we have the case where the sensible representation of a concept *B* contradicts the sensible representation of a concept *A*; the notion of this form of compatibility is the category of negation, and it dictates under the law of compatibility that *A* and *B* cannot both be in the composition of the *materia in qua* of the intuition. Finally, we have the case where the sensible representation of *A* and *B* are compatible within the *materia in qua* of the intuition but are nonetheless contrary representations and must therefore be *aesthetically distinguished* in consciousness. However, since an intuition is *repraesentatio singularis* and an *objective* perception, the aesthetic distinction of the *materia in qua* arising from concept *A* from that of the *materia in qua* arising from *B* can only be a distinction presented in the aesthetic Idea. The 'foreground – background' distinction in our earlier example is an illustration of this idea. The notion of this form of compatibility is the

² In Chapter 16 we will see that the aesthetic Idea is a synthesizing *function* for judicial continuity serving to forge an organic unity between the power of reflective judgment and *psyche* in the Organized Being.

category of limitation.

We can summarize all this succinctly as follows: From the transcendental perspective, *the categories of Quality are notions of the form of compatibility in the determination of the materia in qua of intuition in the synthesis of comprehension and apprehension*. As primitive notions of compatibility, the categories of Quality provide the positive *a priori* ground for the principle of compatibility in the synthesis of imagination, i.e. for agreement (reality), opposition (negation), or distinction (limitation) in the synthesis, respectively.

§ 6.4 The Categories of Relation from the Transcendental Perspective

We now turn from the mathematical categories of composition to the dynamical categories of *nexus*. The idea of *nexus* is the idea of connection by synthesis of “that which is manifold insofar as they necessarily belong to each other,” and so it will not be surprising if the categories of Relation and those of Modality have a somewhat different ‘look and feel’ from the categories of composition.

The Idea of Relation in Rational Psychology is the Idea of unconditioned unity of all relationships (Chapter 4, §4.2). We used this Idea earlier to disestablish Descartes’ real division of mind and body. Our concern with the Idea at present centers on understanding our thinking Nature with regard to the categories of Relation and, in particular, with *real unity* that underlies the *logical multiplicity* of our constructs in cognition.

Now in every judgment I am always the *determining* subject of that relationship that constitutes the judgment. However, that I, the I think, can always be regarded in thinking as *subject*, and as something not merely regarded as predicate adhering in thinking, must be valid - this is an apodictic and even self-identical proposition; but it does not mean that I as *Object* am for myself a self-*subsisting being* or *substance*. The latter goes very far, hence also demands data that are not encountered at all in thinking (so far as I consider that thinking merely as such), perhaps more than I will ever encounter (in it) everywhere [KANT1a: 445-446 (B: 407)].

What Kant seems to have in mind here is Descartes’ dictum *I think, therefore I am*. For Descartes, the certainty that “I exist” is conveyed by the fact that I know I think and if I didn’t exist then I couldn’t “exist thinking.” From this point Descartes’ argument moved forward but he was unable to establish his ‘corporeal’ being with the same ‘apodictic certainty’ he saw as possible for his thinking Nature. Consequently, he divided himself into a *res cogitans* (“thinking thing”) and a *res extensa* (“extended thing”) and divorced the latter from his “true” self. While this is permissible as a merely logical division, we have already seen that it is not permissible to regard this division as a real division.

Equally, we cannot take the logical divisions of our mental anatomy for real divisions. But if we are to understand our thinking Nature, it is not enough to merely say “oh, this is only a logical division.” We must also have a *positive* principle of the *Existenz* of this real unity.

Now in all our conscious representations, the pure intuition of inner sense (the pure intuition of time) is the function of the *unconditioned* unity of these representations. Subjective time is not an object of perception but, rather, is a pure form of all our perceptions. We have spoken of ‘moments in time’ but let us recall that by a ‘moment in time’ we mean only a mark placed on a sensible perception, by reflective judgment, that distinguishes this perception as being formally expedient. A moment is not something that “divides time” as if time were a line that could be segmented into pieces. Indeed, the idea of a ‘timeline’ is the idea of an extended object, and while we might need to so visualize our *ideas* of time in order to explain its function, extension is no real characteristic of the pure intuition of time.

While Bergson seems to have seriously misunderstood a number of the key principles of the Critical Philosophy, he *did* clearly understand that it is incorrect to regard time *per se* in terms of extension. This, indeed, is the key idea of Bergson’s ‘duration’:

There are, indeed, as we shall show a little later, two possible conceptions of time, the one free from all alloy, the other surreptitiously bringing in the idea of space. Pure duration is the form which the succession of our conscious states assumes when our ego lets itself *live*, when it refrains from separating its present state from its former states. For this purpose it need not be entirely absorbed in the passing sensation or idea; for then, on the contrary, it would no longer *endure*. Nor need it forget its former states: it is enough that, in recalling these states, it does not set them alongside its actual state as one point alongside another, but forms both the past and the present states into an organic whole, as happens when we recall the notes of a tune, melting, so to speak, into one another. Might it not be said that, even if these notes succeed one another, yet we perceive them in one another, and that their totality may be compared to a living being whose parts, although distinct, permeate one another just because they are so closely connected? . . . We can thus conceive of succession without distinction, and think of it as a mutual penetration, an interconnection and organization of elements, each one of which represents the whole, and cannot be distinguished or isolated from it except by abstract thought. Such is the account of duration which would be given by a being who was ever the same and ever changing, and who had no idea of space. But, familiar with the latter idea and indeed beset by it, we introduce it unwittingly into our feeling of pure succession; we set our states of consciousness side by side in such a way as to perceive them simultaneously, no longer in one another, but alongside one another; in a word, we project time into space, we express duration in terms of extensity, and succession takes the form of a continuous line or a chain, the parts of which touch without penetrating one another [BERG1: 100-101].

When we say we “understand time,” what this really means is that we have made an object of the succession of appearances in perception. Since the cognition of the appearance of any object requires its representation in intuition, and since for us this representation is always dressed up by the pure form of outer sense (space), this idea of *objective time* must take on the character of extension – and this, of course, is a perambulation and an impediment to understanding subjective time in a theory. James tried to combat this problem with his idea of the stream of thought. We have just seen Bergson’s musical analogy. In the present treatise, we have thus far employed the analogy of a ‘stream of apprehension’ and a ‘stream of comprehension.’ All these *analogies* must call upon extension in the representation of the appearance of the idea.

Let us look at Kant's description of the idea of subjective time:

Time is nothing other than the form of inner sense, i.e., of the intuition of our self and our inner state . . . And just because this inner intuition gives no *Gestalt* we also seek to make up for this lack through analogies, and present time order through a line progressing to infinity, in which the manifold constitutes a series that is only of one dimension, and infer from the properties of this line to all the properties of time apart from a few things: that the parts of the former [the line] are at the same time but those of the latter [time] always follow one another . . .

Time is the *a priori* formal condition of all appearances in general . . . But since all representations, whether or not they may have outer things as their objects, nevertheless, as determinations of the mind, themselves belong to the inner state; yet this inner state belongs under the formal condition of inner intuition, thus of time; so time is an *a priori* condition of all appearances in general, and indeed the immediate condition of the inner intuition (our souls), and just for this reason the mediate condition of outer appearances . . . All appearances in general, i.e. all objects of the senses, are in time and likewise necessarily stand in relationships of time [KANT1a: 180-181 (B: 49-51)].

We may, and do, grant to time empirical reality when by 'empirical reality' we understand this phrase to mean "objective validity in representation." The pure intuition of time is empirically real from the viewpoint that it is a valid and unconditionally necessary power of representation for all objects that may ever be presented in our senses. But this does not mean we may regard time as a 'something' *external to* one's Self that somehow "attaches itself" to objects; such an idea is not an idea pertaining to the pure intuition of time but, rather, an idea of the *noumenon* we call objective time. We measure objective time using clocks, and physics treats its "nature" in the theory of relativity (which, by the way, makes objective time into the 'fourth dimension' of a thing that is given the name "space-time"). Thus, while we grant empirical reality to the pure intuition of time, we must also understand the *transcendental ideality* of time, "according to which it is nothing at all if one abstracts from the subjective conditions of sensuous intuition" [KANT1a: 181-182 (B: 52)].

It is in the pure intuition of inner sense and its product, time, that we find the ground of the Idea of unconditioned unity of relationship. As 'the intuition of our Self and our inner state,' it is to subjective time we turn in seeking the positive principle under Rational Psychology for our explanation of the categories of Relation.

Anticipation

In our earlier discussion of the synthesis of imagination we introduced three laws into our discussion. These are: 1) the law of association; 2) the law of anticipation; and 3) the law of compatibility. In the previous sections we saw that, from the transcendental perspective, the categories of Quantity are notions of association, and the categories of Quality are notions of compatibility. What, though, do we mean by 'anticipation'?

At first glance this seems a trivial question. Do we not all know what ‘anticipation’ means? Probably this is so *from an empiricist’s point of view*, but because this view always involves the priority of ontology over epistemology – it is an ‘external’ viewpoint – we should not be quick to assume that the empiricist’s meaning given to anticipation is necessarily the same meaning we must attach to it under the Copernican hypothesis. Let us then make a distinction between *empirical anticipation* and *anticipation in general*. It is with the latter that we are concerned here.

The common (dictionary) definitions of (empirical) anticipation are:

- anticipation**, n. 1. the act of anticipating; the act of taking up, placing, or considering something before the proper time in natural order.
- 2. foretaste; foreknowledge; expectation; previous view or impression of what is to happen afterward; as, the *anticipation* of the joys of heaven.
- 3. expectation.
- 4. something anticipated.

There are four different views of the noun ‘anticipation’ on display here. The first is anticipation viewed as an *act*; the second views it as a prior representation; the third views it as a state of feeling; and the fourth views it in terms of the object being anticipated. What all these descriptions have in common is the idea that anticipation has to do with “something before it happens.”

Now, in the synthesis of imagination the representations ‘in the stream of comprehension’ unmarked by reflective judgment have this same ‘before the fact’ character. They have, therefore, the flavor of being anticipations in the sense of the second dictionary definition given above. *Objective* perception means *cognition* and therefore refers only to intuitions and concepts. Thus these *subjective* representations ‘between moments’ are ‘anticipations’ regardless of whether they will come to cognition as representations of ‘the past’, ‘the present’, or ‘the future’. Until the representations come to intuition they are *a priori*, merely subjective factors in time and no more than *materia ex qua* for a possible perception.

But, as *materia* for the synthesis of imagination and the *materia ex qua* of a possible intuition, these representations stand apart from the first dictionary definition given above; they are not *acts*. Because we *make* our mental representations through synthesis, the possibility of perception presupposes the ‘know-how’ necessary to make them. Such ‘know-how’ viewed as knowledge *a priori* constitutes yet another ‘kind’ of anticipation and so we require a broader explanation of the idea of anticipation. Not surprisingly, Kant provides one:

One can call all knowledge through which I can recognize and determine *a priori* what belongs to empirical cognition an anticipation [KANT1a: 290-291 (B: 208)].

This is the general explanation. We must next put it in the context of the categories.

Transcendental Anticipations and the Categories of Relation

To do so we narrow our focus and consider only those perceptions brought to sensibility through the power of spontaneity. We will even take this one step further and give our attention to those anticipations of *a priori* 'know-how' necessary for the possibility of making perceptions through the synthesis of reproduction. These we will call the *transcendental anticipations*.

One class of transcendental anticipation is found in the transcendental schemata of Relation, i.e., the schemata of:

- 1) persistence of the real in time;
- 2) the real after which, as soon as discretion grants it, something else follows; and
- 3) coexistence of the determinations of the accidents of one substance with those of another substance.

With regard to the third of these schemata, we must keep in mind that these determinations are always reciprocal determinations of the accidents of appearance and they speak to the *Existenz* of the form of representation.

As schemata of the power of imagination these transcendental anticipations need no assistance from determining judgment because they have nothing to do with the manifold of concepts but instead with the re-production of concepts brought to imagination for presentation in sensibility. Yet we must still attend to the *nexus* of the manifold of concepts. Although the threefold synthesis of imagination is necessary for the formation of this *nexus*, it is determining judgment and not imagination that gives concepts back to the transcendental schemata of Relation in the free play of imagination and understanding. We require more than the merely aesthetic anticipations of imagination; we also require *discursive rules* that govern thinking in the selection of which concepts will come to imagination *and the manner in which* their reproduction is imagined. Only thus is it possible to achieve the harmonization in the free play of imagination and understanding that unites the Object and the transcendental Subject, and *this* unity is that which we call the real unity in cognition.

This brings us to the categories of Relation. From the logical perspective we saw these categories as the notions of the scheme of connection in the manifold of concepts, specifically as notions of the forms of propositions, i.e. Relation in propositions as categorical, hypothetical, and disjunctive. When we view them from the transcendental perspective, they must be notions that stand for the Idea of unconditioned unity of relationships, and thereby for real unity in cognition. But the ground of this unity is to be found nowhere else than in subjective time.

The synthesis of imagination is not bound to moments in time but, rather, to what we have called the stream of apprehension and comprehension. For the unity in the manifold of concepts to be also a unity in *all* relationships, subjective as well as objective, we must require *as a*

principle that this unity take its *Existenz* as a form of connection in time. While the categories of composition show themselves to be notions pertaining to the *materia ex qua* of possible intuitions (and therefore of the *materia in qua* of subjective perception in the stream of comprehension), we likewise see the dynamical categories in terms of the *materia circa quam* of this same synthesis of the stream of comprehension. This is the ***principle of transcendental anticipation***: the unconditioned unity of all relationships is grounded in the *a priori* anticipation of the form of the connection of perceptions in time according to the *modi* of persistence, succession, and coexistence.

Our understanding of the categories of Relation follows directly from this principle. From the transcendental perspective, ***the categories of Relation are the notions of the materia circa quam of transcendental anticipation in the determination of how concepts are to be connected in inner sense in the synthesis of reproduction, i.e. as immanent, transeunt, or reciprocal.***

These notions provide us with no *a priori* knowledge of *objective* time. Indeed, “past, present, and future” are not ideas contained *in* the categories of Relation; these ideas are marks tied to the idea of objective time, which is an empirical idea derived from experience. In the category of substance and accident we find the ground for thinking *things* through the *modus* of persistence in time. In the category of causality and dependency we find the ground for *empirical* anticipation in all its forms (from which the idea of objective time becomes possible). This ground is made actual through the *modus* of succession in time from one moment to the next. Finally, in the category of community we find the ground for the apparent manifold of Nature through the *modus* of coexistence in time, upon which the logical structuring and division of manifold sensations in terms of distinct objects is based. These are the *objective* consequences of the categories of Relation.

The *subjective* consequences are, however, equally important in the phenomenon of mind. We spoke in the previous sections of the interrelationship of the aesthetic Idea and the categories. Through seeming, we said, we have an inducement to determining judgment, and we said these judgments in turn affect the aesthetic Idea. This is the ‘what’ or ‘matter’ of the *a priori commercium* between affective and objective perceptions. The ‘how’ or ‘form’ of this *commercium*, on the other hand, gets its *materia circa quam* in the anticipations of the synthesis of imagination and, from the side of the spontaneity of understanding, this *materia circa quam* is given form via the categories of Relation. Transcendental anticipation concerns only the *a priori* determination of the form of the manifold of perception; empirical anticipation is concerned with objects.

In some ways, the principle of transcendental anticipation and the explanation of the categories of Relation from the transcendental perspective involves what is among the most abstract lines of reasoning we have employed thus far. It might be good, then, if we paused for a moment to inquire if the explanation outlined above exhibits any consequence of fact in

experimental psychology. We have previously reported, several times, that children still in the sensorimotor stages of mental development have no *a priori* recognition of permanent external objects (the word ‘external’ here taken to include even the objectification of the parts of their own bodies). Yet children do come, after a long apprenticeship, to form these cognitions and, when they do so, they do so as uncompromising little realists. That the formation of permanent object concepts relies upon the category of substance and accident is, I think, so obvious that it requires no deep discussion here.

But the thinking of these objects in realist terms is the thinking of the *Dasein* of these *as things*, and this sort of thinking clearly implies the empirical anticipation that what was perceived previously will be perceivable again under the same conditions. The theory we have just presented is such that we have no choice but to regard such empirical thinking as a phenomenon we should expect to see exhibited in behavior as soon as concepts of categorical propositions are sufficiently well developed to permit the structuring of hypothetical propositions at a level of refinement sufficient to be observable in infant behavior. The very youngest infants display no behaviorally observable and unambiguous indications of the anticipation of successions, although we should not expect this from them prior to the establishment of a sufficiently developed manifold of concepts. But do such behaviors show up *prior to* the fourth stage of sensorimotor development¹ (when the beginning of the establishment of permanent object concepts is unequivocally established [PIAG2: 91])? Piaget has demonstrated, quite clearly, that the answer to this question is: *yes*. While there are many examples documented in Piaget’s work, let us pick one that illustrates the behavior with particular clarity.

Obs. 5. A reaction slightly more complex than these is that of the child who stops looking at a certain picture and directs his glance elsewhere and who then returns to the first picture; that is the equivalent, in the realm of primary circular reactions, of the deferred reaction which we shall analyze in connection with the second stage [of sensorimotor development].

Thus Lucienne, at 0;3 (9) sees me at the extreme left of her visual field and smiles vaguely. She then looks in different directions, in front of her and to the right, but constantly returns to the place in which she sees me and dwells on it every time for a moment.

At 0;4 (26) she takes the breast but turns when I call her and smiles at me. Then she resumes nursing, but several times in succession, despite my silence, she turns directly to the position from which she can see me. She does it again after a pause of a few minutes. Then I withdraw; when she turns without finding me her expression is one of mingled disappointment and expectation [PIAG2: 10-11].

This simple example does not, of course, illustrate the rather sophisticated intellectual idea of “cause and effect relationships” we usually have in mind when we speak of causality and dependence. Instead, however, we see something much less developed, namely an anticipation on young Lucienne’s part that the turn-and-look game she is playing while nursing will produce the same result each time. When this anticipation fails to be realized, her reaction clearly

¹ About 8 to 12 months of age.

demonstrates that ‘she knew what she was expecting’ and was taken aback when events failed to fulfill the expectation. We see demonstrated here both empirical anticipation and the linkage of the anticipated intuition with the aesthetic reaction. At the same time, it is also absurd to suppose a child of less than five months of age perceives these events in terms of any idea of objective time, which would mean that the linkage of successive events for the child is perceived only against the substratum of subjective time, i.e. in the connection in intuition. A child at this age does not distinguish between the activity and the object – a fact Piaget has established quite solidly – and so this behavior contains much more of the flavor of Bergson’s musical analogy of duration than of the ‘spatialized’ concept of objective time that we, as adults, have so ingrained in our habits of thinking.

§ 6.5 The Categories of Modality from the Transcendental Perspective

From the logical perspective we called the Modality of determinant judgments “judgments of judgments” and explained the categories of Modality as notions of the schemes for determining problematic, assertoric, and apodictic propositions. When we view these categories from the transcendental perspective, we find our exposition of their Nature to be much more involved than is the case for the other three titles of determinant judgment. This is not so much because the categories themselves are all that complicated in terms of our thinking Nature. Indeed, we will discover in this section that they are, in a sense, “bit players” in the overall manifold of concepts. But the stage upon which they act spans the greatest scope in the phenomenon of mind that we have so far had to deal with.

This situation is a consequence of the Idea of Modality in Rational Psychology. I will begin by stating this Idea exactly as Kant expressed it (and, in doing so, add to our previous statement of this Idea a innocuous-looking qualification that now looms large and so can no longer be neglected). We will then undertake to closely examine this Idea to understand what it means for the transcendental perspective.

The fourth (modal) Idea of Rational Psychology is the Idea of *unconditioned unity of Dasein in space*. In our previous discussion of this Idea (Chapter 4 §4.2) the qualification ‘in space’ was of no concern to our discussion; I omitted it at that time because we had not yet introduced the ideas of the pure intuitions of sensibility. For reasons that will become clearer shortly, we must now deal with the implications of the little phrase “in space.”

“Space” is the name Kant gave to the *a priori* intuition of outer sense. The pure intuitions of space and of time are pure syntheses *a priori* of forms of sensibility; an empirical intuition is constituted as sensation (its matter) and the *a priori* intuition of space (its form). The pure intuition of time is, of course, as we described it earlier but since an intuition is a perception marked (by reflective judgment) at a moment in time, we need not linger upon it for *an* intuition.

The phrase “*Dasein* in space” tells us two things about the fourth Idea of Rational Psychology. First, we recall that *Dasein* denotes ‘existence’ in the sense of a *somewhat*, a subject-matter said “to be.” It is the ‘thing-like’ connotation of our English word, as opposed to *Existenz*, which is ‘existence’ in the sense of a *somehow*, i.e. the mode or manner (subject’s-*form*) “in which something exists.” Now, what or whose *Dasein* is this Idea addressing? The answer here is quite simple. The *I* of transcendental apperception is the unconditioned Subject of everything we talk about in Rational Psychology. The implicit “*I* think” (or one of its other modal forms) is ‘present’ in every act of thinking. When we speak of representation of any kind, “*I* represent” is always implied. When we speak of consciousness, it is this *I* who “is conscious.”

In Chapter 4 we used the fourth Idea merely to debunk the so-called mind-body problem and to show that the mind-body division can only be viewed with objective validity as a logical, not a real, division. In *this* section, our application of this Idea is of considerably broader impact for *here* we are concerned in detail with our thinking Nature. In this context, the word *Dasein* in the statement of the Idea is telling us that we are to investigate not merely the synthesis of imagination in apprehension and comprehension but, in addition, the *synthesis of apperception* – the *making of the state of consciousness*.

We have been content up to now with regarding the *I* of apperception in terms of *Dasein* without *Existenz*. In order to speak of the “unconditioned unity of *Dasein*” in the context of our thinking Nature, we must come to grips with *Existenz* as a form of conscious representation and see what is required of this *Existenz* if there is to be a unity (one-ness) in the state of consciousness. The scope of such an inquiry is vast because such a unity must be a complete unity involving the entire faculty (organization) of the processes of cognition, i.e. sensibility and imagination, determining judgment, reflective judgment, practical judgment, and the organization of their outcomes – understanding, reasoning, and judging.

Second, that this Idea of unconditioned unity of *Dasein* is unconditioned unity of *Dasein in space* tells us that we are dealing with unity in perception and, in particular, with what is required of *intuitions* to produce such a unity in the synthesis of apperception. The pure intuition of outer sense synthesizes an *a priori* form of intuition (space). Space belongs solely to the faculty of sensibility; there is no “pure intuition of space” in the manifold of concepts (understanding is not the same as intuition) nor do we say that the feeling of *Lust* and *Unlust* is an intuition. Rational Psychology is the pure metaphysic proper of our thinking Nature, and our thinking Nature has for its Object *cognition*. (The Object of Rational Psychology is the Nature of our thinking Nature).

Finally, though, we have at least one simplification to the discussion in this section, namely that our topic of discussion here is aimed at understanding only the categories of Modality from the transcendental perspective. We will, to be sure, have to come to grips with the details of all the other considerations just mentioned, but we will take them on one by one at the appropriate

places in this treatise.

The *Nexus* of Perception

We have described time as both an *a priori* intuition of inner sense and as the pure intuition of the state of consciousness. The adjective ‘pure’ distinguishes the intuition of time from empirical intuition in general, and so we should ask about the way in which time, as a pure intuition, is involved in empirical intuitions. If we say, as we do, that time is an *a priori form* of an intuition, in what sense can we mean this when every intuition is a *repraesentatio singularis* ‘at’ a ‘moment in time’? This might seem to be a paradox in our terms. To understand the meaning and resolve the paradox, we must look at the synthesis of apprehension & comprehension in which imagination produces the perception that will be marked as an intuition by reflective judgment.

Every intuition is a magnitude, both extensive (intuition of space) and intensive (the unity in which multiplicity can be thought only by approximation to negation). Let us look at the extensive character of intuition – magnitude ‘built up’ from the ‘successive addition’ of a multiplicity of homogeneous parts. The form given to an extensive magnitude, in an intuition, is what we basically understand by the idea of the pure intuition of space. It is the apperception of this character of extensive magnitude with which the fourth Idea of Rational Psychology is concerned. In our description of the synthesis of imagination that produces an intuition, we have pictured this process as a kind of assembly line for perceptions. We described the “next” intuition as “growing out of” the previous intuition so that “between moments” in time we imagine an evolution of representations being combined with one another to form new magnitudes¹. Borrowing from James, we are picturing this process of imagination as a “stream of apprehension & comprehension.” At various ‘places’ in this ‘stream’ reflective judgment pronounces that *this* magnitude is ‘now’ an intuition by marking it at a moment in time.

‘Between’ successive moments there can be other perceptions of imagination’s representations, although not of this particular cognition² (which requires its intuition). These are, in the Critical Philosophy, the correlates of James’ idea of ‘transitive parts’ and ‘substantive parts’ of thinking in his ‘stream of thought’ model. A moment in time marks perception as ‘being an intuition’; *it does not divide time* except only as a logical division. *Empirically*, however, we can hardly resist calling that which went on ‘before’ the ‘present’ moment “the past” and that which follows it “the future.” Furthermore, the category of causality and dependency makes it

¹ It is likely a good idea to remind ourselves here that in describing this synthesis ‘in time’ we are forced to make analogies by which we can come to a sensible understanding of the phenomenon - i.e., make a theory. This we can do in no other way than by trying to represent, as best we can, the appearance of this synthesis even if this description must be a circumlocution of a what is *per se* a pre-cognitive process.

² This is discussed in Chapter 21. Here let it suffice to say that synthesis in time is a “parallel process.”

possible to represent ‘now’ the appearance of “things that haven't happened yet” – i.e., empirical anticipation.

When we think we do not think “this representation, that representation”; rather, we think *objects*, and so, for empirical consciousness, this appearance of a distinction between “past, present, and future” in our cognitions poses for us a problem. How do we come to impute the objective yet empirical *significance* that “past, present, and future” holds for us without, in the process of doing so, shattering the unity of the state of consciousness? This question, while a question only concerning our ‘merely empirical’ thinking Nature, still demands a transcendental explanation. What *in the a priori* organizing of our cognitions makes it possible for our understanding of empirical Nature to be held together in the acts of representation?

This question does not really touch upon the synthesis of apprehension and comprehension, for the power of imagination certainly “sees to it” that its own process “holds together.” (It can not do otherwise and still be called a process). But just because the work of imagination is self-consistent, this is not enough to guarantee this “unity of self-consistence” is *also* going to be a unity for the *synthesis of apperception*. There is more to ‘knowledge’ than merely intuition, and the synthesis of apperception – because it *is* a synthesis – ‘adds’ something of its own to our thinking Nature. This something is, in a word, *meaning*. The positive principle implicit in the fourth Idea of Rational Psychology is simply this: ***the principle of unity in apperception of all perceptions in the interrelationships of meaning***. Every conscious representation – that is, every perception – must stand in a relationship of meaning, either immediate or mediate, with all other perceptions.

This at once brings up again a question we asked (but did not answer) earlier: *What does ‘meaning’ mean?*³ We will be a long time in fully answering this question, and for now we will have to content ourselves with a partial idea – an exposition of one of the characteristics of ‘meaning’ in the context of the phenomena of sensibility and understanding. Because we came to this question abruptly and through an empirical consideration, we will turn to Kant’s *Anthropology* to establish our footing for the transcendental considerations that will follow. It was in the empirical cognition of “past, present, and future” where this problem began, so let us look at the pragmatic relationship among them.

The capacity for knowledge of the present as a means of the connection of the representation of what is foreseen with that of the past is the power of designation. The act of the mind in effecting this connection is designation (*signatio*), which is also called signaling, of which the greatest degree

³ You may have noticed by now that whenever a question like this, one which threatens to lead to a vicious circle of reasoning, comes up in this treatise, we are dealing with a topic “close to the core” of the theory of mental physics - a primitive or a phenomenon of mind very close to the primitives of the theory. Such ideas *never* have a short one or two sentence explanation, as this chapter's exposition of the categories demonstrates. This is the “nature” of all *Realdefinition* and distinguishes it from *Realerklärung*.

is called the mark of distinction.

*Gestalten*⁴ of things (intuitions), so far as they serve only as a means of representation through concepts, are symbols, and knowledge through them is called symbolic or figurative (*speciosa*). *Characters* are not even symbols: because they can also be mere mediate (indirect) signs, they mean nothing by themselves, but only [acquire meaning] by association with intuitions, and through these [associations] lead to concepts; therefore symbolic knowledge must come to be set against the discursive, not the intuitive, in which the latter sign (character) accompanies the concept only as a watchman (*custos*) on the occasion of its being reproduced. Symbolic knowledge is therefore not contrary to the intuitive (through sensuous intuition), but rather to the intellectual (through concepts). Symbols are merely means of understanding; but they are only indirect, through an analogy to certain intuitions to which the concept can be applied, so that through the presentation of an object it obtains meaning [AK7: 191].

When we speak of “knowledge of the present” what we typically mean by ‘the present’ is “right now, give or take a little while.” The unspoken qualification here is that ‘the present’ is to be regarded as including that which has more or less immediately preceded ‘now’ plus that which will follow in the next little while, assuming nothing “significant” takes place that “changes the situation.” If we carry this pragmatic idea into our theory, ‘now’ means the present or most recent moment. But a ‘moment’ is nothing but a mark inscribed on perception by reflective judgment, and so for us ‘now’ can only be understood as designating the “current” intuition. It is this intuition, and the concepts attached in it, that constitutes the “knowledge of the present.”

The power of designation is likewise seen as the ability to *connect* – i.e., form a *nexus* – in which is combined all ‘present perceptions’ (which includes not only the affective perceptions but also perceptions that are anticipatory through their representation in a hypothetical proposition in the scope of the represented sphere). To designate – to make a “mark of distinction” – is to make a representation *symbolic* and thereby make of a mere presentation *symbolic knowledge*. The power of designation, however, does not belong to imagination. Neither the synthesis of apprehension, nor the synthesis of reproduction, nor the synthesis of re-cognition contain even the ghost of an idea that the representations of imagination are in any way symbolic. A symbol “stands for something” – that is, it *means something*. But “means something” is an empty phrase unless we amend it to: *means something to me*. The power of designation is a power that belongs not to imagination but to *the synthesis of apperception*.

But, Kant tells us, only an intuition can be a symbol and then only if it is an intuition of comprehension (and therefore takes some of its *materia in qua* from concepts). There is a question which, if we haven’t asked it of ourselves before, we should ask now. What is the use of having a perception singled out by reflective judgment and marked as an intuition? There is, of course, a purely technical benefit: if perceptions were not made into intuitions we would have no rational ground for speaking of a synthesis of re-cognition in a concept. But this explanation most surely has an *ad hoc* reek about it. It does not serve to prove that the processes of imaginative

⁴ The plural of *Gestalt*.

synthesis we have described are *necessarily* the processes at work in the phenomenon of mind.

But the great and marvelous thing about the phenomenon of mind is not that we can say it “makes representations” but, rather, that its most definite and fundamental characteristic is *knowledge*. Up to now, we have discussed at length the ideas of cognitions, intuitions, concepts, and so forth. But, until now, we have presented nothing in the theory which connects any of these things to the phenomenon of knowledge. When does empirical cognition become knowledge? **When an empirical intuition is *made symbolic*** by endowing it, via the synthesis of apperception, with a meaning. *This* is the use of the act of reflective judgment in marking a mere perception as an intuition at a moment in time, for it is at this step, *in the coalition* of the synthesis of apperception and the *Verstandes-Actus* of reflexion, when mere cognition becomes a representation of knowledge. **A meaning is that which subsists in this coalition.**

Thus, to go with the *logical perspective* of intuition that we have employed for the past few hundred pages, we now have the *transcendental perspective* of intuition. By bringing together all the subjective and objective elements of representation at the point of an intuition and connecting this intuition in apperception (making it *symbolic*), we see that *intuitions contain the matter of the nexus of all perceptions*. Unconditioned unity of *Dasein* in space is the Ideal of the *nexus* of all perceptions. The matter of this Ideal is empirical intuition; its form is subjective time regarded as the unity of inner sense and the state of consciousness.

The Categories of Modality from the Transcendental Perspective

Yet, although we say that an intuition made into a symbol is knowledge because it has been endowed with meaning, we must still make inquiry into the question: *What kind of meaning?* We can clearly *not* mean any sort of empirical meaning because the cognitive faculty is an organization that must be governed by principles and rules which are both pure and *a priori*. We must consequently draw a distinction between the idea of empirical meaning and the idea of *transcendental meaning*. Now, perhaps a bit to our surprise, transcendental meaning *is not a primitive*. We will see soon enough that the idea of transcendental meaning is describable in terms of characteristics of a more ‘abstract nature’ which, therefore, stand as coordinate concepts to the idea of transcendental meaning. Two of these coordinate ideas, which we have briefly introduced already, are *purpose* and *Zweckmäßigkeit* (expedience). The categories of Modality provide yet another.

Referring back to our earlier quote from *Anthropology*, let us now give our attention to what Kant called signs and characters. A concept, says Kant, cannot be or represent a symbol but, rather, is to be regarded as providing a “sign” that does not by itself “mean anything” but which can *acquire* a meaning “by association with intuitions.” The sign is said to “accompany” the

concept but “only as a watchman on the occasion of” the concept “being reproduced.”

How shall we interpret Kant’s watchman metaphor? What does this ‘watchman’ watch? In Kant’s empirical metaphor, it is plain that the role assigned to the sign is a custodial one (the Latin word *custos* being the root of our English word “custodian”). In Kantian terminology, a *characteristic* of a concept is any higher concept that serves as a mark for the recognition of that concept. But, as we saw in §5, this definition only has a context when the characteristic concept and the concept for which it is a mark of recognition have been combined in a determinant judgment. (Two concepts uncombined in judgment cannot be called higher and lower with respect to one another). **The manner in which two or more concepts are combined in determinant judgment to form a proposition is the character of that proposition.** Since the categories are notions of the scheme of combination in judgment, a character is defined by the rules (categories) under which the judgment has been carried out.

For the categories of Quantity, Quality, and Relation, this character determines the proposition with respect to that proposition’s role as a mediate representation of the appearance of an object. The categories of Modality, on the other hand, “add nothing to our knowledge of the object” but only determine the manner in which the concept is employed in the process of thinking. Thus, the character of the concept (that is, the concept as a proposition) given by the categories of Modality *is also a sign* for that concept (a sign is always a character, but a character is not always a sign), provided that the “sign of the category” can be shown to contribute to the meaning of a symbol when the concept is reproduced by the synthesis of imagination.

Now, the word “meaning” has connotations of *purpose*, *intent*, and *significance*. Under the Copernican hypothesis, and from the transcendental perspective, we can say that an intuition “has” any of these things *a priori* only through some connection of the intuition to a purpose of practical Reason. The reflective judgment of *Zweckmäßigkeit* is nothing else than a judgment of the expedience of the cognition – represented in an intuition – for a purpose of pure Reason. There are, however, only three modalities by which expedience can be determined during the act of reflexion. First, the expedience may be problematic (i.e. merely *possibly* expedient). Second, the expedience may be assertoric (i.e. *actually* expedient). Finally, the expedience may be apodictic (i.e. *necessarily* expedient). It is obvious that these three cases are precisely those of the possible *conscious* character of the concept, and so we see that the Modality of a determinant judgment is, from the transcendental perspective, to be viewed as a *sign*.

From this, we conclude: from the transcendental perspective, ***the categories of Modality are notions of the determination of a sign in the determined concept which can be made part of the symbolic meaning vested in an intuition in the synthesis of apperception.*** This sign determines the intuition as: 1) possibly expedient or possibly inexpedient; 2) actually expedient or inexpedient; or 3) necessarily expedient or merely contingently expedient for a purpose of pure

Reason.

§ 7. The Perspectives with regard to Determining Judgment

In this chapter we have dealt with the categories of understanding from the two perspectives relating to the metaphysics proper of *sensible objects*. Metaphorically speaking, we have been dealing with the ‘mechanisms’ of the matter of the process of determining judgment inasmuch as we have regarded the categories in terms of their role of the composition of judgments and how these judgments are interrelated to representation in sensibility and tied in with the process of reflective judgment. The logical and transcendental perspectives of the *Realdefinition* of the categories are, in a manner of speaking, “outcome oriented” inasmuch as the categories are viewed as primitive notions that ground the possibility of sensible cognitions of experience in terms of the representation of compositional matter of a manifold of objective perceptions.

We have not, however, dealt with the question of what ‘drives’ understanding through determinant judgments. Our discussion has been one of what determinant judgments do rather than how they come to do it. The process of determining judgment is, before all else, a *process* and the ontology of determinant judgments deals merely with the matter of this process. In regard to our *Realdefinition* of the rules by which we come to the outcome, we have so far neglected the rules under which this power of objective representation is *regulated*. In short, the idea of a process of judgment is also the idea of a mental *action* and we have described the composition of the action without describing the rules that bring a *nexus* to these actions. In this Chapter, we have looked at the categories as they pertain to Quantity in the 2LAR of the process of determining judgment (in terms of ‘propositions’) and the Quality in the 2LAR of the process of determining judgment (experiential concepts, notions, and ideas).

We must now deal with the categories in terms of Relation and Modality in determining judgment. These pertain to ideas of *inference* and *truth* and, in particular, with the *making* of inferences and the *determining* of truth in cognitive representations. The former has to do with the construction of the representation of Nature (the unity of the *Existenz* of all objects); the latter has to do with Reality (how we hold-to-be-true the connection of all objects as the matter of Nature). Thus our attention must next turn from the consideration of the representation of sensible objects (Rational Physics and Rational Psychology) to the representation of *objects of Reason* (Rational Cosmology and Rational Theology). With this purpose in mind, let us proceed to Chapter 9 and the ontology of speculative Reason.