Chapter 3   Epistemology-Centered Principles

1. Metaphysics

Since the 19th century it has been the developed tradition of science to ignore and exclude "metaphysical" considerations from the practice of science. This tradition began with the positivists and was in no small measure due to a reaction against the philosophy of Hegel [Seelye & Smith (1886), pg. 423]. In physics, Ernst Mach (1838-1916) was among the most influential spokesmen for positivism in science [Mautner (1997)]. Somewhat ironically, Mach's "anti-metaphysics" thesis is itself predicated on a metaphysic (specifically, Mach's personal metaphysic) for the fundamental reason that *every* metaphysic is nothing more and nothing less than literally "the way one looks at the world."

Mach's "way of looking at the world" was quite different from, e.g., how Aquinas "looked at the world" but it was no less a metaphysic than was Aquinas' (or anyone else's). Beginning in infancy, *every* human being develops his or her own personal metaphysic. Without one a person's perceptions would become an incomprehensible chaos and reasoning would be impossible. Your personal metaphysic is your systematic structure of unquestioned core beliefs that go into the presuppositions you make as you think and reason.

A person's metaphysic is in some ways similar to others' (because people's experiences are not entirely diverse) and in other ways it is unique and specific to the individual. It is, of course, possible for a person to redevelop and change his personal metaphysic; but most adults who do so only do so after encountering paradoxes - experiences that contradict the person's metaphysic[1] in ways that can be called traumatic. For Copernicus the impelling paradox was contradiction between his astronomical observations and the predictions of Ptolemaic astronomy. For Kant,

> I freely admit that the remembrance of *David Hume* was the very thing that many years ago first interrupted my dogmatic slumber and gave a completely different direction to my researches in the field of speculative philosophy. [Kant (1783), 4: 260]

Most people most of the time never encounter a reason to reexamine their personal metaphysic or even to have any reason to become conscious that they *have* a metaphysic that colors and biases how they think about and understand things. For example, Boyle's notion of "corpuscles" was a key part of physicists' "way of looking at the world" for centuries. The discovery of quantum phenomena at the start of the 20th century shattered physicists' unquestioned belief in "corpuscles" as a fundamental bedrock of nature and shook physics to its very foundations. Physicist Sir James Jeans wrote,

> A fourth such revolution has occurred in physics in recent years. Its consequences extend far beyond physics, and in particular they affect our general view of the world in which our lives are cast - in a word, they affect philosophy. The philosophy of any period is always largely interwoven with the science of that period, so that any fundamental change in science must produce reactions in philosophy. This is especially so in the present case, where the changes in physics itself are of a distinctly philosophical hue; a direct questioning of nature by experiment has shown the philosophical background hitherto assumed by physicists to have been faulty. [Jeans (1943), pg. 2]

Metaphysics matters. A person's metaphysic profoundly affects how he understands the world; and the world as it comes to be understood through one particular metaphysic can and does looks very different when viewed from another. For example, Einstein as a young man was profoundly influenced by Mach's view of physics, and his special theory of relativity reflects this. However, by 1917 Einstein had come to reject "Machism" in its simplest form as he developed his general theory of relativity [Bernstein (1973), pp. 130-132].

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[1] For me, this happened when I was a student studying quantum physics.

If philosophy is ever to earn back the prestigious standing it held in the times of the ancient Greeks it is going to have to start by rehabilitating how it practices and researches metaphysics systematically. If philosophy is ever to regain its old title of "queen of the sciences" it must again become scientific. Bloom angered many of his contemporaries when he wrote,

> Most interesting of all, lost amidst the collection of disciplines, modestly sits philosophy. It has been dethroned by political and theoretical democracy, bereft of the passion or the capacity to rule. Its story defines in itself our whole problem. Philosophy once proudly proclaimed that it was the best way of life, and it dared to survey the whole, to seek the first causes of all things, and not only dictated its rules to the special sciences but constituted and ordered them. The classic philosophic books are philosophy in action doing precisely these things. But this was all impossible, *hybris*, say their impoverished heirs. Real science did not need them, and the rest is all ideology or myth. Now they are just books on a shelf. Democracy took away philosophy's privileges, and philosophy could not decide whether to fade away or to take a job. [Bloom (1987), pg. 377]

This is a harsh judgment harshly worded, but it is not without strong elements of truth. The revolution the 20th century saw in physics, as Jeans' remarks express, was a manifest expression that "real science" does in fact need "real philosophy"; and "real philosophy" must begin with metaphysics.

Metaphysics, by the general consensus, first appeared in the philosophy of Parmenides in the late 6th or early 5th centuries BC [Marías (1967), pg. 20]. His central interest was to understand not "things as things" but rather "things for what they are," i.e., as "entities." The Greek word we translate as "entity" in English was *óv*, from which we get the name "ontology" for this branch of systematic metaphysics. For Parmenides the most fundamental quality of any "thing" was that it "had a being" of some kind and ontology was therefore "the study of being *as* being." Aristotle made this study the center of his philosophy and so here we will call this approach to metaphysics an "ontology-centered" metaphysic. 20th century philosophers often describe "ontology" as "the theory of what there is" [Mautner (1997)].

From this beginning and right up to the present day, almost all proposed systems of metaphysics have been ontology-centered. The fundamental issue and problem with any ontology-centered metaphysic is and always has been well illustrated by the question, "okay, if x is 'what there is' then what is x made of?" The potential for an infinite regress is obvious: if x is made of y then what is y made of? Like a little child who responds to everything you say to him with "Why?", the only way to bring the series of "Whys" to any end is to more or less say, "Because *I said so*!" (Many parents will recognize this situation; many children discover "the 'Why?' game" and delight in playing it - sometimes to the exasperation of their parents). Physicists' version of "because I said so" is called "a fundamental particle." How do we know if something - let us say an "electron" - is a fundamental particle? Well, because no one knows how to "split it up" into "more fundamental" fundamental particles. Most physicists will tell you an electron cannot be split into smaller "pieces" but, in fact, no one knows if it can be or not[2]. If anyone ever manages to split an electron he will instantly win a Nobel Prize - and a scramble to study "new particles" will begin. Any ontology-centered metaphysic must have its "fundamental particles" and searching for them eventually leads to ideas of what Newton called "occult qualities." For human beings in general the historically most popular "cutoff point" at which the search is terminated by fiat has been a deity of some sort. And *that* earned ontology-centered metaphysics its dismissal from science as "mere ideology or myth."

It is not necessary to begin with or to center metaphysics on ontology. The alternative is to begin with and center metaphysics on epistemology (the theory of knowledge). Epistemology inquires into the nature and possibility of knowledge as human beings come to possess (or think they possess) it. It deals with the scope and limits of human knowledge and how it is acquired and possessed [Mautner (1997)]. It is not

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[2] Leonard and Martin wrote, "Winston Churchill might have been talking about an electron rather than Russia when he termed it 'a riddle wrapped in a mystery inside an enigma.' It must be accepted at the outset that no one knows what an electron is." [Leonard & Martin (1980), pg. 2]

known for certain who first introduced epistemology-centered metaphysics. There is some reason to think it might have been the Greek philosopher Protagoras sometime in the 5th century BC. We do not know what his philosophy was because his works have been lost. But he is remembered for his assertion "Man is the measure of all things" and we know he did not speak of Parmenides' *όv* [Marías (1967), pg. 37]. This at least hints Protagoras' metaphysic might have been epistemology-centered. Whatever the case may be, we also know no major epistemology-centered metaphysic appeared again in philosophy until Kant around the year 1770.

Two effects the total dominance of ontology-centered personal metaphysics have had on philosophers since the time of Kant have been: that many of them do not recognize that Kant's metaphysic is epistemology-*centered* (although most do recognize its epistemological character); that in it ontology is demoted from its lofty pinnacle and "the theory of things" is made subordinate to "the theory of knowledge." Marías, for example, wrote "Kantianism has . . . a very pronounced tendency to become a theory of knowledge" [*ibid*., pg. 300]. He is mistaken; Kant's Critical Philosophy *is* a theory of knowledge (epistemology). Kant himself tells us it is in *Critique of Pure Reason* without ever using the word "epistemology" (for the simple reason that the word "epistemology" was not invented until 1854 by James Frederick Ferrier - 50 years after Kant died). Kant's word for it was *Kritik* [Kant (1787), B: xxii-xxv].

To some degree these shortcomings in understanding Kant's theory are both understandable and excusable. Kant himself is to blame for part of it. His work is deeply technical, he was inventing a subtle technical vocabulary on the fly as he went[3], his style of writing is filled with complicated run-on sentences that are difficult to follow, and he rarely gave examples to illustrate his meanings. C.E.M. Joad, who was a critic of Kant and his philosophy, said of Kant:

> Kant's thought is intrinsically difficult, and makes use of conceptions which are not readily apparent. Secondly, his exposition is exceedingly obscure. His method of writing is abstract and diffuse; he rarely condescends to use examples to illustrate his meaning, and his meaning itself changes in bewildering ways. . . . Nevertheless, such is the obscurity of Kant's writing that the reader can never feel quite sure that it *is* different. It always remains a possibility for the reader to reckon with that he has simply failed to understand what Kant is saying. [Joad (1936), pg. 359]

I think Joad renders a fair judgment here; certainly Kant's writings pose tremendous challenges for those who would translate his works from Latin and German into English. I have personally found myself becoming very irritated with Kant and his writing style from time to time as I have studied his works. I think only Aristotle rivals him in degree of difficulty to comprehend, and only Hegel tops him in this.

But there is a second and even more formidable barrier to overcome in understanding Kant. That barrier is the powerful influence of ontology-centered habits of thought the reader brings with him to his study of Kant. Many of the criticisms of Kant's philosophy I have seen arise directly from ontology-centered presuppositions. In trying to explain Kant's work to other people, these explanations often strike me as being like trying to explain Copernicus' theory using Ptolemaic astronomy. You simply cannot understand Kant's Critical Philosophy using ontology-centered presuppositions or an ontology-centered metaphysic. Putting it succinctly, a core premise of ontology-centered thinking is "our cognitions conform to the objects of cognition"; In an epistemology-centered metaphysic, this is reversed and becomes "objects conform to our cognitions." This reversal is what is often called "Kant's Copernican Revolution" in philosophy. The treatise now in front of you has a very special and important object to treat: Justice. The metaphysic with which we treat this object makes all the difference in how we come to understand it.

It is one thing to say "make your metaphysic epistemology-centered." It is quite another to know how to

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[3] Much of this vocabulary is buried in Kant's lectures or in his unpublished notes that only appeared posthumously in publications of Kant's collected works. During my Kant studies, I have often wondered if Kant presumed that the people who read his major books had also attended his classes.

do so. Toward achieving the latter, Palmquist made a great contribution in his 1993 book, *Kant's System of Perspectives*. His thesis is that there are various fundamental modes of thinking about and judging objects, and that the mode employed in a person's thinking and reasoning about anything has essential consequences and outcomes for, as I would put it, "the way a person is looking at the world." Palmquist calls these modes "perspectives." He wrote,

> For the Transcendental Perspective in general includes within it several levels of subordinate perspectives which are equally important in guiding the development of the various systems and sub-systems which compose Kant's System. Thus, what I shall call the '*principle* of perspective' (i.e., the universal rule that the truth is always relative to some perspective) can be seen functioning throughout the System; transcendental philosophy begins by giving the knowing subject the determining power formerly given to the object on general matters, such as questions regarding the nature and form of knowledge; so any *change* in the conditions adopted by the subject as the System develops will have a profound effect on the way the subject characterizes the object. [Palmquist (1993), pg. 28]

These perspectival changes during the course of Kant's writings are what cause Joad (and others) to think Kant's meanings "change in bewildering ways." Within Kant's "several levels of subordinate perspectives," Palmquist calls the three highest levels "Standpoints." These are: the theoretical Standpoint; the practical Standpoint; and the judicial Standpoint. They are reciprocally co-determining in processes of active synthesis in reasoning.

2. The Modern Critical Theory

It would be an error to suppose Kant's Critical Philosophy was in a finished state as he left it. His coverage of the theoretical Standpoint in *Critique of Pure Reason* was very thorough but the same cannot be said for the practical and judicial Standpoints in *Critique of Practical Reason* [Kant (1788)] and *Critique of the Power of Judgment* [Kant (1790)]. A very large portion of Wells (2006) and Wells (2009) is devoted to applying Kant's mathematical methodology, which he demonstrated in Kant (1787), to a technical deduction and explanation of the other two Standpoints.

That Hume had an important influence on Kant's development of the theoretical Standpoint has long been recognized. Beck wrote,

> Hume had argued that the causal connection was not a logical relation of ideas (that is, *a* is the cause of *b* cannot be deduced from the concept of *a*), nor, on the other hand can it be derived from experience, which merely tells us that *a* and *b* are usually conjoined. Its claims to be either logically or empirically justified as a necessary principle, therefore, had to be given up. The consequence was not only metaphysics but also the logic of the established sciences was threatened because no justification could be found for the basic law of nature that all events have causes. Kant discovered that Hume's problem was by no means limited to the justification of the causal law; he found that all the propositions claimed by metaphysics - for example, that substance is permanent, the soul is immortal, God exists - are like the law of causal connection in that they cannot be derived from experience nor from logic . . . Natural science, or at least the philosophical understanding of the natural sciences, was suffering under Hume's skeptical attack . . . Kant's problem, therefore, was two-fold:
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>   1. *To show that mathematics and the natural sciences are defensible against Hume's skeptical conclusions* . . .
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>   2. *To expose the illusions of speculative metaphysics and substitute for them metaphysics as a science* [Beck (1950), pp. xii-xiv].

However, Hume also wrote that "passions" actually "govern" human behavior [Hume (1739), Bk II] and that ethics are based on emotion and sentiment rather than stern rational reasoning [*ibid*., Bk III]. Kant was not such a cold fish as to fail to recognize that "feelings" played a role in behavior but he, like almost

every scientist and philosopher going back to the ancient Greeks, appears to not have taken Hume's point very seriously. Indeed, psychology did not begin paying scientific attention to "emotion theory" until near the end of the 19th century, and it did not become a respected subtopic in the field until the latter half of the 20th century. This lack of seriousness on Kant's part left the larger part of the judicial Standpoint untreated.

As for "ethics" and "moral theory," Kant rejected Hume's thesis. As Palmquist has pointed out, Kant manifested a "theocentric orientation" in his philosophizing [Palmquist (2000), pp. 7-13]. He explains,

> 'Theocentric' here does not mean Kant requires human *knowledge* of God to serve as the basis of or center for all other types of knowledge. On the contrary, it means the problems surrounding our understanding of the nature and reality of God serve as the central driving force of his philosophy. [Palmquist (2000), pg. 8]

As Wells (2006) explains, this theocentric orientation led Kant to make a serious and fundamental error in his treatment of the practical Standpoint. This error was the equating of the fundamental practical law of Reason (called "the categorical imperative") with a speculative thing he called "the moral law within me." The objectively valid explanation of the categorical imperative developed in Wells (2006) is a law in the strict sense of a principle of nature (as Newton would call it); "the moral law within me" is, in fact, not a law at all but rather is a speculative principle of an "ought to." It furthermore necessarily requires a supposition about human nature that is psychologically and psycho-physiologically testable. These tests have been made in modern times, and the outcomes of this testing refute the supposition. Human beings have no *innate* "moral law within me." Human nature does exhibit, as other neuropsychology research discloses, a manifest principle of a categorical imperative. The mathematical "flavor" of this principle has a close analogy to what physics calls Hamilton's principle [Clugston (2004)]. It can even be regarded as psychology's version of Hamilton's principle.

One of the manifested effects of the categorical imperative *is* "moral" in nature provided we understand the adjective "moral" to mean "pertaining to judgments of 'right vs. wrong' and 'good vs. evil'." It is a theorem of Critical epistemology that every human being *self-develops* what may be properly called a private and *practical* "moral code" as a result of his or her personal experiences [Wells (2012), chap. 4]. We all come to have such codes but they are neither innate nor do any two people have precisely the *same* moral code. The moral code of a sociopath is very different from that of a law abiding everyday citizen.

But although every person's private moral code is unique to that person, it is also true that it shares some practical precepts in common with others in his or her Society. The basic reason for this is that, in a Society, people have many experiences in which there is very little difference from one person to another. They generally share the same physical climate, customs established before the person was born, habits born of what people have to do in order to get food, shelter, and generally supply their everyday needs, and a common institution of government regulating at least some aspects of their lives. Every person is raised by at least one parent or caregiver, and most people grow up with one or more siblings. Differences in these exist but the differences are slight enough to be accommodated in the commerce of everyday life. There are, in a word, *norms* by which behaviors are judged, symbolism arises, and ideals are constructed. These norms produce societal mores, folkways, and normalized habits of thinking by which people judge "right vs. wrong" and "good vs. evil." These constitute what we call the *moral customs* of a Society.

These moral customs are not the same for every Society, of course, and, as we shall see, this has implications for the idea of justice. Critical epistemology is therefore challenged to explain the origins of these social phenomena and to understand how and why differences from one Society to another occur. One consequence of Kant's error noted above was that he was unable to arrive at a satisfactory explanation of these differences. It is a fault in his original theory that the modern Critical theory corrects.

This understanding has immediate pertinence for one of the most basic issues of law and justice, *viz.*, the conflict between personal liberty vs. social order and general welfare. Mill wrote,

> There is a limit to the legitimate interference of collective opinion with individual independence; and to find that limit and maintain it against encroachment is as indispensable to a good condition of human affairs as protection against political despotism.
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> But though this proposition is not likely to be contested in general terms, the practical question, where to place the limit - how to make the fitting adjustment between individual independence and social control - is a subject on which nearly everything remains to be done. All that makes existence valuable to anyone depends on the enforcement of restraints upon the actions of other people. Some rules of conduct, therefore, must be imposed, by law in the first place and by opinion on many things that are not fit subjects for the operation of law. What these rules should be is the principal question in human affairs; but if we except a few of the most obvious cases, it is one of those which least progress has been made in resolving. No two ages, and scarcely any two countries, have decided it alike; and the decision of one age or country is a wonder to another. Yet the people of any given age and country no more suspect any difficulty in it than if it were a subject on which mankind has always been agreed. [Mill (1859), pg. 4]

Resolving this difficulty calls for an objectively valid understanding of how and why the difficulty comes to exist in the first place. Achieving this understanding calls for understanding its source in human nature and, so, calls upon the Critical theory to provide this.

In what follows, this treatise is based exclusively on the modern Critical theory. This immediately raises an issue of its own. Educators call it "the prerequisites issue." A somewhat light-hearted way to describe the prerequisites issue is to phrase it as a question, *viz*., "How do we teach somebody anything without first teaching him everything?" For example, in most engineering colleges a student majoring in, let us say, electrical engineering doesn't see his first course in electrical engineering until his sophomore year. He spends his first year taking courses in analytic geometry, calculus, basic physics, chemistry, language, economics, and, perhaps, some "core curriculum" humanities course. Only after these has he "satisfied the prerequisites" for taking the introductory "core" courses in his major. Department, college, and university curriculum committees spend a considerable amount of time deciding upon what students have to take and when they have to take it in order to be able to satisfy the degree requirements of their majors.

The same issue is found in K-12 public education, although oftentimes it is easier to recognize what is a necessary prerequisite for learning something else. For example, a small child must first learn how to read before he can be taught almost any other academic subject. He must be taught how to count before arithmetic is within his grasp. Usually not mentioned but still vital to the learning process is the development and habituation of paying attention in class, doing classroom exercises, and other such "routine" social abilities that keep a room full of children from descending into playground-like free-form chaos[4].

The body of Critical theory is very technical, has a very large scope, has a specialized vocabulary, and is inherently mathematical in its fundamental character. The theory is epistemologically-centered and *this* means that most of its key ideas and theorems are based on a very different "way of looking at the world" than readers of this treatise are habituated to using. At the same time, the sheer size of the theory makes it impractical to present it in this treatise because that presentation would utterly distract us from the topic of justice. There is indeed a "prerequisites issue" in play here. The question is: How best to address it?

Some people have a preferred "learning style" that can be described as "bottom up": start with the root

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[4] My office at the university was across the street from one of our local elementary schools. It was often very instructive - and more than a little entertaining - to watch how the children behaved and interacted during their recess periods. There were discernable aspects of organized behaviors within different little knots of children playing together, but these organizations, and the knots, were apt to undergo spontaneous and abrupt disintegration and reformation into entirely different activities and behaviors. About the only fairly persistent behaviors I saw were that the kids rarely walked when they could run, rarely talked when they could shout, and the levels of energy displayed far exceeded what is typically expressed by adults.

essentials and gradually work up to more abstract and comprehensive topics. That would mean, in the present case, starting with the material provided in Wells (2006) and proceeding from there to successive works. This approach, if taken here, has the principal disadvantage of concealing "where this is all going" and obscuring the significance of most of the topics insofar as their pertinence to the idea of "justice" is concerned. In, for example, K-6 education this is the approach almost always taken in curriculum design (and for the obvious reasons just stated). Young children don't tend to question why they have to learn the things their teacher is teaching them nor "why is school is important?" if their parents tell them that it is. They are pupils and they learn the way pupils do. Youngsters in grades 7-12 do begin to ask themselves these sorts of questions, and this is when the next type of "learning style" begins being manifested.

By the time a person reaches the high school level, and thereafter, an important shift in "learning style" has set in. The style becomes more of a mixture of *initial* "top down learning" followed by "bottom up" learning. The person isn't a pupil anymore but, instead, is a student[5]. He or she is becoming gradually more "self-educated" and less dependent on concrete direct instruction. "Why do I need to learn this?" and "How will I use this?" become more central to and important for the learner's *interests* to be engaged in a topic [Wells (2012b). chap. 5].

What do I mean by this phrase "top down learning"? I refer by it to a method of investigation and learning that has been employed by scientific research since at least the days of Aristotle. He wrote,

> The natural way of [determining what relates to the principles of nature] is to start from the things which are more knowable and clear to us and proceed towards those which are clearer and more knowable by nature; for the same things are not knowable relatively to us and knowable without qualification. So we must follow this method and advance from what is more obscure by nature, but clearer to us, towards what is more clear and knowable by nature. [Aristotle (date unknown), Bk I, 184^{a}16-21]

By "things more knowable and clearer to us," Aristotle means phenomena we easily observe, study, and empirically characterize or understand. By "clearer by nature" he means concepts we deduce from principles of science (e.g., "the law of conservation of linear momentum" in physics). In this treatise I will follow Aristotle's dictum and try to keep things at the "level of phenomena" as we discuss topics. I will relegate the "things more clear and knowable by nature" to citations from the Critical corpus as much as I feel I am able without introducing misunderstandings or excessive ambiguities.

3. Mathematics

One of the things that looks very different with an epistemology-centered metaphysic is *mathematics*. Most Westerners, when they see a line of algebra written out, recognize it as "math" even if the odd little symbols in it are as obscure to them as Egyptian hieroglyphics at Luxor. And yes, certainly, a line of algebra is an example of *a particular **practice** of mathematics*. But what does "mathematics" mean *in general*? I don't wish or intend to offend my colleagues in the College of Education or the Mathematics Department, but *mathematics is not what most people habitually think it is*. Children in school generally take the pragmatic attitude that mathematics is whatever the teacher says it is. And what the teacher "says

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[5] In my lifetime I have seen the distinction between a "pupil" and a "student" become so blurred that today in the U.S. the two words are usually regarded as synonyms. It wasn't this way when I was a boy. Dictionary and Internet usages of the terms are really very unsatisfactory because they boil down to saying "pupil denotes a young student" as if "being young" is somehow a deficiency. A pupil is a learner who primarily acquires his knowledge directly and almost exclusively from being *told* things by a teacher. A student is a learner who, in addition to learning just the things he is told, is expected to go beyond this and acquire additional depth of learning by additional reading (study), reflection on the topic, additional practice, and reasoning [Wells (2012b), chap. 5]. "Pupil" is not a pejorative "word. Kant said, "the human being is first infant, then pupil, and then apprentice" [Kant (1803), 9:441]. I have known a number of adults who, despite their age, still learn many things in the way of pupils and not of students.

it is" in presenting the topic to them consists of examples of particular practices of mathematics. Some of these children go on to become future teachers and, when they do, they teach their pupils and students what they themselves have been taught. There might be no finer example of the faith children have in their teachers than this generation-after-generation shortfall in understanding of mathematics. But what schools teach is not the same thing as *mathematics per se*. It's just a little piece of it. To use one of Plato's oft-quoted examples, "knowledge of shoemaking" is not the same as "knowledge *per se*."

Kant defined mathematics as

Mathematics is the science of the construction of concepts. [Kant (1776-1795), 18:141]

In order to fully understand the technicalities of this definition one must know what "concepts" are [Wells (2006), chap. 3, pp. 194-203], the processes by which concepts are constructed in the "mental physics" of the phenomenon of mind [*ibid*., chap. 8, pp. 615-622], [Wells (2009), chap. 3], and what the roles of concepts are (that is, what they *do*) in thinking and judgment [Wells (2016b)]. As you might glean from the chapter and page numbers in the citations here, the "prerequisite" of "concept" has many other prerequisites standing in front of it. The Webster's Dictionary definition of "concept" as "something conceived in the mind" doesn't really tell us much about that "something," does it? The term is, as Newton and others liked to say, "well known by all"; I remark, "yes, the term is but the meaning is not."

Luckily, our goal in this treatise is not a thoroughgoing review of Critical Epistemology; nor is it to study mathematics [Wells (2020)]. Rather, our goal is to understand the idea of "justice." If we keep in mind that it is not necessary to study mechanical engineering in order to become an auto mechanic, or to have a degree in accounting in order to become a bookkeeper, perhaps you might find it plausible that we can make a Critical examination of the idea of justice without becoming Critical epistemologists or professional mathematicians if we pursue a careful enough "top down" inquiry into our topic.

My purpose in introducing the idea of "mathematics" just now is not merely to add another illustration of the idea that "things look different in an epistemology-centered metaphysic." The motivation for doing so is this: *most of the concepts of our inquiry are mathematical concepts*. I don't mean by this that you face a bleak and dreary trudge through algebraic hieroglyphics or have to solve trigonometry problems. What I mean is that we will be constructing abstract concepts in order to finally attain to the abstract concept of "justice." To do so is to *practice* mathematics as "mathematics" is Critically understood. This practice does not involve exercises in solving polynomial equations, doing calculus problems, finding areas of polyhedrons, extracting square roots, or any of the other types of things most often associated with "everyday mathematics." All these practices have their useful applications; that is *why* the practices were *invented* in the first place. But *our* application is different; it requires a different mathematical practice. We will develop it as we go.

The word "mathematics" derives from the Greek word μάθημα (*mathema*), which means "that which is learned; lesson." Viewed in this context, to practice mathematics is to embark upon learning about something systematically. Let me give you an example. In my career of scientific and engineering work, I was sometimes faced with figuring out solutions to problems that were not well described by the standard, everyday types of mathematics routinely taught in schools. It is an important principle in both engineering and science that you "should make the math fit (properly describe)" the problem you want to solve and you should never try to "force the problem to fit the math." If you do that, you can get a "mathematical solution" but it will be the solution to some other problem that is *not* the one you wanted to solve. If your typical, everyday math "tools" don't fit the problem then the thing to do is lay them aside and design a new "tool" that does. When I taught my students how to do this, I sometimes called it "rolling your own math." Mathematicians do it all the time but, for reasons I've never quite understood, seem to be reluctant to tell anyone outside the math department that this is what they're doing.

Kant had to do the same thing as he developed the Critical theory. Evidence of his doing so is seen in his handwritten notes from the 1760s to the 1780s. It's not clear whether or not he was cognizant that he

was doing this; the man never slowed down enough to jot down "I need some 'new math' here." But if you watch what he was *doing* - constructing new abstract concepts - you can see him "rolling his own math."

4. Some Basic Epistemological Definitions, Principles, and Theorems

Like every science, the science of Critical Epistemology has its own peculiar technical vocabulary and, again like every science, many of the words it uses are words commonly used in ordinary language. It is true enough that the words a scientist chooses to describe and explain his scientific topic often originate as metaphors of dictionary words. But in the science these words are given specific technical meanings and connotations that differ from the usages found in ordinary dictionaries. To take one widely known example, in physics the word "work" means something very different from what most people mean when they use this word. In physics, "work" is "the transfer of energy as a consequence of a force acting through a distance" [Clugston (2004)]. This meaning is clearly something quite different from what, let us say, a TV news anchor means when she says, "I'm going to work now." A technical vocabulary presents a splendid example of what Aristotle called "the homonymous usages of words." A good deal of what a student studies in an introductory college science course is devoted to learning its specialized vocabulary.

Critical Epistemology is no different. A glossary of its technical terms is provided by Wells (2016). There are, however, a number of terms and principles that come so frequently, and are so fundamental to understanding it, that it is expedient to set them out here so that when they come up in this treatise their meanings and implications are better understood. Along with them, there are also some basic principles and theorems of Critical Epistemology that are important to know because they constitute background suppositions underlying many epistemological arguments and deductions.

4.1 Our Empirical Knowledge of Human Beings is Holistic

This statement is a theorem of Critical Epistemology [Wells (2006), chap. 6, pp. 487-490]. Everything each of us learns about himself or about other people through experience is empirical. Furthermore, all one's experience of either him- or her-self is experience of the entirety of a whole human being. We never have an experience of a disembodied mind nor an experience of a living human body without a mind. We experience *only* whole persons. As a consequence, it is not objectively valid to make a *real* division between body and mind, such as Descartes did with his *res cogitans* and *res extensa*, and to then regard them as two distinct real entities. Doing so gives rise to the famous "mind-body" problem but this is a false problem because the idea itself lacks objective validity. It is a stillborn child of ontology-centered metaphysics.

At the same time, one *can* make a merely *logical* division between "mind" and "body" as a convenient and scientifically useful means of *mathematically classifying* one's experience into a class of "mental phenomena" and a class of "physical phenomena." People were doing this long before Descartes was born. Historically the most frequent of such logical divisions has been that of "body and soul" or "body and spirit." Plato and Aristotle both wrote at some lengths about "soul" and its supposed relationship with body. Aristotle was a bit less religious about it in his philosophy than Plato was in his; but, for all of that, both men's theories were transcendent speculations without objectively valid grounds.

If you are a religious person and hold-it-to-be-true that you have a soul, there's nothing wrong with that. No one can ever prove you are wrong. The Critical definition of *faith* is "holding-to-be-true that of which you know you *might* be mistaken." Critical *belief*, on the other hand, is unquestioned holding-to-be-true. It is a tiny little tincture of doubt that distinguishes faith from belief [Wells (2019)]. But, at the same time, you can never prove to anyone else you are right. The "object of your faith" is empirically unknowable (transcendent). It is the same in the case of an atheist who holds the proposition "I have no soul" to be true. His opinion is just as much a matter of faith as yours and just as much lacking in objectively valid grounds. Indeed, there is no conflict between science and religion, as I have shown elsewhere [Wells (2019)], provided each sticks to its own knitting and doesn't trespass into the other's domain.
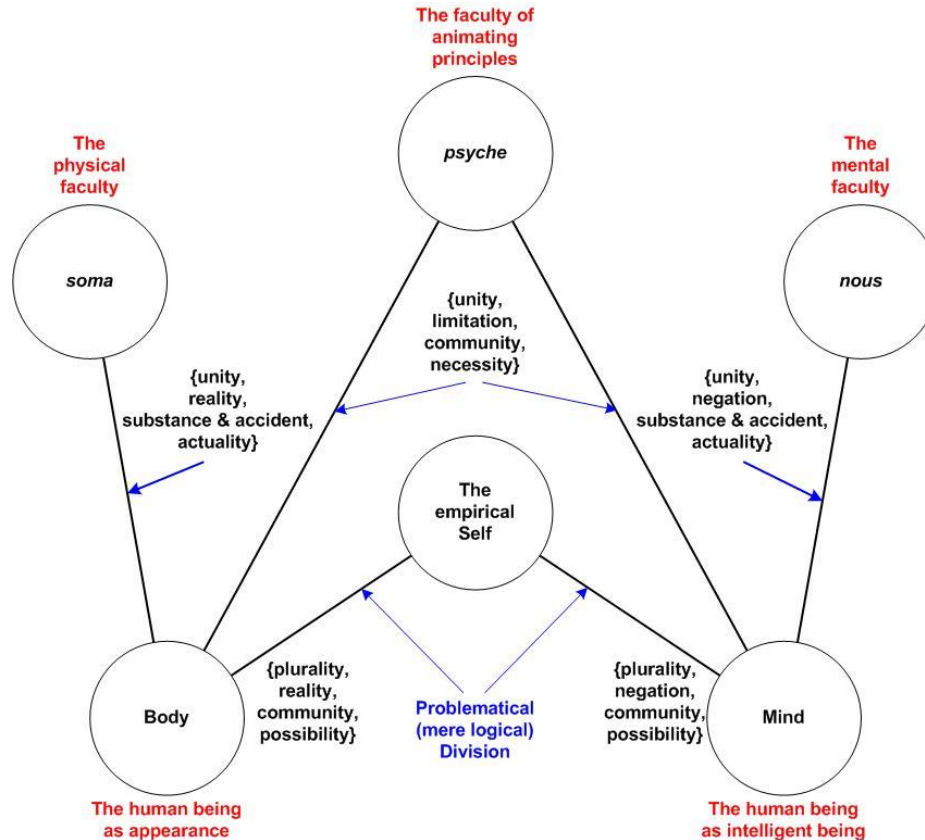
Figure 1: Illustration of the mathematical concept of an Organized Being and it relationship to the empirical Self. The circles represent concepts within the overall Organized Being concept. The connecting lines represent the types of determinant judgments by which these relationships are understood. In the mathematics Kant developed for his theory, a structure of connected concepts such as illustrated here is itself also a concept. The bracketed 4-tuple of words describing each connection of determinant judgment are called categories of understanding. In Kant's theory the word "faculty" means "the form of an ability insofar as that ability is represented as an idea of organization." The Critical word "faculty" has no *ontological* significance whatsoever. I said earlier that Kant "rolled his own math" to develop his theory. This figure is our first illustration of what this math looks like.

Because the logical mind-body division is a merely mathematical division into two mathematical Objects, there are other mathematical consequences that follow from its problematic logical modality[6]. These consequences pertain to how one must mathematically model the phenomenon of mind and restrict how the phenomenon of body may be modeled, with objective validity, in biology and physiology. These requirements and restrictions lead to a model of human nature which Kant called the Organized Being. Figure 1 illustrates the mathematical structure of the Organized Being model. The figure caption explains what the figure is "saying" to you.

Three of the concepts depicted in figure 1 - *soma*, *nous*, and *psyche* - are mathematical requirements of Kant's doctrine of the human being as an Organized Being [Wells (2006), chap. 1, pp. 77-78]. *Soma* (from the Greek word for 'body') represents the idea of the person's physical organization (the physical faculty). The Object of this concept is a topic of study for the physical sciences - physics, chemistry, biology, and their offshoot sciences. *Nous* (from the Greek word for 'mind') represents the idea of the person's mental faculty. It is the Object studied by the science of psychology. Neither of these technical terms appear in

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[6] In logic, problematic modality means a proposition is made with consciousness that it is possibly true but not known to be true.

Kant's works but were instead introduced in Wells (2006) for the purpose of providing better clarity in organizing and presenting Kant's theory. Indeed, I personally suspect Kant himself did not originally set out to develop the theory of mind that eventually emerged from his epistemology. Instead, I think this just happened to end up being the destination to which his research eventually took him. Hutchins & Adler (1952), pg. 177, remarked, "The notion of mind seems to have significance, for Kant, primarily in a collective sense," and I personally agree with this assessment. In Kant's day the word "psychology" was used, when it was used at all, by rationalist philosophers like Leibnitz or Christian Wolfe for what was essentially "soul theory" - a usage and theory Kant had fundamental disagreements with. As for "body," Kant rarely gave this even a passing mention, e.g.,

> A doctrine of the cognizance of the human being, systematically drawn up (anthropology) can be either in a physiological or in a pragmatic point of view. Physiological cognizance of the human being concerns the investigation of what *nature* makes of the human being, pragmatic of what he makes of himself, or can or should make of himself, as a free-acting being. [Kant (1798), 7:119]

Physiology, such as it was in Kant's day, was a far cry from the science we know today, was thoroughly laced with vitalism, and Kant was more or less dismissive of it as it was presented by, e.g., Descartes. It is little wonder that he seems to have regarded expending effort on "body theory" as a waste of his time.

The third concept, *psyche*, enters the Organized Being model as an immediate mathematical necessity of Kant's theory. Note in figure 1 that the judgment lines connecting the empirical Self with body and mind each carry the category of community as the category of Relation in these judgments. This category pertains to objects that coexist with each other in time. What this means is that body and mind are said to be co-determining in the empirical Self; this is to say that each is regarded as a partial cause of and, *at the same time*, a partial effect of the other. Relations of community are fundamental features in the idea of an Organized Being. Physics makes a practice of ignoring Relations of community in its ontology-centered way of looking at the world. Indeed, this neglect leads to confounding paradoxes, e.g., Rohrlich (1983), which, like the "mind-body problem," do not exist in the epistemology-centered way of looking at the world.

As soon as the concepts of *nous* and *soma* are introduced into the model, something else is required by Kantian mathematics to *enforce* the Relation of mind-body community because, as you can note in figure 1, this Relation is not presented in the connecting judgment lines between *nous*-mind and *soma*-body. This necessary "something else" is the concept of *psyche*, which is defined as the concept of a faculty of *animating principles* by which the reciprocity (co-determination) of *nous* and *soma* is restored. Without such animating principles, there would be nothing in the model by which goal-directed physical movements or the ability to feel physically stimulated sensations would be possible. Kant's discussions of this aspect of his theory were rather fragmented and presented non-systematically (not surprising due to his neglect of the body). Wells (2006) filled in this unsatisfactory lack of adequate discussion in chapters 6 and 15, and an augmenting discussion of the topic was presented in Wells (2009), chapter 4.

4.2 Mental Representation and Depictions

Kant's idea of **representation** (in Latin: *repraesentatio*; in German: *Vorstellung*) is the primitive foundation of his entire mathematics of epistemology. It is at the root of Kant's "Copernican hypothesis" that objects conform to our cognitions rather then the other way around. Failure to properly understand this idea means failure to understand Kant's theory.

As is true in other languages as well, most German words have multiple and homonymous definitions. Over the years this has sometimes sparked controversies among translators of Kant's works over the translations of *Vorstellung* as "representation." Here is where Kant's pairing up of this word with the Latin word *repraesentatio* is crucially important. Kant thought and taught that, if a writer wanted to preserve his meaning against becoming corrupted by subsequent changes to the language, he should write it in a dead

language because in such a language the meanings of words were no longer being changed, as they are over time in living languages. Kant used Latin for that purpose. This is why it is always a cardinal error for a translator (or a copyist) to either paraphrase or "grammar correct" Kant's Latin phrases (as some Kant translators do). It was not an uncommon practice for 18th century authors to suddenly insert Latin phrases into the middle of a sentence[7]. They weren't doing it just to show off.

The pertinent connotations of *repraesentatio* are: (1) the act of bringing before the mind; and (2) a re-embodiment, an image. The first connotation is an act (the act of representing), the second is the outcome of such an act. The English word "representation" carries these same two homonymous connotations. The German word *Vorstellung* likewise carries two pertinent connotations: (1) presentation; and (2) representation. Some scholars do make a strong case that Kant should have used the word *Darstellung* instead of *Vorstellung* in his writings, but the fact is he paired *Vorstellung* with *repraesentatio* and tended to use the word *Darstellung* to mean "depiction." Furthermore, he taught that

> what representation is in itself is inexplicable. A definition of that cannot be given because a representation can be explained only and in no other way than when one again represents a representation to oneself, hence there is lacking grounds of cognition in the logical sense. This act of the mind can be described as something in me that refers to something other. This reference to this something other in me is representation now taken subjectively. The representation is aimed in part *at the Object* to which I am referring, in part *at that action of the mind* through which I compare something in me with the Object. [Kant (1794-1795) 29: 970]

Here Kant is explicit in saying representation is "an act of the mind" (connotation 1 of *repraesentatio*). However, it is sometimes important to be able to distinguish between connotations 1 and 2, and in order to do so I have introduced a new term, not found in Kant's writings, for this. The term is *parástase*, which is Greek for "depiction."

Although "representation in itself" is an inexplicable primitive when regarded ontologically, that doesn't mean *what it does* is indescribable or that there is no methodology for understanding its *practical essence* [Wells (2006), chap, 3, pp. 153-175]. After all, primitive bases are also found in physics and in mathematics, and their "inexplicability" does not prevent physicists or mathematicians from using these ideas. Indeed, a theory of nature is nothing else than an intelligible representation of nature.

While Kant never explicitly presented a formalized methodology of "representation," one nevertheless can be found by watching *how he used* the notion of representation and reflecting upon these practical manifestations. Palmquist was the first to present a mathematical methodology providing a symbolic way of representing the practical essence of representation [Palmquist (1993), pp. 81-103]; Wells (2006) adopted this methodology with some alterations to the symbolism. I use the term practical essence to mean what the outcome of an act of representation (its *parástase*) must have if that outcome is to be usable in practice.

Minimally, any act of representation must provide two things. First, a representation must provide a "what" that a representation *represents*. This "what" is called *the matter of the representation*. Secondly, it must provide for "how" this matter is represented. This "how" is called *the form of the representation*. Any represented *parástase* can, therefore, always be analytically divided into a *matter* and a *form*. Analytic division is a process by which one seeks to obtain knowledge through the construction of concepts and, therefore, "matter" and "form" are mathematical ideas when regarded epistemologically. These two terms have no *ontological* significance whatsoever.

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[7] Modern scientists are doing the very same thing except they use mathematics in place of Latin, and they do it for the same reason. As languages go, math is a very peculiar language but there is none better for saying things precisely and in a way such that consequences of what is said can be deduced. In my opinion, "math as language" should be part of every high school science course. I say more about "math as language" in Wells (2020).
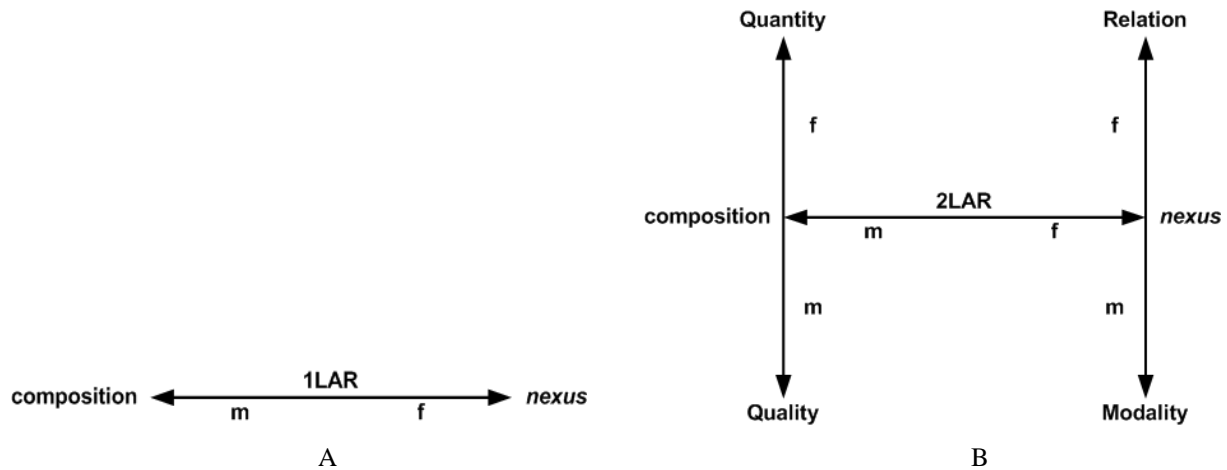
Figure 2: Illustrations of analytic divisions of matter (m) and form (f). A: a first level analytic representation (1LAR); B: a second level analytic representation (2LAR). Any number of analytic divisions are possible, resulting in 3LAR, 4LAR, etc. representations. Notice that each additional division is an additional matter-and-form division. Kant called the matter term in a 1LAR its "composition"; the form term he called its "*nexus*" [Kant (1787) B: 201-202fn]. Dividing a second time to make a 2LAR gives us a "matter of the matter" (Quality), a "form of the matter" (Quantity), a "form of the form" (Relation) and a "matter of the form" (Modality).
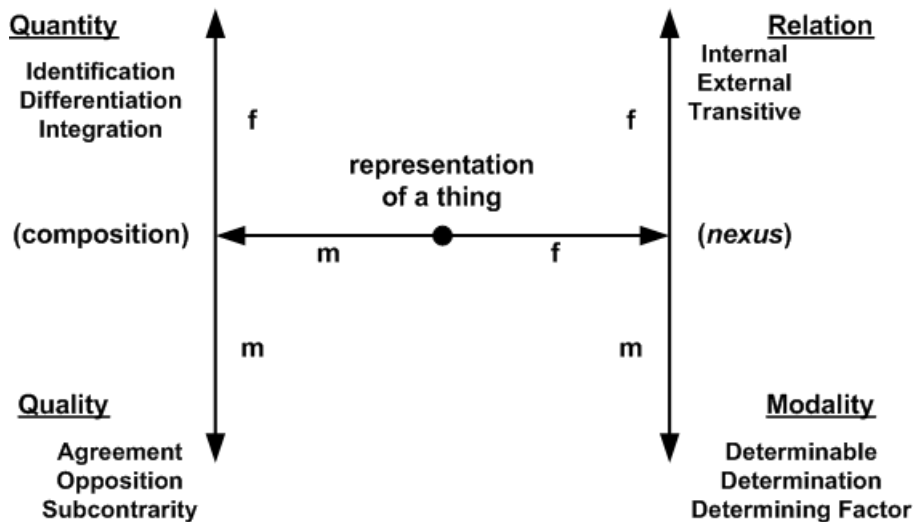
Figure 3: 2LAR for the general representation of a thing with its twelve required synthesizing functions (*momenta*).

Analytic division is the first tool in our mathematical toolbox of representation theory. Figure 2 provides illustrations of how this tool is used to represent representations. The figure's caption provides descriptions for understanding the symbols and applying the tool. Note, though, that analytic division is used for "taking a *parástase* apart." We also need to be able to "put one together," and doing this is called "synthesis." Performing a synthesis requires *synthesizing functions* for "going back" from the endpoints of an analytic representation (1LAR, 2LAR, etc.). Kant's name for functions of this kind was *momenta*. Figure 3 illustrates one particular case, namely that of the representation of a thing at the 2nd (2LAR) level of representation. Each endpoint or "heading" of a 2LAR requires three *momenta* [Wells (2006), chap. 3, pp. 166-175] because synthesis operations are always threefold. For a 2LAR one *momentum* is required from each of the four "heads" for a total of four functions (refer to figure 1 again). This means $3^4 = 81$ different kinds of synthesis are possible at this level. A 3LAR would have eight "heads" and, therefore eight *momenta* per synthesis for $3^8 = 6,561$ possible kinds of synthesis.

The twelve *momenta* named in figure 3 are generalized classifications of more specialized mathematical synthetic functions found in 2LAR representations that apply to particular representational processes within the Organized Being model [Wells (2006). chap. 3, pp. 166-175]. As functional *genera*, these twelve *momenta* are not primitive functions in the theory. They are instead analogous, in terms of the roles they play, to what are called "functionals" in formal mathematics. Furthermore, they pertain to *practical mental acts* and make no *ontological* determinations despite the word "thing" in the name of the 2LAR. Their deduction was made starting from Kant's analytical treatment of what he called "transcendental relationships" [Kant (1787) B: 316-325]. In Critical epistemology, a **thing** is "an object when regarded in terms of the possibility of an actual or necessary existence independent of the Organized Being who represents that object." The "object" of a representation is, simply put, "what the representation is aimed at." The combination of the object and its representation is called the **Object**.

"Matter" and "form" are two notions of obvious importance in Kant's theory of representation. Under ontology-centered metaphysics, both of these terms have long posed major problems for philosophers. Many of today's philosophers, in fact, seem to have altogether run away from both terms (and, indeed, from metaphysics as well). *The Penguin Dictionary of Philosophy* [Mautner (1997)] omits any entry for the term "matter" and discusses only Plato's theory of "forms." *The Oxford Dictionary of Philosophy* [Blackburn (1996)] discusses "matter" only for the context of physics theory and, likewise, "form" only in regard to Plato's theory of forms. Perhaps this speaks to the enormous difficulties both terms have confronted ontology-centered metaphysics with.

In the Critical Epistemology neither term carries any ontological significance. Crudely put, matter pertains to "the what" representation is *representing* and form pertains to "how" it *represents* it. Kant put this a little more verbosely by saying,

> *Matter* is the *datum*, what is given, thus the *stuff* - But *form* is how these *data* are fixed, the manner in which the manifold stands in combination. . . . In *transcendental* understanding every *datum* is matter but form [is] the *relationship* of the *dati*. Transcendental matter is the *determinable*; but transcendental form the determination or the act of determining. Transcendental matter is the reality or *datum* for all things. But the limitation of reality makes transcendental form. All realities of things lie as if in infinite matter, where one thus separates some realities for a thing, which is the form. [Kant *c*. (1790-91) 28: 575]

The terms reality and existence have always been very troubling words in ontology-centered metaphysics. Snugly tucked away inside the quote is a hint of one of the most important theorems of the Critical metaphysic: *All things are real in some contexts, unreal in other contexts, and non-real in still other contexts* (which is to say the thing does not pertain to the context). The ghost of Hamlet's father is real in the context of characters in Shakespeare's play, *Hamlet*; he is unreal in the context of an actual tormented spirit who haunts a castle in Denmark; and he is non-real (has no pertinence) in the context of molecular biology (which doesn't mention him at all and excludes him from that topic).

Matter and form are also pertinent notions for the notion of "existence." Here the German language has an advantage over English because it has *two* words that both translate to English as "existence" but have very different connotations. The first is ***Dasein***, a declaration of some "what" that exists; the second is ***Existenz***, pertaining to "how it exists." Logically, *Dasein* declares the subject of a predication; *Existenz* declares the predicate. The notion of matter speaks to *Dasein*; that of form speaks to *Existenz*.

4.3 Mathematical "Anatomy" of *Nous*

One of the factors that makes understanding Kant's overall theory challenging is that his books and lectures focus in on specific pieces of it but he never did present an overall summation of it. He left it to his readers and students to assemble the pieces and put together a systematic "picture" of it. All the necessary hints and clues are present within the Kantian corpus, but it has no roadmap to the destination. His theory most certainly came with "some assembly required" but no assembly instructions.
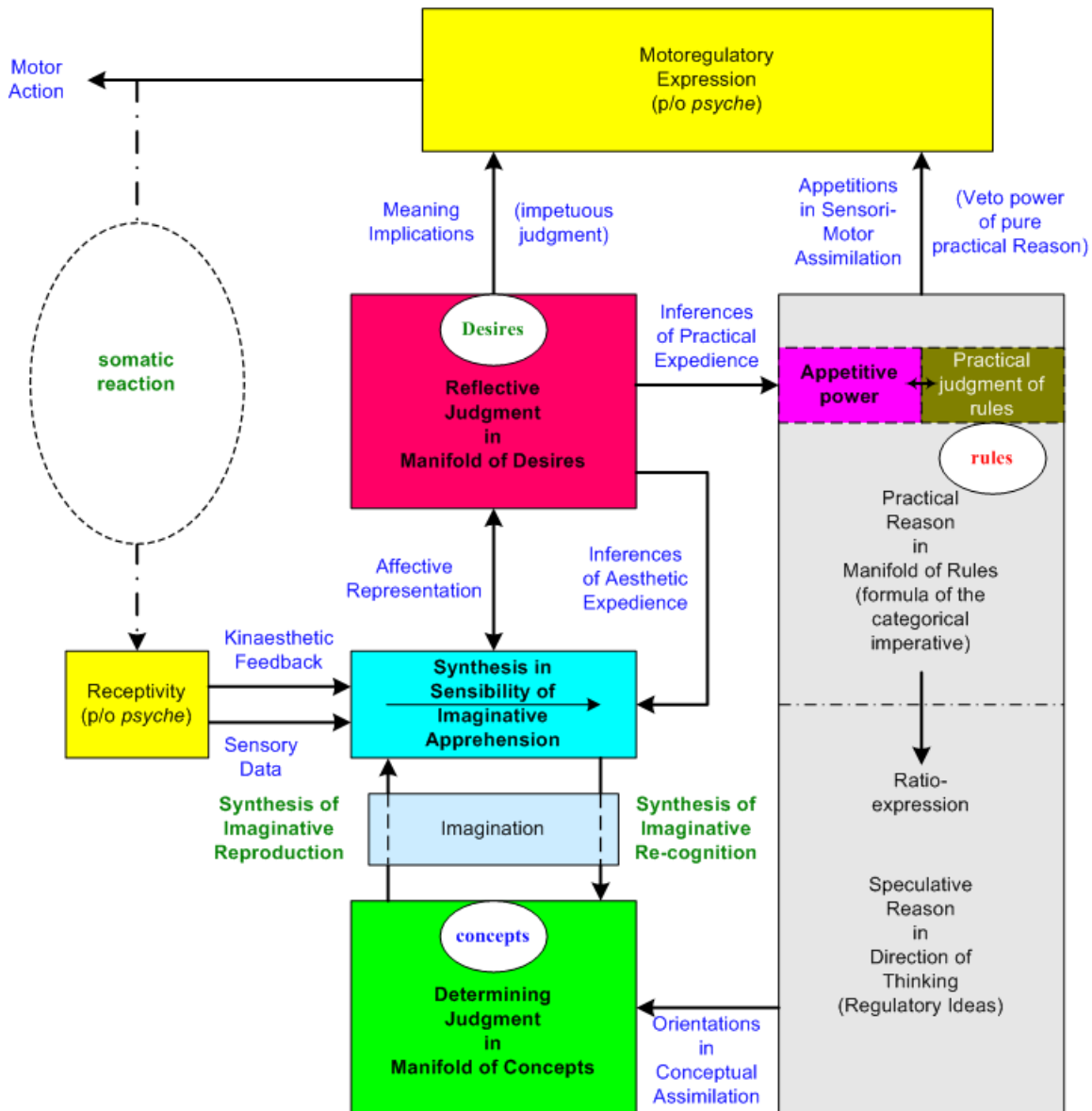
Figure 4: The mathematical organization of *nous* and its interrelationships with *psyche* and *soma*. The blocks represent the major mental processes of representation and judgment for the construction, structuring, transforming, and cognizance of information. Directed arrows show information pathways flowing from parts of this "anatomy" to other parts. None of the depictions in this figure have ontological significance; all of them have epistemological significance. It can be anticipated, owing to the principle of mind-body reciprocity, that future empirical research should eventually allow science to establish specific empirical connections between the mathematical processes in this figure and corresponding biological structures of human anatomy. However, we should also expect full accomplishment of this to lie many years into the future from the date of this treatise.

With the aid of much insight supplied by 20th century psychology, the sciences of system theory and information theory, and modern systematic neuroscience research, an overall "picture" of the organization of the theory of *nous* was pieced together over a three year period from the findings of Wells (2006). This was first presented in Wells (2009). The resulting mathematical "mental anatomy" of Kant's system is summarized in figure 4. A post-1950s engineer would call this figure a "block diagram" of Kant's system.
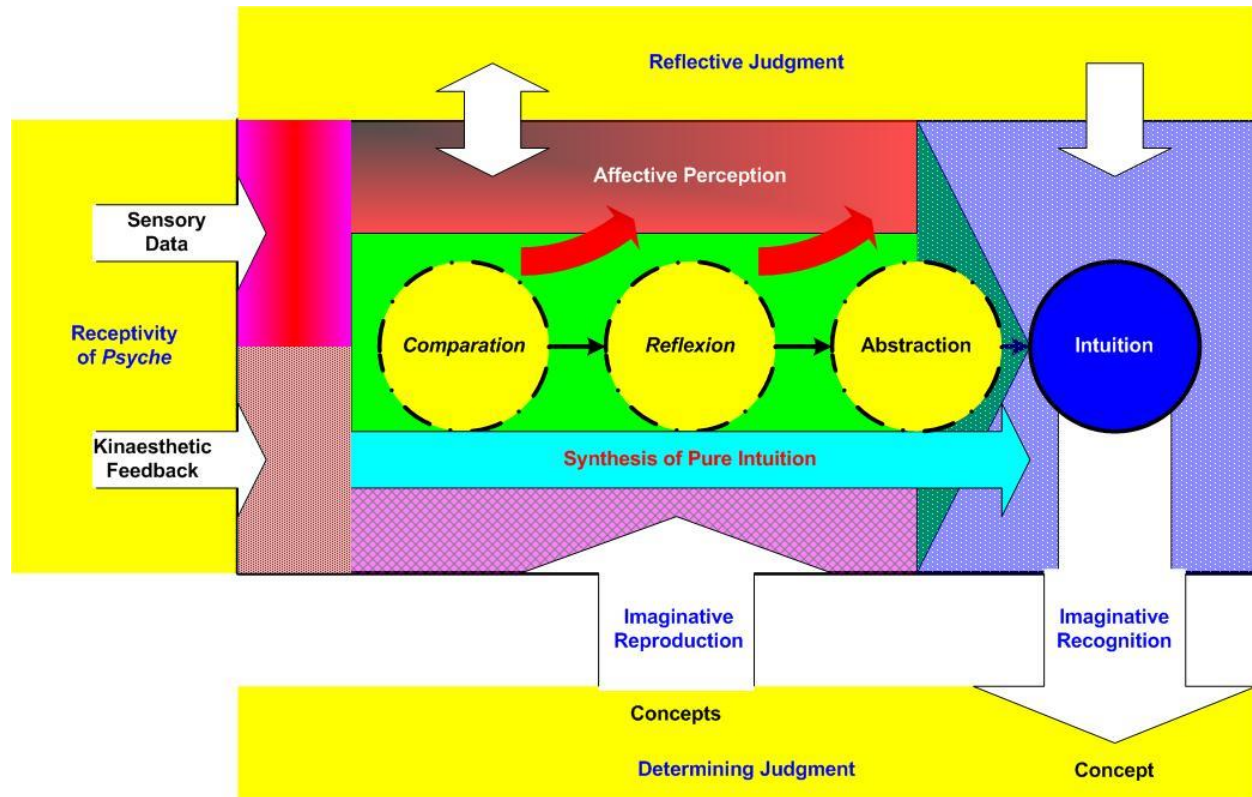
Figure 5: Detailed description of the synthesis in sensibility.

My purpose in introducing figure 4 is that it will aid us in following and sufficiently understanding what in human nature underlies such phenomena as societies, the idea of justice, and how human nature gives rise to mores, folkways, and systems of social morality - all of which are important aspects of that-which-we-call justice. The "what" of justice is the determinable (matter) of our topic. Figure 4 will be helpful in understanding its determination (form).

I do not expect or require any reader of this treatise to need comprehensive and detailed knowledge of the theory represented by figure 4 in order to understand the arguments and discussions that follow in the subsequent chapters. One does not need to know the detailed chemistry and construction of a battery in order to know how to use a flashlight. One does need to know flashlights require batteries, that batteries eventually run down, how to replace them when needed, where to buy them, and what kind to buy. Figure 4 and the text below are offered in a similar "spirit of instruction" for dealing with the "prerequisites issue" mentioned earlier in this chapter.

4.3.1 Let us begin with the **synthesis in sensibility** in figure 4. Of all the processes depicted in figure 4, this is the one that is arguably "clearest to us" because it is the one which deals with *sensuous* matter and the determination of *constructed* possible perceptions. Figure 5 provides a more detailed description of this process. I call this process the "clearest to us" because, after all, what is more direct, familiar, and immediate in all your experience than the sensations you perceive and the feelings you feel? Indeed, you may be so familiar with these that under most circumstances you take them entirely for granted. I doubt if you would be surprised if I said sensuous matter "has something to do with the senses." As for perception, Critical Epistemology defines perception as "representation with consciousness." "Consciousness" in this context means *the representation that another representation is in me* [Kant (1800), 9:33]. The first is produced by the process called sensibility; the second is produced by the process of reflective judgment.

Sensuous matter is introduced into the synthesis in sensibility from two sources. The first is *receptivity*,

which belongs to the logical division of *psyche* in the Organized Being model. It is convenient to logically divide this class of information into data of *soma*'s external senses (seeing, hearing, etc.) and kinaesthetic feedback (e.g. muscle tension, balance, etc.). Data from this source provides the empirical matter of experience. The second source is data introduced by the process of determining judgment from a structure called the *manifold of concepts*, about which I have more to say below. This source of data is called *spontaneity* because its *re*introduction into sensibility is an outcome of active reasoning.

Perceptions are classified into two general types: objective perceptions and affective perceptions. The first type is called objective because the *parástase* of representation refers to an object (more precisely, to an *appearance* of an object). Such a *parástase* is called an **intuition**. Sensuous matter contained in an intuition is called *sensation*. The second type of perception is non-objective - indeed its *parástase* is never part of any intuition - and underlies that stubbornly indefinable part of experience we mean when we use the words emotions, moods, passions, etc. The sensuous matter contained in an affective perception is called *feeling*.

Perceptions are synthesized. That is, they are not "impressed upon the mind" (as the British empiricists thought) nor are they somehow fetched from some closet of "innate ideas" (as the continental rationalists thought). Referring to figure 5, the matter in an intuition is the outcome of a three-step process Kant called the *Verstandes-Actus* (acts of understanding): *comparation - reflexion -* abstraction. Kant was only able to provide vague descriptions of these acts because the mathematics necessary to more crisply explain them had not yet been invented in his day. He described **comparation** as "the comparison of representations among one another in relationship to unity of consciousness" [Kant (1800), 9: 94]. What does this mean? In the language of modern mathematics, its practical implication is: synthesis of what are called *equivalence classes* [Preparata & Yeh (1973), pg. 45]. In the simplest terms as applied to Kant's "unity of consciousness," an equivalence class is a set of sensuous matters that are all in some way equal to one another. But, because there are many different ways by which things might be said to be equal, the question arises at once: equal in what way?

This is not a question that is or can be put to the process of the synthesis in sensibility. Perhaps that is evident merely from the fact that "things" can be "equal" in many different ways. Furthermore, the idea of "comparison" in Kant's description implies some kind of judgment be made and sensibility is not a process of judgment. Kant called the sort of judgment being called for here a *judgment of perception* [Kant (1783), 4: 298]. Such judgments "do not need a pure notion of understanding, but only the logical connection of perceptions in a thinking subject" [*ibid*.]. Judgments of perception belong to the process of reflective judgment in figures 4 and 5. I'll say more about them below where this block is described. For the moment, let it suffice to say that sensuous matters can be "equal" in various ways.

The second step in the *Verstandes-Actus* is **reflexion**. Kant described this as "reconsideration as to how various representations can be comprehended in one consciousness" [Kant (1800), 9: 94]. Mathematically, *reflexion* is: synthesis of what are called *congruence relations* [Preparata & Yeh (1973, pg. 136]. What a mathematician means by "congruence relation" is a little more difficult to explain than an "equivalence class" is. Dictionaries of mathematics tend to "explain" this idea only through examples and these examples are always specific and rather abstract mathematics applications. First, in order to have a congruence relation, the sensuous matters must be "equal" (belong to the same equivalence class). The Critical definition of "congruence" is *agreement and suitability without contradiction or real opposition* and *practically* means matters can be "put together" without negating or cancelling each other out. Suppose sensuous matters a, b, c, and d all belong to the same equivalence class. Their equivalence relation, $a = b$ and $c = d$ is a congruence relation if a and c can be put together (ac) and likewise b and d can be put together (bd) and we still have the same equivalence relation $ac = bd$. Mathematicians call this a "substitution property." A congruence relation is sort of a "super-equivalence" relation, which is what is implied by Kant bringing the word "comprehended" into his description of *reflexion*. Again, a judgment of perception is required here and, again, this act belongs to reflective judgment.

The third step in the *Verstandes-Actus* is **abstraction**, which Kant described as "separation of every-thing else in which the given representations differ" [Kant (1800), 9: 94]. In mathematics "abstraction" is "the process of considering certain features of objects while discounting other features that are not relevant. Abstraction is the basis of classification" [Nelson (2003)]. Kant means pretty much the same as this but we do need to say a little more about it. Referring to the previous paragraph, suppose when *ac* and *bd* are "put together" they do not quite satisfy whatever criteria is being used to judge equivalence. In other words, suppose *ac* ~ *bd* but not *ac* = *bd* results. Read the expression "*ac* ~ *bd*" as "*ac* is 'kind of like' *bd* but also 'kind of different'." But further suppose that by removing some part of the sensuous matter of *ac* or some part of *bd* or both, i.e. re-representing one or the other or both of them as (*ac*)' or (*bd*)', the equivalence criteria is satisfied and the equivalence (*ac*)' = (*bd*)' is recovered. Then (*ac*)', let us say, is lacking something the representation *ac* had and we say (*ac*)' is represented "by abstracting from" *ac*. The word Kant used for doing this was *praecision*, which in Latin literally means the act of lopping off or amputating [Kant (*c*. 1773-79), 15: 271].

These *Verstandes-Actus* pertain to the representing of sensuous *matters* in sensibility, but figure 5 also contains a synthesis of sensible *forms* called the **synthesis of pure intuition**. More precisely, there are two such pure intuitions, which Kant called the pure intuition of external sense and the pure intuition of inner sense [Wells (2009), chap. 3]. In Critical Epistemology the adjective "pure" means "containing nothing that belongs to sensation or experience"; the synthesis of pure intuition is that which *provides* the forms by which perceptions are given representations, without which no experience at all is possible.

The pure intuition of external sense is also called the pure intuition of *subjective space*. Subjective space is not the mysterious no-thing we call *objective* space[8], measure using various instruments, describe using geometries, and that human infants require the first two years of life to conceptualize and elaborate [Piaget & Inhelder (1967)]. Subjective space is also not a geometry. Human beings are born without any knowledge, innate or otherwise, of geometry, and how we come to conceptualize and develop our ideas of geometry is itself a fascinating study [Piaget *et al*. (1981)].

Rather, the pure intuition of subjective space is the synthesis of topological structuring. In mathematics, topology is a topic we can describe as "pre-geometry" because it underlies the ideas that go into the development of geometries. The *Penguin Dictionary of Mathematics* defines topology as "the study of those properties of geometric figures that are invariant under continuous deformation." It is concerned with such notions as proximity, continuity, connectedness and separation, boundedness, and enclosure. The pure intuition of subjective space is not itself "a topology"; rather, it is a *synthesis* out of which are produced representations of intuition that make these ideas possible. Indeed, one of the more fascinating findings made by Piaget & Inhelder is that infants *exhibit* manifestations of perceptions of proximity, continuity, etc. [Piaget & Inhelder (1967), pp. 5-9]. The Bourbaki mathematicians of the 1950s proved that topological structure is one of three so-called "mother structures" they identified out of which all of mathematics can be constructed. The pure intuition of subjective space is the foundation of topological structures. All objective perceptions (intuitions in figure 5) are represented "in" a subjective space.

Mathematics had not yet invented topology theory during Kant's lifetime. He was, therefore, forced to try to explain subjective space without the benefit of that theory. Certainly the idea of "topology" would not leap off his pages for the modern reader and, to a person coming at it from an ontology-centered meta-physic, much of his explanation does seem controversial at best.

The pure intuition of inner sense is also called the pure intuition of *subjective time*. Subjective time is not the stubbornly puzzling no-thing we call *objective* time[9] that we measure using clocks, that physicists like to make "run backwards" in the theory of antiparticles, and Einstein found to "slow down" at high velocities or in strong gravitational fields. Neither is subjective time describable as a "time line." Instead its mathematical description is better called a "timescape."

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[8] see Wells (2006), chap. 17.
[9] see Wells (2006), chap. 21.
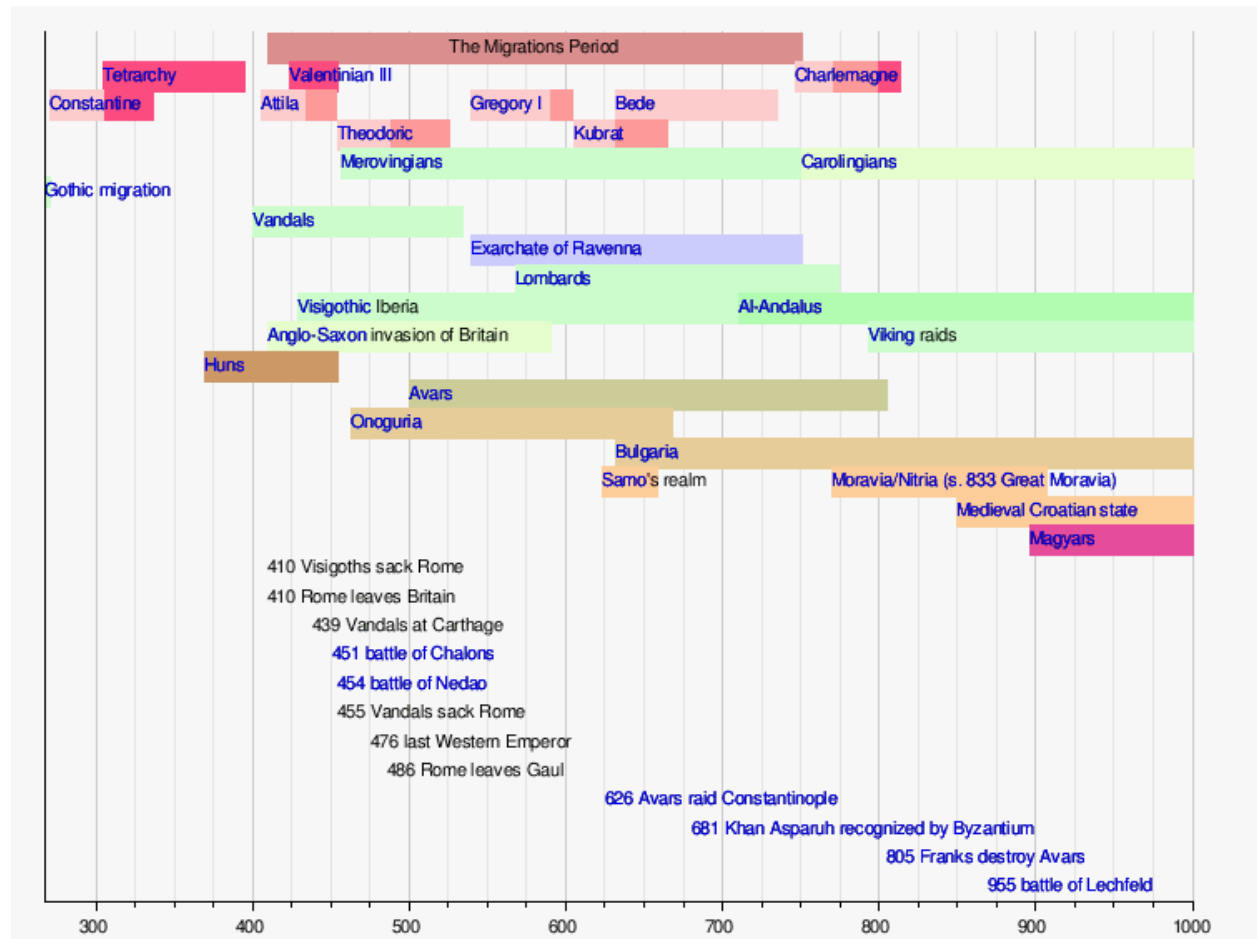
Figure 6: Historical "timeline" of the European migration period. As you can see, this figure isn't depicted as a "line" at all. It is, rather, an example of what I mean by "timescape."

This idea of a "timescape" is not so unfamiliar as you might initially think. Historians tend to use them frequently. Figure 6 illustrates one such timescape. The word "timescape" was chosen for this sort of depiction by analogy with the idea of a "landscape." In more formal terminology, the synthesis of the pure intuition of subjective time is the synthesis of *mathematical order structures* [Wells (2009), chap. 3].

Referring again to figure 5, all perceptions are perceived represented in subjective time. Where affective perception and objective perception (intuition) differ is that the latter is always represented with *both* subjective space *and* subjective time in its form of representation. Affective perception, on the other hand, has *only* subjective time in its form of representation. This is why affective perception can never be part of the cognition of an object and relates only to the subjective state of the person. Affective perceptions are, in this sense, autistic representations.

4.3.2 **Apperception and Imagination.** When we look at mathematical explanations such as those we have been considering above, it is very easy to let our thinking fall into a subtle psychological trap. This trap is the tendency to see the pieces while forgetting that these pieces are only part of a whole we will call *mentality*. This is a general tendency of the phenomenon of mind: we forget the cliff face is part of the mountain; we forget the sky is part of the earth; we forget the phenomena of body and the phenomena of mind are parts of the phenomenon of being a human being. William James wrote,

Experience, from the very first, presents us with connected objects, vaguely continuous with the rest

of the world, which envelops them in space and time, and potentially divisible into inward elements and parts. These objects we break asunder and reunite. We must treat them in both ways for our knowledge of them to grow . . . The noticing of *any part* whatever of our object is an act of discrimination. . . . Where the parts of an object have already been discerned, and each made the object of a special representative act, we can with difficulty feel the object again in its pristine unity; and so prominent may our consciousness of its composition be that we may hardly believe that it ever could have appeared undivided. But this is an erroneous view, the undeniable fact being that *any number of impressions from any number of sensory sources falling simultaneously on a mind* WHICH HAS NOT YET EXPERIENCED THEM SEPARATELY *will fuse into a single undivided object for that mind*. The law is that all things fuse that *can* fuse, and nothing separates except what must. [James (1890), vol. I, pp. 487-488]

These intertwined characteristics of the phenomenon of mind are well known to psychologists and have been given the names syncretism and juxtaposition [Piaget (1928), pp. 221-232]. Both can be clearly observed in the behaviors of very small children. Children manifest juxtaposition in their drawings (e.g., an eye drawn alongside a head; the child knows the eye "goes with" the head but doesn't draw it as *part* of the head). As for syncretism,

Syncretism therefore permeates the thought of the child. . . . Before the age of 7-8, syncretism may be said to be bound up with all mental events and with nearly all the judgments that are made. For any two phenomena perceived at the same moment become caught up in a schema which the mind will not allow to become dissociated, and which will be appealed to whenever a problem arises in connection with either of these two phenomena. Thus when children of 5-6 are asked: "Why do the sun and moon not fall down?" the answer does no more than evoke the other features appertaining to the sun and the moon because these features, having been perceived *en bloc* and within the same whole as the feature requiring explanation, seem to the child a sufficient reason for the latter. . . . Here are some examples: The sun does not fall down *"because it is hot. The sun stops there. -* How? - *Because it is yellow"* (Leo, age 6). [Piaget (1928), pp. 228-229]

Now, because syncretism and juxtaposition are two factual characteristics of the phenomenon of mind, any mathematical model of this phenomenon *must include* a representation of the capacity to produce them. The general whole, in which syncretism and juxtaposition are contained as parts, Kant calls *apperception*. To the frequent irritation of Kant scholars, he never actually provides a clear definition of what he means by this word; perhaps this might be because the term was originally coined by Leibniz, was well known in Kant's day, and meant "a final clear phase of perception characterized by recognition, identification, or comprehension of what has been perceived" [Reber & Reber (2001), def. 1]. Psychology has added additional connotations for the word since Kant's day, but Leibniz' connotation is the one Kant would have used.

Figures 4 and 5 do not depict any block labeled "apperception" (just as, likewise, there is no block labeled "understanding"). This is because "apperception" is an "outcome" or "result" of mental acts and not a piece of the functioning of the "parts" depicted in these figures. "The unity of understanding" is Kant's most frequent phrase he uses to *describe* (not "define") the outcome we call apperception. What has been presented above for the synthesis in sensibility is not sufficient, though, to produce apperception. One of the things we are still missing is what Kant calls *the power of imagination*.

The idea of imagination, Kant tells us, is an empirical idea. This is because it is a mental phenomenon and, were this not so, we would never know about it at all. Because it is an empirical idea, it cannot be defined (no empirical object can be defined; it can only be described and explicated). Mathematical objects are all defined objects (because they are the way they are because *we* make them so), but Kant is talking about imagination as a mental phenomenon when he calls it "empirical." He tells us,

Imagination is the capacity for representing an object, even ***without its presence***, in intuition. Now since all our intuition is sensuous, imagination . . . belongs to **sensibility**; but insofar as its synthesis is

> still an exercise of spontaneity, which is determining and not, like sense, merely determinable . . . imagination is to this extent a capacity to determine sensibility *a priori*[10], and its synthesis of intuitions . . . must be the transcendental synthesis of **imagination**, which is an effect of understanding on sensibility and its first application . . . to objects of intuition that is possible for us. [Kant (1787), B: 151-152]

Figure 5 mathematically splits up imagination into three parts: apprehensive imagination (the processes explained in sec. 4.3.1); imaginative re-cognition (to "cognize again"); and imaginative reproduction. Imaginative re-cognition is a *transformation* of representation that remakes a *parástase* of intuition into *a **rule** for the reproduction of that parástase*. That rule is called a **concept**, and so here we have our definition of what "concepts" are, i.e., rules for the reproduction of intuitions. Imaginative reproduction is likewise a transformation (a reverse transformation) that transforms a concept back into an intuitive *parástase*. It reintroduces, via spontaneity, information into the synthesis of sensibility and, as engineers put it, "closes the loop" of sensibility → determining judgment → sensibility. For reasons I will explain a bit later on, this loop is called **thinking**. Thinking is "cognition through concepts."

Kant tended to describe imagination as "figural," but this is misleading at best. In order to represent objects of the external senses, a *parástase* of intuition must be representable in any or all of the five classic sensory modalities: visual; auditory; haptic; gustatory; and olfactory. Consider thunder, for instance. A person's objective perception of thunder is the loud rolling boom he hears. As William James liked to describe it, he perceives "thunder-breaking-on-silence-and-abolishing-it." As another example, during the first days and weeks of life most of an infant's earliest and most important objective perceptions are haptic and gustatory; he puts things in his mouth, sucks on them, and explores them with his tongue and lips. As noted earlier, these earliest *parástase* are syncretic; the infant doesn't yet know he is sucking on the nipple of his bottle or on his thumb; his objective perceptions are of the whole of his sensations.

As transformation functions from sensibility to determining judgment and from determining judgment to sensibility, imagination obviously requires synthetic *momenta* for accomplishing this. At the 2LAR level of representation there have to be twelve of these (as we discussed earlier) and these *momenta* must effect the kind of transformations mathematicians call "isomorphisms" because re-cognition and reproduction in imagination are inverse functions of each other. Kant gave them the name "transcendental schemata" and very briefly discussed them in one small chapter of *Critique of Pure Reason* [Kant (1787), B: 176-187]. Rather unfortunately, he decided to save his readers from "a dry and boring analysis of what is required" for them and instead merely "presented" aspects of them. I think I would not be wrong to say most Kant scholars have regarded this "presentation" as unsatisfactory.

Neither is any deduction, nor even a "dry and boring analysis" of them, to be found elsewhere in the Kantian corpus. I personally harbor an unkind suspicion that this "analysis" never actually existed. In Kant's day, isomorphic transformations were, if not a new idea, at least a relatively undeveloped one. Napier had invented the logarithm in the early 17th century, and other notables such as Kepler made some contributions to its development, but generalized mathematical development of isomorphic transformations only came much later. It seems doubtful if Kant even thought of his transcendental schemata in terms of isomorphic transformations. But it is at least clear enough, to someone with a good mathematics background, that isomorphic transformation was what Kant intended for transcendental schemata to do. Wells (2006), chapter 8, pp. 621-648, provides a deduction of the twelve *momenta* of transcendental schemata. This is a level of technical detail not really needed for this treatise, but the interested reader can go to this citation to see these details if desired. I do not think they are dry or boring.

4.3.3 Reflective Judgment.

In Critical Epistemology, judgment is a process for: (1) operationally, the act of subsuming a particular

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[10] In Critical Epistemology, *a priori* just means "prior to experience."

mental representation under a general one that serves as a rule; and (2) functionally, is the representation of the unity of consciousness of various representations or the representation of their relationships insofar as they constitute a concept. There are three distinguishable processes of judgment shown in figure 4: (1) determining judgment, which is the principal process of judgment in the theoretical Standpoint; (2) practical judgment, which is the principal process of judgment in the practical Standpoint; and (3) reflective judgment, which is the principal process of judgment in the judicial Standpoint. These three processes are reciprocally co-determining via the clockwise loop labeled in figure 7 as the judgmentation loop[11]. Figure 7 is a simplified re-depiction of the same thing shown in figure 4.

Judgmentation is the overall process of exercising reasoning, determining judgment, reflective judgment, the synthesis in sensibility and the regulation of motoregulatory expression (figure 4) by which understanding is achieved and the law of the categorical imperative in practical Reason is satisfied. The three processes of judgment are co-determined by synthesis operations illustrated in figure 8. The figure 8 diagram is called a "first level synthetic representation" or 1LSR [Palmquist (1993), pg. 85].

These three synthesis operations can be described symbolically using a notation patterned after forms of notation commonly employed in chemistry. These operations are:

cognition + belief → purpose (construction of reasoning; practical Standpoint);
purpose + cognition → belief (construction of consciousness; judicial Standpoint);
belief + purpose → cognition (construction of experience; theoretical Standpoint).
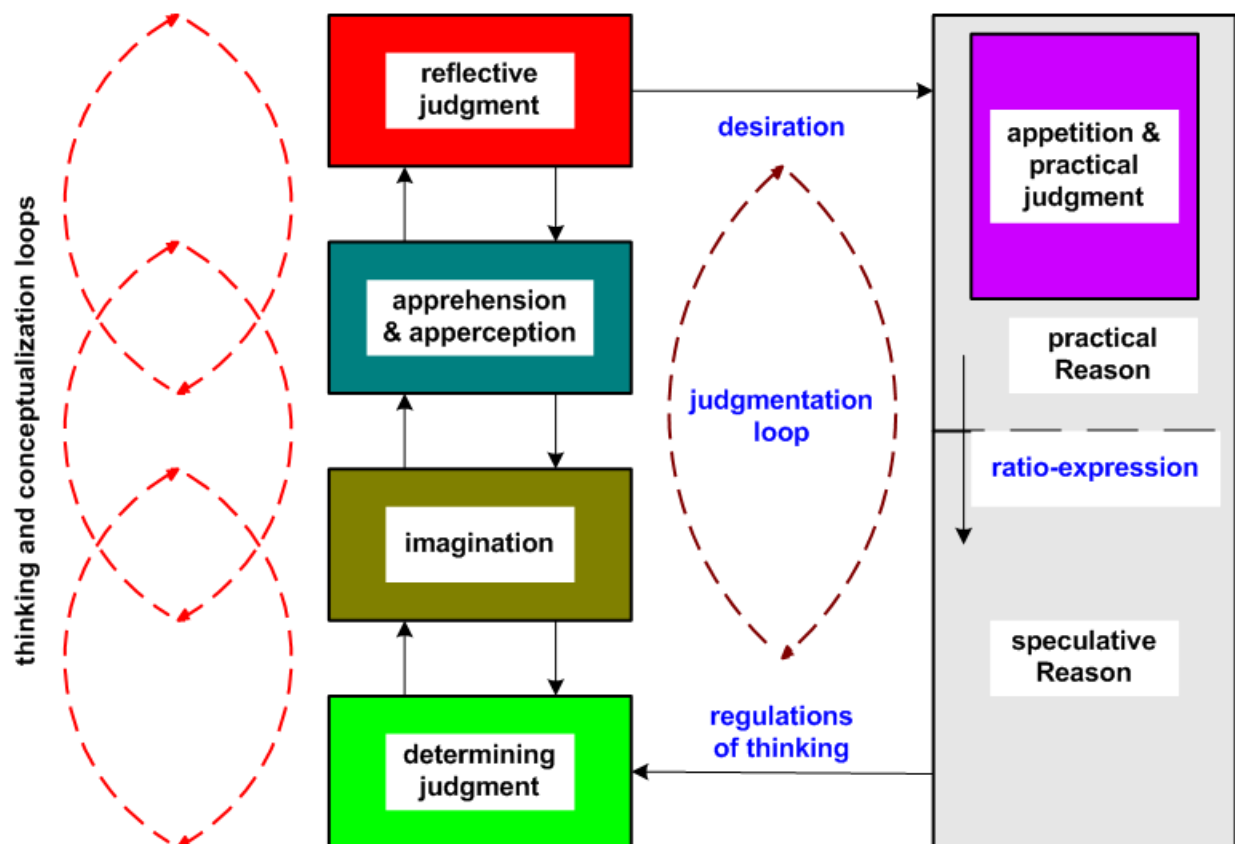
Figure 7: Simplified re-depiction of figure 4. The synthesis in sensibility is relabeled apprehension & apperception in this depiction. The purpose of this re-depiction is to more clearly identify the information loops involved in thinking, conceptualization, and judgmentation.

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[11] Judgmentation is the word I use to translate Kant's word *Beurtheilung*.
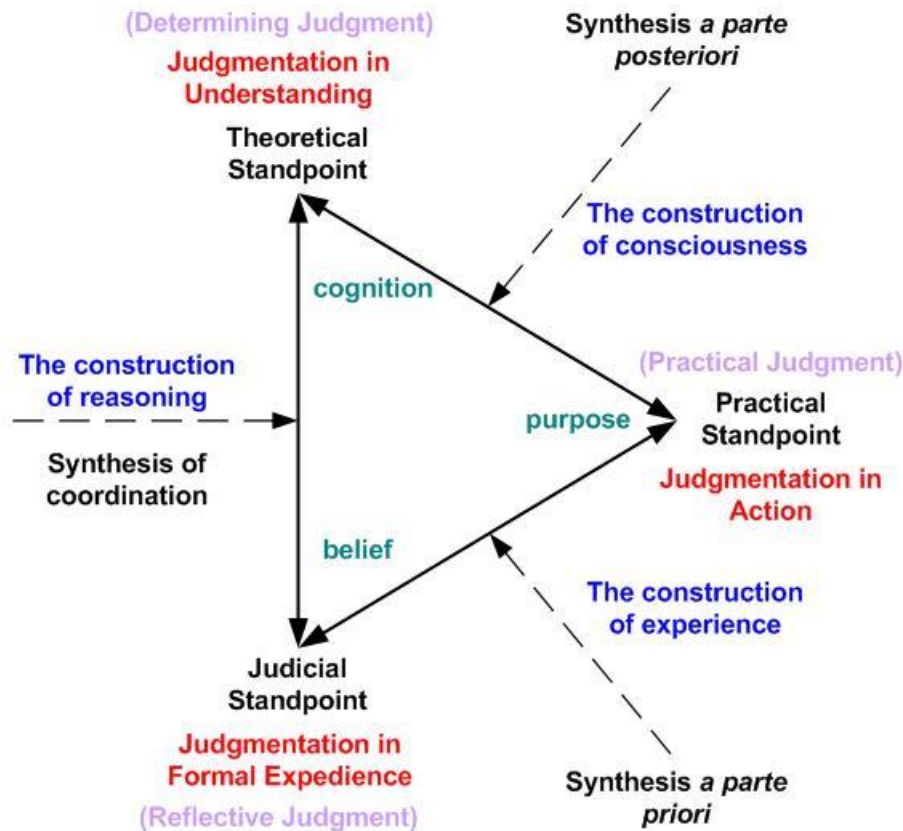
Figure 8: 1LSR illustration of the synthesis of the Standpoints and co-determination of judgments.

In these "formulas" you read "+" as "synthesized with" and you read "→" as "produces."

The synthesis of judgmentation is regulated, as are all non-autonomic acts of the Organized Being, by the process of practical Reason. This regulation is governed by the fundamental law of pure practical Reason: *the Organized Being always acts **unconditionally** to achieve a state of mental equilibrium*. This law is called **the categorical imperative of pure practical Reason**. In due time we will discuss what this Idea of equilibrium requires and what conditions mark its achievement, but for now let it suffice to say it has connotations of stability, homeostasis, and "everything being in balance." Research in developmental psychology carried out over many decades supports this interpretation of Kant's categorical imperative, e.g. Piaget (1975). I mentioned earlier that Kant made a fundamental error by confounding the categorical imperative with his idea of "the moral law within me," and that this confounding leads to experimentally testable conclusions that psychology research unequivocally refutes. Kant was correct in his deduction of the *logical* character of the categorical imperative; but he slipped back into ontology-centered thinking with his "moral law within me" idea, and that was where he made his error.

This subsection is addressed to reflective judgment and the judicial Standpoint. As I mentioned earlier, Kant does not seem to have taken the practical and judicial Standpoints nearly as seriously as he did the theoretical Standpoint. Nowhere in his Critical Philosophy is this more apparent than for the judicial Standpoint and his *Critique of the Power of Judgment*. Palmquist remarked, and I agree, that the third Critique is generally unsystematic in its organization [Palmquist (1993), pg. 291]. One signpost of this is that Kant wrote *two* introductions for this work. It was the second, and significantly abbreviated, version that was published with *Critique of the Power of Judgment*. He had made a last minute decision to replace the longer and more thorough first introduction "because it was too long" [Guyer (2000)], [Kant (1789-94), 11: 441]. Guyer gives an account of the somewhat rushed and mildly chaotic process of the third Critique's publication. In my opinion, Kant's "First Introduction" [Kant (*c*. 1789)] is crucial for

understanding the judicial Standpoint, for deducing figure 4, and for deducing the Critical elements Kant failed to develop in Kant (1790). That missing deduction is supplied in Wells (2006), chapters 14-16 and 18. According to Guyer's account, we seem to have been lucky Kant's First Introduction survived. Kant had sent his unpublished manuscript to his friend and former student, J.S. Beck. From there it passed on to the library at Beck's university, where it was rediscovered and eventually published in 1914.

The judgments made by the process of reflective judgment are subjective (pertain to the effect sensibility has on the Organized Being) and are non-objective judgments of perception. As you can see in figure 4, reflective judgment "faces" the synthesis in sensibility on one side and "faces" the process of practical Reason and motoregulatory expression in *psyche* on the other. The one "face" deals with subjective feelings (affectivity) of the subject; the other face pertains to appetites and somatic expression of actions. Taken together, this functional organization can be called the sensorimotor faculty. Kant made a logical division of the process of reflective judgment distinguishing what is called *aesthetical* reflective judgment and *teleological* reflective judgment. Accordingly, basic mathematical representation of reflective judgment requires a 3LAR structure as shown in figure 9. The eight "heads" shown in this figure each require three synthesizing *momenta* for a total of twenty-four *momenta*. These are explained in Wells (2009), chap. 8, but we will be able to get by with a less technical discussion in this treatise.

The matter (determinable) of reflective judgment is called **desire**. This is the relationship of sensibility to the subjective state of the person. A desire consists of a combination of affective perceptions that are associated with each other by the synthesis of aesthetical reflective judgment and belongs to the judicial Standpoint of Critical Epistemology. The *form* of a desire is called a **value**. The *matter* of a desire is what Kant called a **feeling of *Lust*** (pronounced "loost") or ***Unlust*** (pronounced "un-loost"). This terminology requires a brief explanation. First, the German word *Lust* has no exact English equivalent and does *not* mean the same thing as the English word "lust." Translators, in conformity with the dainty manners of the Victorian Age, habitually translate *Lust* as "pleasure" (and *Unlust*, therefore, as "pain") but *this is not an accurate translation*. Kant described a feeling of *Lust* as a "feeling of the promotion of life"; the "flavor" of this feeling is idiomatically conveyed by the American colloquialism "I'm up for that!" Similarly, the feeling of *Unlust* is a "feeling of the hindering of life" and is idiomatically conveyed by the colloquialism "That's a bummer." In our mathematical model, *Lust* and *Unlust* "themselves" are ideas belonging to the logical division of *psyche* in the Organized Being model [Wells (2006), chap. 15] but the *feeling* of them belongs to affective perceptions judged by the process of aesthetical reflective judgment.
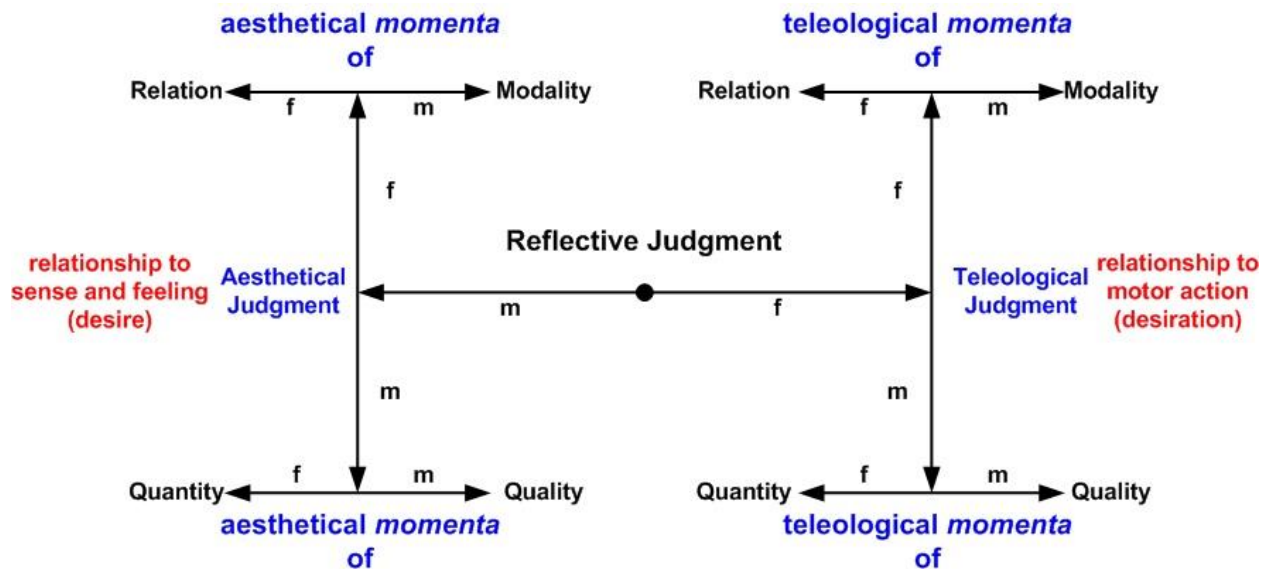

Figure 9: 3LAR organization of the process of reflective judgment.

While a desire is never part of the representation of an intuition, the Object of the latter is frequently said to be "an object of desire." How can this expression be understood according to epistemology? Kant provided the following description:

> By the designation "an aesthetic judgment about an Object" it is therefore forthwith indicated that a given representation is certainly related to an Object but that what would be understood in the judgment is not the determination of the Object but of the Subject and its feeling. For in the power of judgment understanding and imagination are considered in relationship to one another . . . but one can also regard this relationship of two faculties of knowledge merely subjectively, as far as one helps or hinders the other in the very same representation and thereby affects *the state of mind*, and so as a relationship which is *sensitive*. [Kant (*c*. 1789), 20: 223]

This second way of regarding "this relationship of two faculties of knowledge" is, of course, a perspective of the judicial Standpoint.

The synthesis in sensibility performs no judgments of any sort. Metaphorically speaking, sensibility is a factory for manufacturing *possibilities*[12]. Reflective judgment judges and selects from among these in conformity with the fundamental law that governs reflective judgment. Kant named this law *the principle of formal expedience*[13] *of nature* [Kant (1790), 5: 181-186]. The first question to ask seems obvious: "Expedience for what?" The answer here is immediate: expedience for the categorical imperative of pure practical Reason, which is the master regulatory law of the Organized Being.

This does not mean reflective judgment is in possession of some innate idea of objects. Reflective judgment has no knowledge of Objects whatsoever and its judgments are entirely subjective. The master regulation regulates for the achievement of global equilibrium. Now, equilibrium does not necessarily mean a *static* condition in which there are no variations or changes. More generally, it denotes a closed cycle of activity in which there are no innovations ("changes"); each cycle is the same as the previous one. Equilibrium in homeostasis in biological cells and organisms is of this type. A constant frequency oscillator is another example. Physicists speak of zero-point energy and ground state oscillation in atoms.

Figure 7 depicts several "loops of information flow" in *nous*. When co-determination of activities in these loops bring them into mutually self-sustaining activity cycles, these cycles are in equilibrium with each other; Kant used the term "harmonization" to describe the condition of such mutual equilibration. In judging expedience, harmonization among these loops is a marker of formal expedience. Its detection by reflective judgment makes any knowledge of objects unnecessary for rendering a judgment. Kant wrote,

> Thus an aesthetic judgment is that whose determining ground lies in a sensation that is immediately combined with the feeling of *Lust* or *Unlust*. In the aesthetic judgment of sense it is that sensation which is immediately produced by the empirical intuition of the object; in the aesthetic judgment of reflection, however, it is that sensation which the harmonious play of the two faculties of cognition, imagination and understanding, produce in the subject insofar as in the given representation the faculty of apprehension of the one and the faculty of presentation of the other are reciprocally expeditious [Kant (*c*. 1789), 20: 224].

I often find it helpful to think of harmonization as analogous to a state of thermodynamic equilibrium, and to think of the process of getting to a state of harmonization as analogous to non-equilibrium thermodynamics. I imagine poets and musicians can find their own appropriate metaphors for harmonization.

Let us now turn to the other side of reflective judgment, namely, teleological judgment. Ever since

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[12] The human ability to generate possibilities underlies the phenomenon of human creativity. Psychological studies of the manifestations and roles of possibility and necessity have returned a number of very interesting findings. For more on this see Piaget (1983).

[13] *Zweckmäßigkeit*. It has long been the tradition in translating Kant to render this word as "purposiveness." That was perhaps not a bad guess but it is nonetheless not a proper word for conveying Kant's principle.

Aristotle employed the idea of "final causes" in his ontology of nature, there has been, and justly so, a great deal of opposition to any attempt to introduce teleology into natural science. Kant himself, in fact, also opposed it. But, once again, things look different from an epistemology-centered metaphysic than they do from an ontology-centered one. Here teleological judgment is admissible as a *regulative principle* of judgmentation [Kant (1790), 5: 359-361]. Wells (2006), chap. 16, presents a thorough discussion of the function and role of teleological judgment. As for the *momenta* of teleological reflective judgment, it takes on that topic in chapter 18. Wells (2009), chapter 8, provides a more succinct summary of them.

The role of teleological reflective judgment is to determine forms of connection for the determinable matter of desire adjudicated by aesthetical reflective judgment. This determination is called ***desiration***, and the combination of the matter of desire in the form of desiration is called Desire. (Note that this term is capitalized in its spelling). Desiration is the form of unity in affective perception in relationship to the overall capacities of the Organized Being.

The direct *practical* object of teleological reflective judgment is *real action* the Organized Being takes, firstly through motoregulatory expression in *psyche* and, secondly, through ratio-expression (acts of reasoning) in practical Reason. (See figure 4). Ratio-expression is a homologue in *nous* of motoregulatory expression in *psyche* and pertains to Reason's employment and direction of determining judgment in thinking. However, it is quite wrong to suppose that teleological judgment is in any way *constitutive* for the empirical representation of objects. Its role is *strictly* regulatory for the dynamics of judgmentation. Its goal is *practical laws* of Reason and no such law is ever an object of perception.
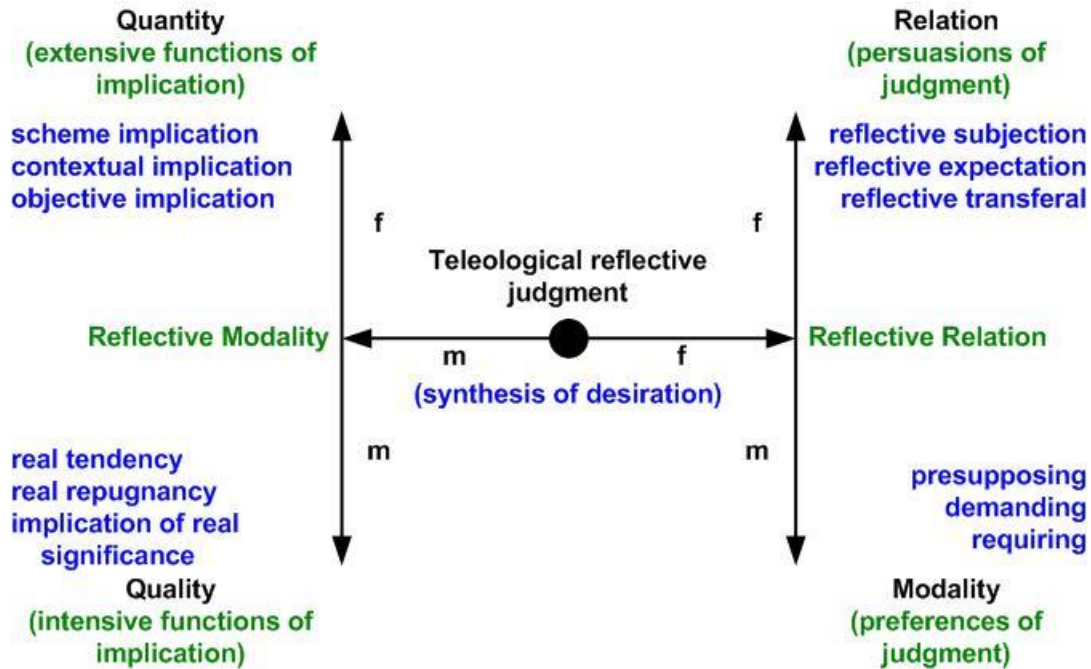
As a process, teleological reflective judgment is proactive, *impetuous*, tied directly to action expression, and serves a "final purpose," namely, maintenance and/or recovery of equilibrium under the *master* regulation of the categorical imperative of pure practical Reason. The form it brings to desires produces what Kant called the manifold of Desires (again, see figure 4). Unlike the other two manifolds in figure 4 (the manifold of rules and the manifold of concepts), the manifold of Desires is not conserved as a structure. Desires are not "remembered"; they are "regenerated" during the on-going synthesis of judgmentation.

In Critical Epistemology a *scheme* is anything that can be repeated and generalized in an act or action. A perception is said to be assimilated into a scheme when that perception "triggers" the expression of that scheme, and so the perception is said to have a ***meaning implication*** [Piaget & Garcia (1987)]. The matter of desiration in teleological judgment is assimilation of perception (the determinable) into an action scheme (the determination) and so the six *momenta* of composition in teleological judgment are called *functions of implication*. In Critical Epistemology, *all real meanings are at root* ***practical***; as Piaget & Garcia put it, the meaning of an object

> amounts to "what can be done" with it, and is thus an assimilation into an action scheme (whether the action is overt or mental). As for actions themselves, their meaning is defined by "what they lead to" according to the transformations they produce in the object or in the situation to which they are applied. [Piaget & Garcia (1987), pg. 119]

To a baby, a rattle is something to shake, something to throw, something to suck on, etc. With the growth of experience and the child's manifold of concepts, more abstract logico-mathematical predications can be conjoined to these root meanings of "what a rattle is," but its basic meanings always refer back to its assimilations into those early action schemes.

Nexus (i.e., connection) in desiration has two general tasks to accomplish in order to make perception expedient for the categorical imperative. The first is to reconcile how sense is affected with the *Existenz* of the Organized Being, or, in other words, how sense is *harmonized* with the other faculties of Organized Being by the process of judgmentation. In this process of harmonization, it is sense that must be accommodated so that it is possible for the Organized Being to assimilate its perceptions. This is Relation (form of the form) in teleological judgment and its *momenta* are called the *persuasions of judgment*.

Figure 10: The 2LAR structure of teleological reflective judgment. See Wells (2009), chap. 8, for details.

But, in addition, when perception produces a disturbance to equilibrium or gainsays an already-existing concept reintroduced into the synthesis of sensibility (and thereby creates a disturbance), then either the action or the concept must be accommodated to bring back harmony in judgmentation. There is here an inherent bias in judgmentation that serves to produce the equilibration required by the categorical imperative. Such bias is Modality (matter of the form) in the 2LAR of teleological judgment and its *momenta* are therefore called *the preferences of judgment*. We can see in this that behavioral phenomena of persuasion and preference are "built into" the foundations of human psychology by *nexus* in desiration. Figure 10 summarizes the 2LAR structure of teleological reflective judgment.

These properties of teleological judgment lead to two important theorems in Critical Epistemology. The first is: *motivation is the accommodation of perception*. The second is: *motoregulatory expression is the assimilation of perception*. The power of the Organized Being to organize and accommodate its own perceptions is called the motivational dynamic of organized being [Wells (2009), chap. 10]. Within this idea lie important keys to removing the mysticism attending the notions of motivations and motivational states in psychology and in neuroscience.

4.3.4 The Process of Practical Reason.

Reason is the power to Self-regulate all non-autonomic acts of the Organized Being. We make a logical division of the process of Reason into a process of practical Reason and a process of speculative Reason in figure 4. The first division is the principal adjudicating process for the practical Standpoint.

Deduction of the structure and elements of the process of practical Reason shown in figure 4 is given in Wells (2006) chapters 12-13 and 19-20. As noted near the beginning of this chapter, Kant made a serious error in his treatment of pure practical Reason, and this error led to his *Critique of Practical Reason* being something of a disappointment to a reader expecting to find in it the sort of thorough development his *Critique of Pure Reason* had provided for determining judgment and speculative Reason. Reath noted,

> Certain remarks in the *Groundwork*[14] suggest that Kant did not originally plan a separate critique of

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[14] Kant's *Groundwork of the Metaphysics of Morals* (1785).

practical reason. He notes that although a critique of practical reason is the only foundation for a metaphysics of morals (i.e., a systematic classification of human duties), the need for a critique is less pressing in the case of practical reason than it is for speculative reason, and that an outline of such a critique would suffice for his purposes. [Reath (1997), pg. vii]

Understanding the *phenomenon* that human beings make moral judgments is important for understanding the Idea of Justice. Kant's unspoken premise, that there should be only *one universal* metaphysic of morals governing all of humankind, was badly misplaced, and it is the fundamental flaw in his *Kritik der praktischen Vernunft*. That people do make moral judgments is a phenomenon of human nature. That a *single* classification of human duties might be possible is a speculation. A critique of practical Reason is, indeed, a necessary foundation for any theory of moral phenomenon but the need for a critique is broader and no less pressing than the need for a critique of speculative Reason. Kant erred in summarily rejecting, instead of trying to answer, Hume's thesis about ethics and morals. The modern Critical theory corrects this mistake. The proper deduction for the process of practical Reason is provided in Wells (2006) in chapters 12-13 and 19-20. A more concise summary is provided in Wells (2009), chapters 9-11.

Figure 11 provides a more detailed depiction of the process of practical Reason and its information flow links to and from the other blocks shown in figure 4. Appetitive power (*Begehrungsvermögen*) is the practical ability of the Organized Being to take actions and thereby be the efficient cause of the actuality of the Objects of such actions. However, there is a curious subtlety here. Explicit possible action expressions connect to motoregulatory expression in *psyche* as meaning implications from teleological reflective judgment. Teleological judgment, though, is impetuous in its nature and expressions of its meaning implications *do not become actualized unless permitted by* appetitive power's appetitions. We say that practical Reason exercises a *veto power* over teleological judgment's pronouncements. In effect, it is more correct to say human beings exercise "free won't" rather than "free will." Kant *was* able to deduce this curiously negative character of practical Reason despite his "moral law within me" error [Kant (1788), 5: 72-73]. In 2004 an interesting neurological experiment led researchers Obhi and Haggard to likewise conclude people exhibit "free won't" rather than "free will" [Obhi & Haggard (2004)].
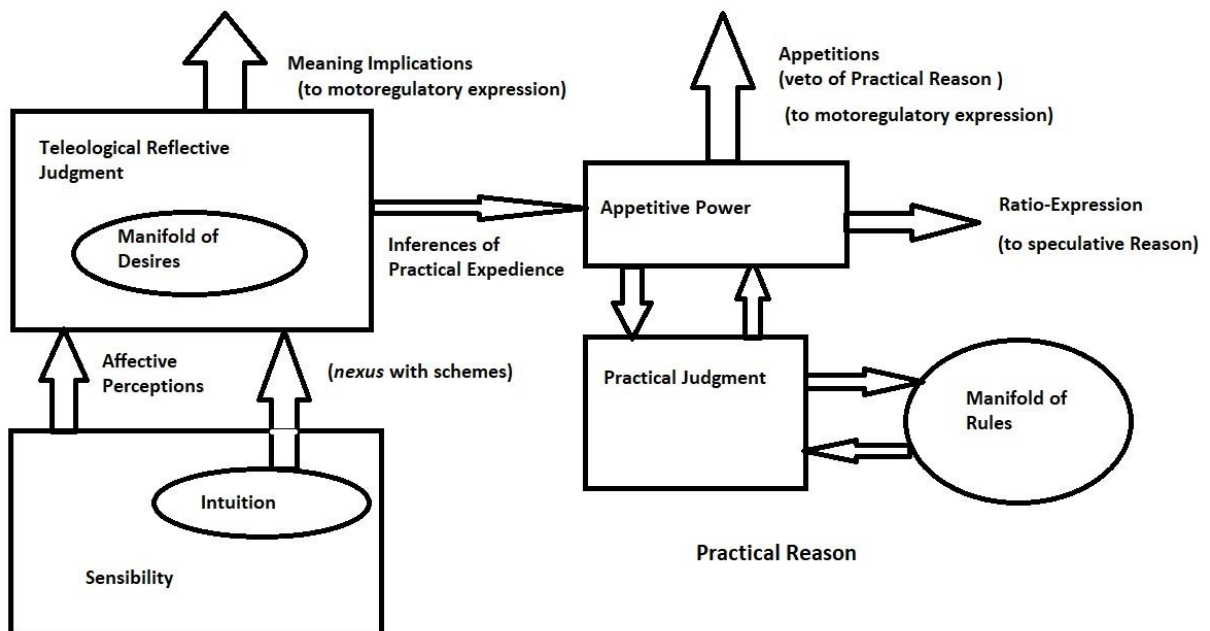
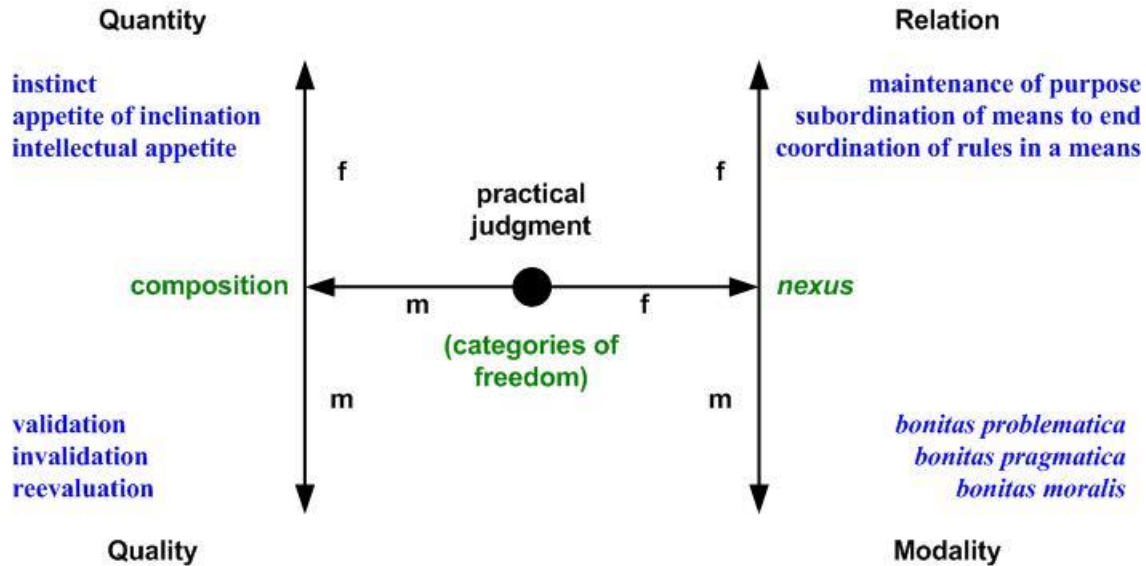**Figure 11:** Practical Reason and Its Information Flow Sources and Destinations.

**Figure 12:** 2LAR structure of the *momenta* of practical judgment.

Maintenance or reestablishment of a state of equilibrium is the practical object of every expressed action. A state of equilibrium is signified by negation of feelings of *Lust* or *Unlust* in reflective judgment. When an expressed action fails to achieve this, practical judgment triggers a reevaluation of the situation by means of ratio-expression. Ratio-expression summons the process of speculative Reason to bring into play the process of determining judgment to effect a change in sensibility by means of reintroducing concepts into the synthesis in sensibility [Wells (2016b)]. (Reevaluation is the third *momentum* of Quality in the 2LAR of practical judgment; see figure 12). This act of practical judgment also produces an accommodation to the practical manifold of rules aimed at trying to prevent future reoccurrences of unsuccessful action expressions. Intuitions in sensibility are made symbolic of meaning implications by the process of teleological reflective judgment, and this is the information pathway by which practical Reason is able to regulate and adapt the non-autonomic action schemes of the Organized Being.

The manifold of rules constructed by practical judgment is a structure[15] in which "old lessons" (previous successful action schemes) are conserved; i.e., new learning does not abolish the old rule structure but, rather, introduces allowable variations in actions by which subjective differences in the manifold of Desires are assimilated as variations by the Organized Being. This is, indeed, one of the manifest properties of human nature observable in psychological research [Piaget (1952), pp. 3-8]. Furthermore, this practical learning precedes cognizance (conceptualization) of what is learned [Piaget (1974)].

At the beginning of life, an infant has no *a priori* manifold of rules and no *a priori* manifold of concepts. It does, however, come into the world with a basic set of innate sensorimotor reflexes and a baseline of affective preferences. A human being spends the first two years of life gradually building up a basic rule structure and the most basic concepts by which he/she comes to understand "reality" [Piaget (1954)]. All of this is accomplished in the dynamics of the judgmentation cycle (figures 7 and 8). Sensori-motor reflexes ("instincts") first seed the manifold of rules and the acquisition of experience extends it through acquired habits and inclinations [Piaget (1952)].

Getting a little closer to the basics, an appetite (*Begierde*) is a represented *parástase* of appetitive power having as its matter a Desire and having as its form a structure within the manifold of rules. Appetitions presented to motoregulatory expression in *psyche* (see figure 13) produce manifested somatic actions.

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[15] A structure is a system of self-regulating transformations such that no new element engendered by their operation breaks the boundaries of the system and the transformation of the system does not involve elements outside it. The system may have subsystems differentiated within it, and have transformations from one subsystem to another.
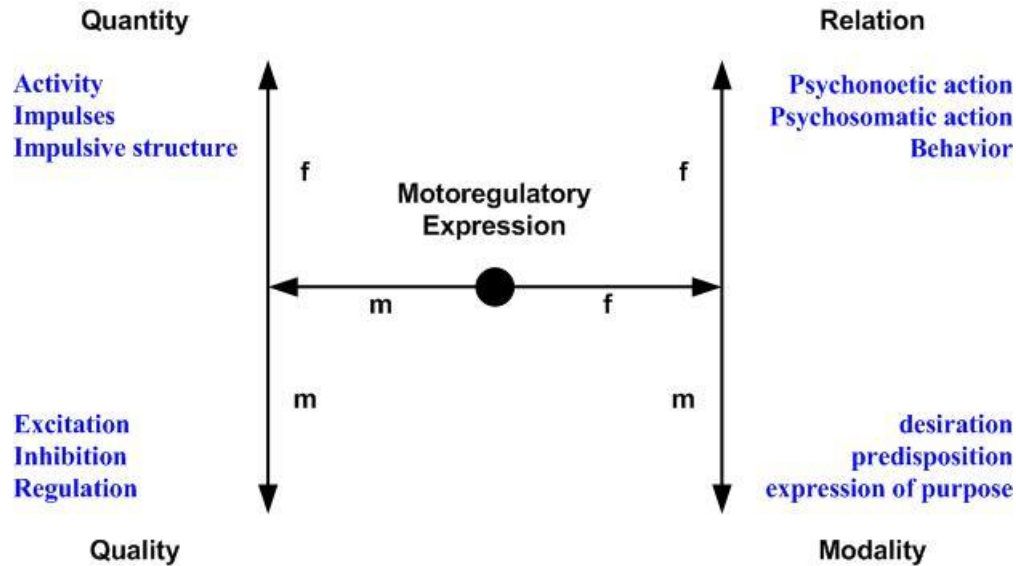
Figure 13: 2LAR structure of motoregulatory expression in the logical division of *psyche*.

These manifest actions are said to be manifestations of an Organized Being's *purposes*. A purpose, regarded from the practical Standpoint, is the object of an Idea of a relationship between reflective judgment and a practical intent of Reason. Now, the Critical explanation of "intent" is that **intent** is the determination of an action expression according to a rule or maxim of practical Reason. The matter of intent is a feeling of subjective expedience and its form is the determined appetite. However, such a dry and "anatomizing" explanation as this tends, I think, to do little to impart the role and importance of intent to human life. A more poetic description, but one that comes closer to conveying the "spirit" of the idea of intent, was provided by George Santayana:

> Intent is one of many evidences that the intellect's essence is practical. Intent is action in the sphere of thought; it corresponds to transition and derivation in the natural world. Analytic psychology is obliged to ignore intent, for it is obliged to regard it merely as a feeling; but while the feeling of intent is a fact like any other, intent itself is an aspiration, a passage, the recognition of an object which not only is not a part of the feeling given but is often incapable of being a feeling at all. What happened to motion under the Eleatic analysis happens to intent under an anatomizing reflection. The parts do not contain the movement of transition which makes them a whole. Moral experience is not expressible in physical categories, because while you may give place and date for every feeling that something is important or is absurd, you cannot express what these feelings have discovered and have wished to confide to you. The importance and the absurdity have disappeared. Yet it is this pronouncement concerning what things are absurd or important that makes the intent of these judgments. To touch it you have to enter the moral world; that is, you have to bring some sympathetic or hostile judgment to bear on those you are considering and to meet intent, not by noting its existence, but by estimating its value, by collating it with your own intent. If someone says two and two are five, you are no counter-mathematician when you conscientiously put it down that he said so. Your science is not relevant to his intent until you run some risk yourself in that arena and say, No: two and two are four.
>
> Feelings and ideas, when plucked and separately considered, do not retain the intent that made them cognitive or living; yet in their native medium they certainly lived and knew. If this ideality or transcendence seems a mystery, it is only in the sense in which every initial or typical fact is mysterious. . . . The fact that intellect has intent, and does not constitute or contain what it envisions, is like the fact that time flows, that bodies gravitate, that experience is gathered, or that existence is suspended between being and non-being. [Santayana (1906), pp. 172-173]

In Critical Epistemology, an Idea (in German, *Idee*) is a regulative principle of actions, and in our

present context the principle is an animating principle of *psyche*. The root intent of all determinations of appetites is dictated by the categorical imperative and it is, again, the negation of feelings of *Lust* or *Unlust* in aesthetical reflective judgment. In relationship to the presentations of reflective judgment, the manifold of rules is to be regarded as a ***value structure*** in relationship to which values (in aesthetical reflective judgment) constitute the conditions for the assertion of these practical rules. Human actions are determined from values the person himself constructs and develops out of his experiences. This basic idiosyncrasy of the *individuality* of action determinations is an important factor in coming to understand the idea of, and the possibility of establishing, justice.

4.3.5 The Process of Determining Judgment.

The process of determining judgment is the principal process of judgment for the theoretical Standpoint. It performs what Kant called judgments of experience (by which representations of phenomena are conceptualized) and judgments of ideas of logico-mathematical objects that we never encounter in sensuous experience (*noumena*). The outcome of an act of determining judgment is called a determinant judgment because the judgment determines concepts of objects and the organized structure of the manifold of concepts.

Figure 14 illustrates the 3LAR structure of determinant judgments. The matter of a determinant judgment pertains to how a person understands the manifold of what is contained in intuitions (their scope) and so what he thinks about an object when those intuitions are reintroduced into sensibility from its concept [Kant (1787), B: 106]. The form of a determinant judgment pertains to the logical structure of concepts in the manifold of concepts. The twelve ontological *momenta* of scope are called the ***categories of understanding*** and they determine the conceptual significance and empirical understanding of objects. The twelve logical *momenta* of form are called the ***logical functions of understanding in judgment*** and pertain to the mathematical form of the manifold of concepts.

Figures 15 and 16 illustrate the 2LARs for the categories and the logical functions. Kant's presentation of the categories in *Critique of Pure Reason* is widely regarded as too brief, and I agree with that opinion. Wells (2006), chapters 8-9, provides a thorough deduction and explanation of them from all four of the reflective perspectives of the theoretical Standpoint (logical, transcendental, hypothetical and empirical). Their explanations are then summarized in chapter 10 of that work.
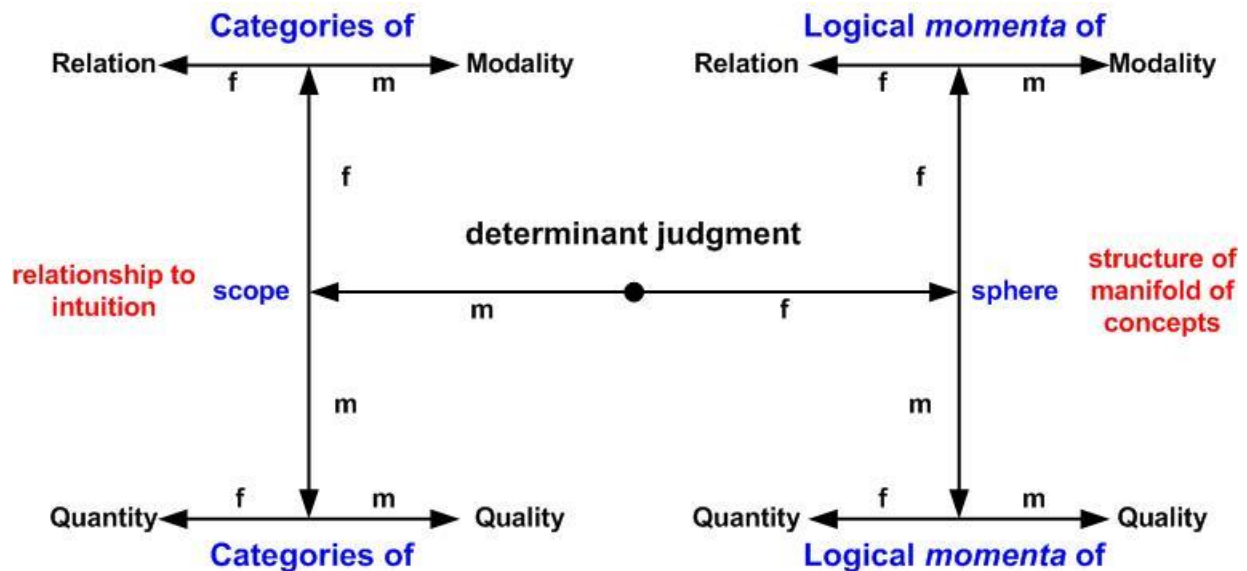

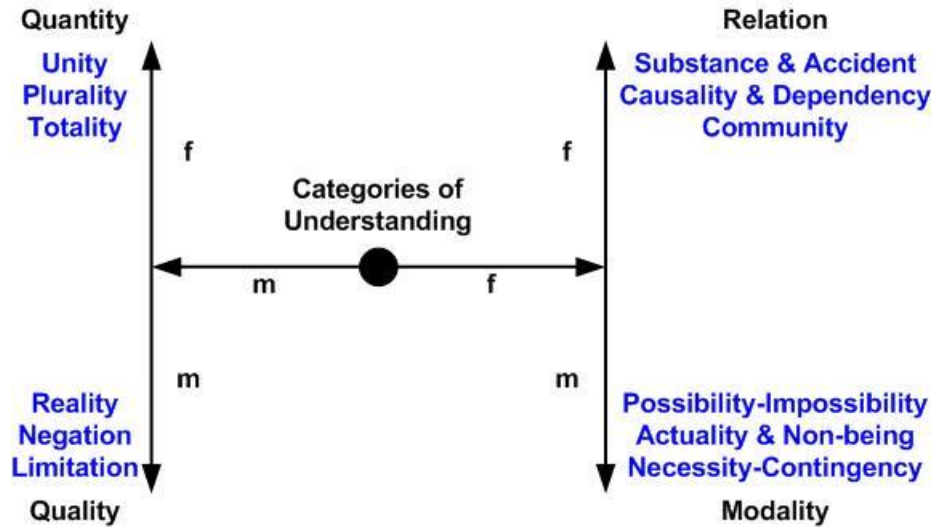**Figure 14:** 3LAR organization of determinant judgments.

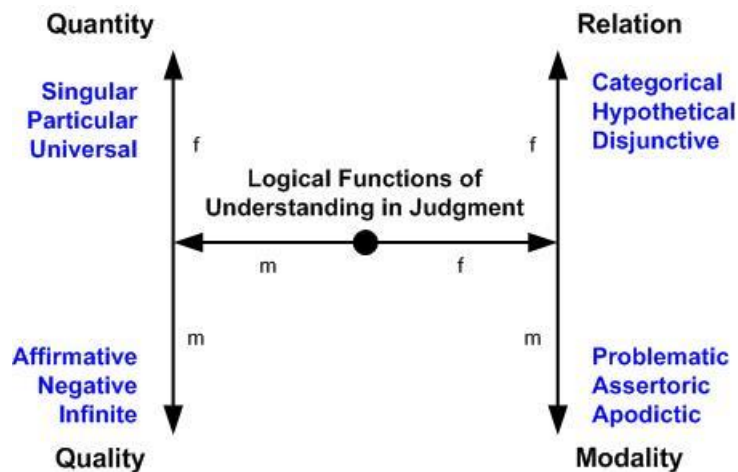**Figure 15:** 2LAR of the categories of understanding.

Figure 16: The logical functions of understanding in judgment.

As for the logical functions, the labels Kant gave them are almost identical to labels given to predications in so-called "Aristotelean logic." This has also created historical difficulties in interpreting Kant's philosophy. First of all, this "Aristotelean logic" should properly be called "Scholastic logic" because it differs from Aristotle's "science of demonstration" in some important ways. Probably the most important of these is that the scholastics divorced "Aristotle's logic" from his metaphysics. Why they did so is not hard to understand. In the first place, Aristotle's logic works came down to them before the rest of Aristotle's corpus of writings did and the scholastics had already eagerly embraced those works. Then came Aristotle's metaphysics, which contains doctrines that are contrary to Christian doctrines. As they regarded Aristotle as a pagan, they tossed out these doctrines but kept his "logic" works. Indeed, Aquinas' great accomplishment was in making Aristotle "safe for Christianity." But Aristotle never divorced his "science of demonstration" from his metaphysics and the latter has consequences for the former.

However, Aristotle's metaphysic is ontology-centered, and that is precisely what Kant changed with his philosophy. Consequently, "Kantian Logic" is not the same thing as "Aristotle's logic," and so the labels for Kant's logical functions of understanding have different epistemological interpretations from those of both scholastic logic and Aristotle's "science of demonstration." Wells (2006), chapter 8, discusses these differences in detail. The logical functions are not representative of a different or separate brand of

"formal logic" but are instead an integral and function part of Critical Epistemology:

> [The] critique of pure reason is moreover to be regarded as a higher logic in that it gives Reason rules at hand as to how it may know Objects *a priori*, and those rules are nothing but *a priori* principles; it is thus different from logic itself, in that this ["logic itself", i.e. traditional logic] abstracts from all Objects, and [Kant's "higher logic"] states the rules of thinking in general. [Kant (1794-95), 29: 949]

Aristotle claimed his science of demonstration was a doctrine for *how* one should reason "from reputable opinions about any subject" while avoiding "saying anything self-contradictory" when "sustaining an argument" [Aristotle (*c*. 350 BC), 100ª20-25]. This is something far less ambitious than stating "the rules of thinking in general."

   Even so, there is a lot of similarity between Kant and Aristotle and, for the most part, if you are familiar with Aristotle's logic terminology your understanding of those labels will not mislead you *too* greatly in understanding Kant's terminology. If you should encounter paradoxes, you can always refer to the Wells citations to clear them up. The key idea to keep in mind is that Aristotle's logic refers to and pertains to *things* (ontology) while Kant's refers to *concepts* (epistemology). I find that a great deal of Kant's philosophy can be well understood by taking Aristotle's philosophy and applying Kant's "Copernican turn" to it. What is indispensable in understanding all this is to understand the characteristics of Kant's manifold of concepts. Figure 17 provides a simplified illustration of it's organization and several of its key ideas.
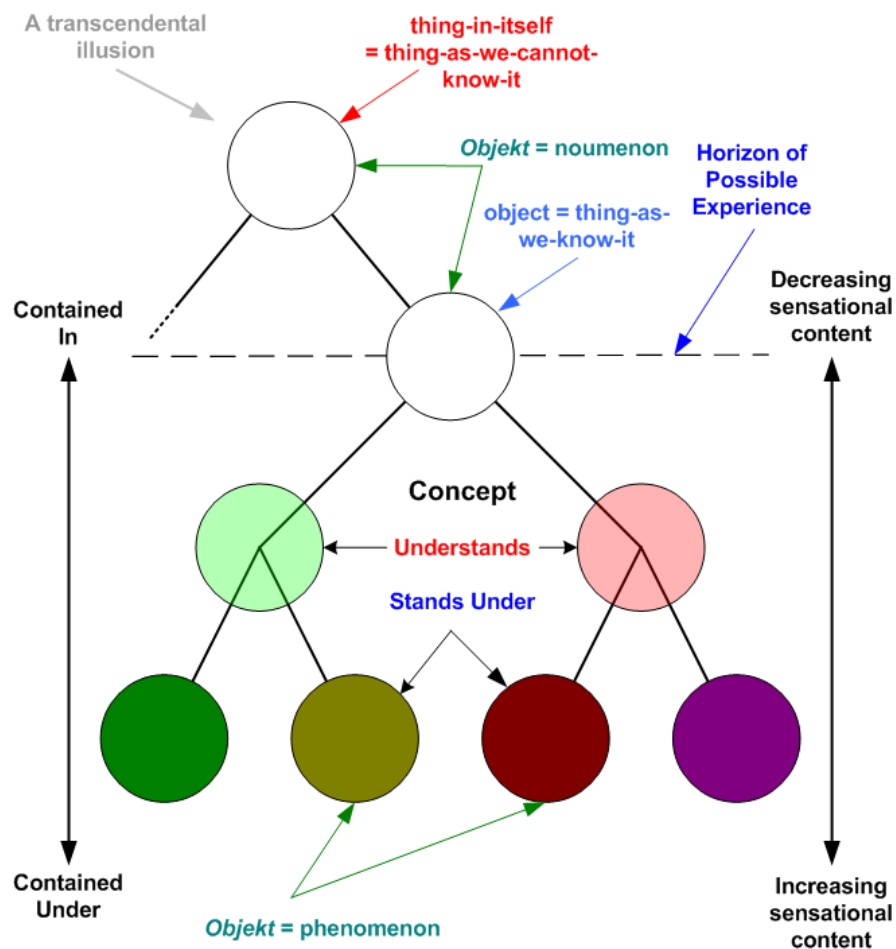

Figure 17: A simple illustration of the structure of the manifold of concepts. *Objekt* translates as Object.

In this figure, the circles depict concepts and the solid lines depict combinations of concepts defined by the categories of understanding. Each such line represents *four* momenta, one from each of the four "heads" of Quantity, Quality, Relation, and Modality in figure 15. Every higher concept in the figure is made by combining two or more lower concepts, which are said to be "contained in" the higher concept and which "stand under" it. The higher concept is said to "under stand" the lower ones (hence the term "understanding"). The colored fill of a circle represents sensational matter "in" the concept. The Object corresponding to the concept is called a phenomenon when the concept contains sensational matter. Because a higher concept is more abstract than its lower concepts, its intuitive representation makes abstraction of the differences between the lower concepts and, generally, each higher concept contains less sensational matter than its lower concepts.

As higher concepts are produced in the process of thinking, eventually a point is reached when the higher concept no longer contains *any* sensational matter. When this happens, the Object is said to be supersensible because it no longer has sensational matter. Such an Object is called a *noumenon*. The first point at which this happens is called the *horizon of possible experience* [Kant (1800), 9: 40-43]. A concept having a noumenon for its Object is called an idea, and an idea at the horizon of experience represents "a thing-as-we-know-it" because all that we can know about it *from experience* is what is represented by its chain of lower *sensible* concepts.

However, the processes of thinking, reasoning, and judgmentation do not come to a halt at the horizon of possible experience. These processes may continue and produce still-higher ideas of more supersensible Objects. *All* noumena are mathematical Objects, are products of pure speculation and definition (by the Organized Being itself), and *ontologically* are transcendental illusions. Objects beyond the horizon of experience would be "things-as-we-cannot-know-them" insofar as ontology is concerned. However, they *can* have *epistemological* significance *as Objects of pure mathematics*. Here their objective validity is *practical* validity rather than theoretical objective validity. You might, at first encounter, think that it is a failing of human reasoning that such non-ontological *noumena* are inevitable eventual products of thinking and reasoning. However, science itself is possible *only* because human beings are capable of thinking up and defining supersensible Objects. Examples include: gravity, mass, electric charge, and energy in physics; allotropy, agonists, and antagonists in chemistry; communities and societies in sociology; emotions and motivations in psychology; utility in economics; lines, points, and surfaces in geometry; infinitesimals in calculus; and nation, citizen, and justice in political science.

The ideas found in scientific theories are all *noumena* of this sort. Of course, we are also capable of thinking up superstitious ideas like ghosts, or that a rabbit's foot brings good luck, or (for that matter) the idea of luck itself. These ideas are harmful *only* if we mistake their Objects as having a false *ontological* reality or mistake epistemological significance for ontological significance. The categories of understanding have real objective validity only up to, and never beyond, the horizon of possible experience.

Technically, the depictions in figure 17 explicitly show only the simplest form of combinations in the manifold of concepts, and the figure suppresses a number of important subtleties producible by the synthesis in judgmentation. I said above that determining judgment is only the *principal* process of judgment at work in the *theoretical* Standpoint. But *all three* processes of judgment are at work in an overall synthesis in judgmentation. One important subtlety suppressed in figure 17 is this: *a combination of concepts can itself be a concept*. Recall the earlier remark by James that "These objects we break asunder and reunite. We must treat them in both ways for our knowledge of them to grow." This "breaking apart and reuniting" happens during the full cycle of judgmentation and is not the work of just *one* process of judgment. Recall, too, that determining judgment does not determine its own employment. This falls to the process of speculative Reason and is regulated during ratio-expression by practical regulatory Ideas (principles) that Kant called the "transcendental Ideas" of pure Reason [Wells (2009), chap. 2]. Synthesis of two or more already categorically combined concepts into a unified concept, i.e.,

Concept 1 + Concept 2 $\rightarrow$ (Concept 1, Concept 2)

is called an *inference of Reason*. There are two main kinds of inferences of Reason. They are called *poly-syllogisms* [Wells (2011)] and *disjunctive inferences of Reason* [Wells (2012c)]. In determining judgment these *forms of combination* are what are implied by the hypothetical and disjunctive logical functions of understanding in judgment (figure 16). Recall that I said Kant's logical function names do not mean the same thing as is meant in "Aristotelean logic" by these names. Instead, Kant's logical functions refer to the *structure* given to the manifold of concepts as it is constructed. Perhaps it is a little more clear from this why Kant's logical functions are called logical functions *of understanding* in judgment? The *structure* is logically assigned to the process of determining judgment but the *synthesis* of this structure is the product of the overall cycle of judgmentation. Wells (2011) and Wells (2012c) explain in much greater detail how syntheses of inferences of Reason work.

4.3.6 Practical Imperatives vs. Theoretical Imperatives

Mathematically, the structure of the manifold of rules in practical Reason has the same mathematical expression as the manifold of concepts in determining judgment with three important differences. The first is that combinations in the manifold of rules are made by the *momenta* of practical judgment rather than by the categories of understanding in determinant judgments. The second is that concepts can be re-introduced into sensibility through imagination and thereby be made *conscious* representations; in contrast, practical rules are reintroduced directly into the synthesis of appetition but are *never* conscious representations. The third, and possibly most important, difference is: *all practical rules stand under and are conditioned by the categorical imperative of pure practical Reason*, whereas the highest concepts in the manifold of concepts are *unconditioned* by any other concept or principle. All practical rules are logically hypothetical (because they are conditioned) but unconditioned concepts are said to be categorical. There is, therefore, a key and most essential difference between a *practical* categorical imperative (and there is only one of these) and a *theoretical* categorical imperative. This is a distinction that Kant totally missed because of his decision not to give the practical Standpoint as thorough a treatment as he gave the theoretical Standpoint.

A theoretical categorical imperative in the manifold of concepts is liable to being "demoted" because it is always possible for new experience to bring about an accommodation in the manifold of concepts that places a *higher* concept above it and, thereby, makes it conditioned. If and when this happens, what at one time was a "categorical imperative" of understanding becomes instead a "hypothetical imperative." Furthermore, no concept in the manifold of concepts has a direct connection to appetitive power in practical Reason; because of this, a theoretical categorical imperative is merely an "ought to" concept. In contrast, the practical categorical imperative of Reason is a **law** and is always obeyed (in acts and actions) by the Organized Being. If a rule in the manifold of rules, that has no higher rule placed above it, is invoked (as a result of reflective judgment), that rule is immediately acted upon by the Organized Being by command of the categorical imperative of practical Reason because it is practically regarded as being made necessary (necessitated) for maintenance or reestablishment of equilibrium.

If evoking a (presently highest) rule (hypothetical practical imperative) in the manifold of rules *fails* to maintain or reestablish equilibrium, that circumstance triggers a reevaluation (of practical judgment) and a ratio-expression of appetitive power that leads to an accommodation in the manifold of *rules*. A new "higher rule" will be established that conditions what had been a rule unconditioned by another rule in the manifold. This is an instance of *practical* sensorimotor learning, the consequences of which are manifest changes in observable somatic actions (whether the change is directly observable or observable by means of appropriate scientific instrumentation).

There are numerous places within the Kantian corpus demonstrating that Kant failed to sufficiently distinguish between practical imperatives and theoretical imperatives. My own opinion is that these instances are most likely harmful consequences of Kant attributing ontological significance to his idea of a "moral law within me." It is a mistake habits of ontology-centered thinking can easily lead one to make.

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