Math 175
Exam 2
Spring 2019

Exam Rules:

- Do not open this exam until instructed to do so.
- No questions will be taken during the exam.
- Calculators or other electronics are not allowed to be used on this exam.
- Calculators, phones, laptops, or other devices must be put away out of sight during the exam.
- Phones must be off or set to "silent".

Note: You must use correct mathematical notation, show all of your work, and present your work in a neatly organized manner in order to get full credit for your answer.

If you need more space during the exam, you may use the back of this sheet.

Exam 2

MATH 175, CALCULUS II, SPRING 2019

NAME / SECTION NO.: Key

Problem 1. (12 points) Evaluate

$$\int \sin^3 x \cos^2 x \, dx.$$

$$\int \sin^2 x \sin x \cos^2 x \, dx = \int (1-\cos^2 x)\cos^2 x \sin x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$= -\int (u^2 - u^4)\, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$$

Problem 2. (12 points) Evaluate

$$\int \tan^2 x \sec^4 x \, dx.$$

$$\int \tan^2 x \sec^2 x \sec^2 x \, dx = \int \tan^2 x (1 + \tan^2 x) \sec^2 x \, dx$$

$u = \tan x$

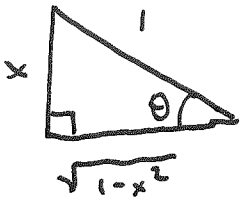
$du = \sec^2 x \, dx$

$$= \int u^2 + u^4 \, du$$

$$= \frac{u^3}{3} + \frac{u^5}{5} + C$$

$$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$$

Problem 3. (15 points) Evaluate $\int x^3\sqrt{1-x^2}\,dx$, by using a trig substitution.



$x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

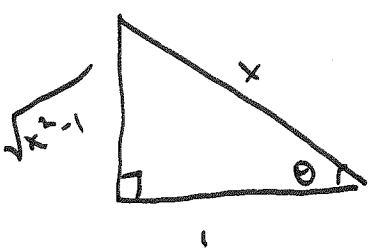
$\sqrt{1-x^2} = \cos\theta$

$$\int x^3\sqrt{1-x^2}\,dx = \int \sin^3\theta \cos\theta \cos\theta\, d\theta$$

(by 1) $$= \frac{\cos^5\theta}{5} - \frac{\cos^3\theta}{3} + C$$

$$= \frac{(1-x^2)^{5/2}}{5} - \frac{(1-x^2)^{3/2}}{3} + C$$

Problem 4. (15 points) Evaluate $\int x^3\sqrt{x^2-1}\,dx$, by using a trig substitution.



$x = \sec\theta \Rightarrow dx = \sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-1} = \tan\theta$

$$\int x^3\sqrt{x^2-1}\,dx = \int \sec^3\theta \tan\theta \sec\theta \tan\theta\, d\theta$$

$$= \int \tan^2\theta \sec^4\theta\, d\theta$$

(by 2) $$= \frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5} + C$$

$$= \frac{(x^2-1)^{3/2}}{3} + \frac{(x^2-1)^{5/2}}{5} + C$$

Problem 5. (13 points) Estimate the value of $\int_0^6 (x+1)^2\,dx$ by numerical integration using the midpoint rule with $n = 3$ (three rectangles). Also, find the error of approximation.

$$\int_0^6 (x+1)^2\,dx = 114$$

$$\Delta x = \frac{6-0}{3} = 2 \Rightarrow x_0 = 0,\ x_1 = 2,\ x_2 = 4,\ x_3 = 6$$

$$M_3 = 2\left[f\left(\frac{0+2}{2}\right) + f\left(\frac{2+4}{2}\right) + f\left(\frac{4+6}{2}\right)\right]$$

$$= 2(4 + 16 + 36)$$

$$= 112$$

Error: $|114 - 112| = 2$

Problem 6. (13 points) Evaluate

$$\int \frac{e^{2x}}{1+e^x}\,dx$$

by using any method(s) you want.

$u = e^x$
$du = e^x\,dx$ $\Rightarrow \int \frac{u}{1+u}\,du$

$$= \int \frac{u+1-1}{1+u}\,du$$

$$= u - \ln|1+u| + C$$

$$= e^x - \ln|1+e^x| + C$$

OR

$u = e^x + 1$
$u - 1 = e^x$ $\Rightarrow \int \frac{u-1}{u}\,du$
$du = e^x\,dx$

$$= u - \ln|u| + C$$

$$= 1 + e^x - \ln|1+e^x| + C$$

$$= e^x - \ln|1+e^x| + C$$

(merge constants)

Problem 7. Evaluate each of the following improper integrals. Remember to use correct notation, writing each as a limit of proper integrals.

(a) (10 points) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

Integrand DNE at $x=1$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin(x) + C$$

$$\Rightarrow \lim_{t\to 1^-} \left[\arcsin(x)\right]_0^t = \lim_{t\to 1^-} \left[\arcsin(t)\right] = \frac{\pi}{2} \quad (\text{goes to } \pi/2)$$

(b) (10 points) $\int_0^\infty \frac{dx}{(3x+1)^2}$

Infinite bound

$$\int \frac{dx}{(3x+1)^2} = -\frac{1}{3(3x+1)} + C$$

$$\Rightarrow \lim_{t\to\infty} \left[-\frac{1}{3(3x+1)}\right]_0^t = \lim_{t\to\infty} \left(\frac{1}{3} - \frac{1}{3(3t+1)}\right) = \frac{1}{3} \quad (\text{goes to } \tfrac{1}{3})$$

Extra Credit. (10 points) Solve

$$\int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}}. \quad (\text{bad bounds})$$

$$\int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}} \left(\frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}\right) = \int_0^1 \left(\sqrt{x+1} - \sqrt{x}\right) dx$$

$$= \frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2} + C \Big|_0^1$$

$$= \frac{4\sqrt{2}}{3} - \frac{2}{3} - \frac{2}{3}$$

$$= \frac{4\sqrt{2}}{3} - \frac{4}{3}$$