Math 175 – Review Questions for Exam 3

1. Write a formula for the $n$th term of the sequence $6, -3, 3/2, -3/4, 3/8, -3/16, \ldots$

2. For each sequence below, either find the limit or show that the limit does not exist. Be sure to justify your answer.

\[
\begin{align*}
\left\{ (-1)^n n \right\} & \quad \left\{ \frac{\sin(n)}{n + \cos(n)} \right\}
\end{align*}
\]

3. Write the first four terms of the sequence of partial sums for the series $\sum_{k=2}^{\infty} \frac{2k}{k^2 + 1}$.

4. Find the sums of the following geometric series (if they converge).

\[
5 - \frac{5}{3} + \frac{5}{9} - \frac{5}{27} + \frac{5}{81} - \cdots \quad \sum_{k=2}^{\infty} \frac{3^k}{2^{2k}}
\]

5. Find the sum of the series $\sum_{k=1}^{\infty} \left( \frac{1}{2k+1} - \frac{1}{2k+3} \right)$ if it converges.

6. Use the Integral Test to determine if the following series converge or diverge:

\[
\sum_{k=1}^{\infty} \frac{k}{(k^2 + 1)^2} \quad \sum_{k=1}^{\infty} \frac{1}{\sqrt{k} + 3}
\]

7. Use the Ratio Test or Root Test to determine if the following series converge or diverge:

\[
\sum_{k=1}^{\infty} \frac{3^{k+1}}{k!} \quad \sum_{k=1}^{\infty} \left( \frac{2k+1}{3k-5} \right)^k
\]

8. Use the Limit Test or Limit Comparison Test to determine if the following series converge or diverge:

\[
\sum_{k=1}^{\infty} \frac{1}{k^3 + \ln(k)} \quad \sum_{k=1}^{\infty} \frac{k}{k^2 + 1}
\]

9. Use any of the series convergence tests to determine whether these series converge or diverge:

\[
\sum_{k=2}^{\infty} \frac{6}{(k^2 - 1)} \quad \sum_{k=1}^{\infty} \frac{(-1)^k}{2k + 5} \quad \sum_{k=1}^{\infty} \frac{5k^2 - 4k + 2}{3k^2 + \ln(k)}
\]

10. For the second series in problem 9, how large do we need to make $n$ so that the $n$th partial sum $S_n$ is within $10^{-3}$ of the actual sum for the series?