Math 175 – Review Questions for Exam 4

1. Find the radius of convergence and interval of convergence for the power series below.
\[ \sum_{k=1}^{\infty} \frac{k x^k}{(k^2 + 1)5^k} \quad \sum_{k=1}^{\infty} \frac{x^{2k}}{3^k} \]

2. Modify a geometric series to give a power series for the function \( f(x) = x^2 \ln(1 - x) \).

3. Find the first four nonzero terms of the Taylor series for \( f(x) = x^{1/3} \) at \( x = 8 \). Use this to give an approximation of the number \( (8.2)^{1/3} \).

4. Use the first three nonzero terms of the Maclaurin series for \( \sin x \) to give an approximation for \( \sin(0.1) \). Use the Taylor Remainder Theorem to give a bound on the error for this approximation.

5. Use a Taylor series to evaluate \( \lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x^2} \).

6. Use the first three nonzero terms of a Taylor series to estimate \( \int_{0}^{0.5} \cos(x^2) \, dx \).

7. Plot each of the parametric curves below. Then eliminate the parameter to give the equation in \( x \) and \( y \) only.
   (a) \( x = 5 - t, \ y = 3 + 2t, \ 0 \leq t \leq 2 \).
   (b) \( x = \cos^2 t, \ y = 2\sin t, \ \pi/2 \leq t \leq 3\pi/2 \).

8. Give parametric equations for each of the following.
   (a) The segment from \((3, -1)\) to \((0, 4)\).
   (b) The arc of the circle centered at \((1, 2)\) that goes clockwise from \((4, 2)\) to \((1, -1)\).
   (c) The portion of the parabola \( y = 5 - x^2 \) from \((-1, 4)\) to \((2, 1)\).

9. Do each of the following.
   (a) Write the Cartesian point \((-\sqrt{3}, 1)\) in polar coordinates in three different ways.
   (b) Write the polar coordinate point \((-2, \pi/4)\) in Cartesian coordinates.
   (c) Convert the equation \( r = 3 \csc \theta \) to an equation in \( x \) and \( y \).
   (d) Convert the equation \((x - 2)^2 + y^2 = 4\) to polar coordinates.

10. Consider the equation \( r = 1 - 2\sin \theta \).
    (a) Graph this equation.
    (b) What is the slope of the tangent line to this curve at the polar coordinate point \((1, \pi)\).
    (c) What is the area inside the curve’s “inner loop”?