Math 175 – Review Questions for Exam 4

1. Find the radius of convergence and interval of convergence for the power series below.

$$\sum_{k=1}^{\infty} \frac{k\,x^k}{(k^2+1)5^k} \qquad\qquad \sum_{k=1}^{\infty} \frac{x^{2k}}{3^k}$$

2. Modify a geometric series to give a power series for the function $f(x) = x^2 \ln(1-x)$.

3. Find the first four nonzero terms of the Taylor series for $f(x) = x^{1/3}$ at $x = 8$. Use this to give an approximation of the number $(8.2)^{1/3}$.

4. Use the first three nonzero terms of the Maclaurin series for $\sin x$ to give an approximation for $\sin(0.1)$. Use the Taylor Remainder Theorem to give a bound on the error for this approximation.

5. Use a Taylor series to evaluate $\displaystyle\lim_{x \longrightarrow 0} \frac{e^{2x} - 1 - 2x}{x^2}$.

6. Use the first three nonzero terms of a Taylor series to estimate $\displaystyle\int_0^{0.5} \cos(x^2)\,dx$.

7. Plot each of the parametric curves below. Then eliminate the parameter to give the equation in x and y only.

(a) $x = 5 - t$, $y = 3 + 2t$, $0 \le t \le 2$.

(b) $x = \cos^2 t$, $y = 2\sin t$, $\pi/2 \le t \le 3\pi/2$.

8. Give parametric equations for each of the following.

(a) The segment from $(3, -1)$ to $(0, 4)$.

(b) The arc of the circle centered at $(1, 2)$ that goes clockwise from $(4, 2)$ to $(1, -1)$.

(c) The portion of the parabola $y = 5 - x^2$ from $(-1, 4)$ to $(2, 1)$.

9. Do each of the following.

(a) Write the Cartesian point $(-\sqrt{3}, 1)$ in polar coordinates in three different ways.

(b) Write the polar coordinate point $(-2, \pi/4)$ in Cartesian coordinates.

(c) Convert the equation $r = 3\csc\theta$ to an equation in x and y.

(d) Convert the equation $(x-2)^2 + y^2 = 4$ to polar coordinates.

10. Consider the equation $r = 1 - 2\sin\theta$.

(a) Graph this equation.

(b) What is the slope of the tangent line to this curve at the polar coordinate point $(1, \pi)$.

(c) What is the area inside the curve's "inner loop"?