

A16

Extra credit module 16

Joint Distributions

Stat 251

Instructions:

Only upload in BbLearn. Follow directions on Assignments link on class website for BbLearn submissions.

This assignment is worth up to 3.5 points **if and only if you complete ALL problems**

- (1) A certain market has both an express checkout line and a superexpress checkout line. Let X denote the number of customers in line at the express checkout at a particular time and let Y denote the number of customers in the line at the superexpress checkout the same time. Suppose the joint pmf of X and Y are found in the accompanying table.
- (a) What is the probability that there is exactly one customer in each line?
 - (b) What is the probability that the numbers of customers in the two lines are identical?
 - (c) Find $p_X(x)$ and $p_Y(y)$ (marginal distributions of X and Y)
 - (d) calculate:
 - (i) $E(X)$, $E(Y)$
 - (ii) $E(X^2)$, $E(Y^2)$
 - (iii) $V(X)$, $V(Y)$
 - (iv) $SD(X)$, $SD(Y)$
 - (v) $E(XY)$
 - (vi) $Cov(X, Y)$
 - (vii) $Corr(X, Y)$
 - (viii) $E(X + Y)$, $E(X - Y)$
 - (ix) $V(X + Y)$, $V(X - Y)$
 - (e) Are X and Y independent? Show work

	y	0	1	2	3
x	0	0.08	0.07	0.04	0.00
	1	0.06	0.15	0.05	0.04
	2	0.05	0.04	0.10	0.06
	3	0.00	0.03	0.04	0.07
	4	0.00	0.01	0.05	0.06

- (2) A service station had both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, let Y denote the number of hoses on the full-service island in use at that time. The joint pmf of X and Y are in the accompanying table.
- (a) Calculate the Marginal distributions of X and Y
 - (b) Calculate EX , EY , $E(XY)$, VX , VY , SDX , SDY , $Cov(X, Y)$, $Corr(X, Y)$. Once $Corr(X, Y)$ is calculated, interpret.
 - (c) Calculate the probability there is exactly one hose in use at each station

- (d) Calculate the probability there is at most one hose in use at each station
- (e) Given that two hoses are in use at the self-service island, what is the probability there is at most one hose in use on the full-service island?
- (f) Given that two hoses are in use at the full-service island, what is the probability there is at most one hose in use on the self-service island?

		Y		
$p(x, y)$		0	1	2
X	0	0.15	0.02	0.01
	1	0.06	0.18	0.07
	2	0.03	0.18	0.30