Example of CI interpretation statement: “We are 95% confident the true mean eruption time of Old Faithful is between 2.5 and 3.5 minutes.”

A consumer testing agency wants to evaluate the claim made by a manufacturer of discount tires; the claim is that its tires can be driven at least 35,000 miles before wearing out. Assume that these tires have a normal distribution, and its standard deviation is 5,000 miles.

To determine the average number of miles that can be obtained from the tires, the agency randomly selects 60 tires from the manufacturer’s warehouse and places the tires on 15 similar cars driven by test drivers on a 2-mile oval track, with sample mean of 31,470 miles. Estimate $\mu$ with 95% confidence (construct a 95% CI).

$$\bar{X} = 31470, \sigma = 5000, n = 60$$

$$\bar{X} \pm bound = \bar{X} \pm z^{\star}(se)$$

$$se = \frac{\sigma}{\sqrt{n}} = \frac{5000}{\sqrt{60}} = 645.5$$

$$CL\ (confidence\ level) = 0.95\ and\ \alpha = 1 - Cl = 1 - 0.95 = 0.05$$

Statdistributions.com for z score: p-value=.05, 2T. $z^{\star} = 1.96$ (it can be negative but you still get same answer)

$31470 \pm (1.96)(645.5) = 31470 \pm 1265.18 = 30204.82, 32735.18$ miles

With 95% confidence, the true mean mileage of theses tires is between 30,204.8 and 32,735.2 miles. Since 35,000 is NOT in CI, there is not way the claim is correct.

Cis are intervals that are basically acceptance interval. If the claimed value is in CI, then the claim has truth; if the claimed value is not in CI, we cannot accept the claim.

Bound: 1265.18 is relatively small compared to the mean and its data values, so it is a relatively small bound. Precision is decent.

If we increase confidence, the z-score gets larger, the interval widens, and precision goes down.

Confidence: example with 95%. In repeated CI construction, out of 100 intervals, 95 will contain the true parameter, and 5 will NOT contain the parameter.

How could we decrease the bound (aka margin of error) without decreasing confidence level?

$$\bar{X} \pm z^{\star}\left(\frac{\sigma}{\sqrt{n}}\right)$$

You can increase the sample size to make the bound smaller and increase precision