Central Limit Theorem (CLT): The sampling distribution of the sample mean is approximately normal with mean μ and standard deviation of the sampling distribution of the sample mean (also called standard error) is $se = \sigma/\sqrt{n}$, provided n is sufficiently large.

"Sufficiently large": if original distribution is normal, there is no sample size requirement (n should be at least 2). If original distribution is NOT normal or you have no information the distribution, then when dealing with the distribution of the sample mean ($\bar{x}$) $n \geq 30$. If looking at distribution of the sample proportion, then $n \geq 60$

Sample mean:

$$\bar{X} \sim N(\mu, se)$$

Probabilities calculated with z-distribution

$z = \frac{\bar{X}-\mu}{se}$ with $se = \frac{\sigma}{\sqrt{n}}$

The level of a particular pollutant, nitrogen dioxide (NO2), in the exhaust of a hypothetical model of car, that when driven in city traffic, has a mean level of 2.1 grams per mile (g/m) and a standard deviation of 0.3 g/m. Suppose a company has a fleet of 35 of these cars.

$$\mu = 2.1, \sigma = 0.3, n = 35$$

What is the mean and standard deviation of the sampling distribution of the sample mean? (state mean and calculate se):

$$\bar{X} \sim N\left(\mu = 2.1, se = \frac{\sigma}{\sqrt{n}} = \frac{0.3}{\sqrt{35}} = 0.0507\right)$$

The mean is $\mu = 2.1$ and standard deviation of sampling distribution (standard error) is $se = \frac{\sigma}{\sqrt{n}} = \frac{0.3}{\sqrt{35}} = 0.0507$.

Describe the distribution of the sample mean: distribution of sample mean should be normal

$$z = \frac{\bar{X} - \mu}{se}$$

Solve for $\bar{X}$: $z(se) = \bar{X} - \mu$ ➔ $\bar{X} = z(se) + \mu$

1. find the probability that the mean NO2 level is less than 2.03 g/m
$$P(\bar{X} < 2.03) = P\left(Z < \frac{2.03 - 2.1}{0.0507}\right) = P(Z < -1.38) = 0.084 = 8.4\%$$
statdistibutions.com: z=-1.38, LT: http://www.statdistributions.com/normal?z=-1.38&tail=3
2. Mandates by the EPA state that the average NO_2 of the fleet of these cars cannot exceed 2.2 g/m, find the probability that the fleet NO2 levels from their fleet exceed the EPA mandate.
$$P(\bar{X} > 2.2) = P\left(Z > \frac{2.2 - 2.1}{0.0507}\right) = P(Z > 1.97) = 0.024 = 2.4\%$$
statdistibutions.com: z=1.97, RT: http://www.statdistributions.com/normal?z=1.97&tail=2
3. At most, 25% of these cars exceed what mean NO2 value?
    a. Find z for top 25%:

b. Plug in z, μ, and se to solve for $\bar{X}$
Need z for top 25%; statdistibutions.com:p-value=.25, RT z=0.675
http://www.statdistributions.com/normal?p=0.25&tail=2

$$\bar{X} = (0.675)(0.0507) + 2.1 = 2.13\ g/mi$$

Distribution of sample proportion: Sampling distribution of the sample proportion will be approximately normal with mean π and standard deviation of the sampling distribution of the sample proportion (standard error) $se = \sqrt{\frac{\pi(1-\pi)}{n}}$, provided n is sufficiently large (with proportions, $n \geq 60$)

$$\hat{\pi} \sim N(\pi, se) \text{ and } z = \frac{\hat{\pi}-\pi}{se}$$