Distribution of sample proportion with CLT: Sampling distribution of the sample proportion will be approximately normal with mean π and standard deviation of the sampling distribution of the sample proportion (standard error) $se = \sqrt{\frac{\pi(1-\pi)}{n}}$, provided n is sufficiently large (with proportions, $n \geq 60$)

$$\hat{\pi} \sim N(\pi, se) \text{ and } z = \frac{\hat{\pi}-\pi}{se}$$

Mars company claims that 10% of the M&M's it produces are green. Suppose that candies are packaged at random in bags that contain 60 candies.

$$\pi = 0.1, n = 60$$

(a) Describe the sampling distribution of the sample proportion (what should the distribution look like?); calculate the mean proportion and standard deviation of the sampling distribution of the sample proportion of green M&M's in bags that contain 60 candies (calculate π and se).

$$se = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.1(1-0.1)}{60}} = 0.0387$$

$$\hat{\pi} \sim N(0.1, 0.0387)$$

distribution should be approx. normal

(b) What is the probability that a bag of 60 candies will have more than 13% green M&M's?

$$P(\hat{\pi} > 0.13) = P\left(Z > \frac{0.13-0.1}{0.0387}\right) = P(Z > 0.775) \approx P(Z > 0.78) = 0.218 = 21.8\%$$

statdistributions.com: z=.78, RT: http://www.statdistributions.com/normal?z=0.78&tail=2

(c) What is the probability that a bag of 60 candies will have less than 9% green M&Ms?

$$P(\hat{\pi} < 0.09) = P\left(Z < \frac{0.09-0.1}{0.0387}\right) = P(Z < -0.26) = 0.397 = 39.7\%$$

statdistributions.com: z=-.26, LT: http://www.statdistributions.com/normal?z=-0.26&tail=3

The level of a particular pollutant, nitrogen dioxide (NO2), in the exhaust of a hypothetical model of car, that when driven in city traffic, has a mean level of 2.1 grams per mile (g/m) and a standard deviation of 0.3 g/m. Suppose a company has a fleet of 35 of these cars.

$$z = \frac{\hat{\tau}-\tau}{se} \; with \; se = \sqrt{n}\sigma = \sigma\sqrt{n}$$

$$\hat{\tau} \sim N(\tau, se)$$

what is the mean and standard deviation of the total amount (sum), in g/m, of NO2 in the exhaust for the fleet?

Mean of total: $\tau = n\mu = 35(2.1) = 73.5$

Standard deviation of sampling distribution (standard error): $se = \sqrt{n}\sigma = \left(\sqrt{35}\right)(0.3) = 1.77$

Shorthand: $\hat{\tau} \sim N(73.5, 1.77)$ ➔ it should be approximately normal

find the probability that the total amount of NO2 for the fleet is between 70 and 75 g/m

$$P(70 < \hat{t} < 75) = P\left(\frac{70 - 73.5}{1.77} < Z < \frac{75 - 73.5}{1.77}\right) = P(-1.98 < Z < 0.85)$$

$$= P(Z < 0.85) - P(-1.98) =$$

stadistributions.com: 1st: z=.85, LT; 2nd: z=-1.98, LT