Real estate ads suggest that 64% of homes for sale have garages, 21% have swimming pools, and 17% have both. Find the following probabilities:

$$P(garage) = 0.64, P(pool) = 0.21, P(garage\ and\ pool) = 0.17$$

This is the perfect setup for the confusion matrix

	P(pool)	P(pool')	total
P(garage)	0.17	0.47	0.64
P(garage')	0.04	0.32	0.36
total	0.21	0.79	1

Pool or a garage: $P(pool\ or\ garage) = P(garage\ or\ pool)$

$$= P(garage) + P(pool) - P(garage\ and\ pool) = 0.64 + 0.21 - 0.17 = 0.68$$

Pool but no garage: $P(pool) - P(garage\ and\ pool) = 0.21 - 0.17 = 0.04$

Neither a pool nor a garage: neither=intersection

$$P(pool'and\ garage') = 0.32$$

Are having a pool and a garage independent?

$$P(pool\ and\ garage)? =?\ P(garage)P(pool) ==>$$

$$0.17? =?\ (0.64)(0.21)$$

$0.17 \neq 0.1344$ ∴ (therefore) having a garage or a pool is not independent

Are the events mutually exclusive/disjoint? The intersection between having a garage and pool exists, therefore they cannot be disjoint

Dr. Peter Venkman wanted to do a test on ESP. He randomly selected his volunteers and they were shown one card of 4 different ones, one card at a time (blank side facing the subject) and were told to guess what shape they thought was on the back side of the card. The test was done for a total of 10 cards per subject.

We have a sample size (10 trials per subject), probability of success is 25% (1 in 4 chance of guessing correctly), trials are independent (outcome of any one card has no impact on outcome of others) ➔ Binomial distribution

$$X \sim bin(n, p) ==> X \sim bin(10,0.25)$$

$P(X = x) = \binom{n}{x} p^x q^{n-x}$ with $q = 1 - p$

Probability of exactly one correct guess

$P(X = 1) = \binom{10}{1}(0.25)^1(0.75)^{10-1} = 0.1877 = 18.77\%$ chance of guessing exactly one

At least 8 correct guesses

$$P(X \geq 8) = P(8) + P(9) + P(10)$$

$$P(8) = \binom{10}{8}(0.25)^8(0.75)^{10-8} = 0.00039$$

$$P(9) = \binom{10}{9}(0.25)^9(0.75)^{10-9} = 0.000029$$

$$P(10) = \binom{10}{10}(0.25)^{10}(0.75)^{10-10} = 0.00000095$$

$$P(X \geq 8) = 0.00039 + 0.000029 + 0.00000095 = 0.0004$$

$$EX = np = 10(0.25) = 2.5$$

$$VX = npq = 10(0.25)(0.75) = 1.875$$

$$SDX = \sqrt{VX} = \sqrt{1.875} = 1.369$$