

The number of daily requests for emergency assistance at a fire station in a medium-sized city has the probability distribution show here.

y	0	1	2	3	4	5	6	7	8	9	10
P(y)	0.06	0.14	0.16	0.14	0.12	0.10	0.08	0.07	0.06	0.04	0.03

- What is the probability that four or more requests will be made in a particular day?  
 $P(Y \geq 4) = P(4) + P(5) + \dots + P(10) = 1 - P(Y < 4) = 1 - P(Y \leq 3)$   
 $= 1 - [P(3) + P(2) + P(1) + P(0)] = 1 - (0.14 + 0.16 + 0.14 + 0.06)$   
 $= 1 - 0.5 = 0.5$
- What is the probability that the requests for assistance will be at least four but no more than six?  
 $P(4 \leq Y \leq 6) = P(4) + P(5) + P(6) = 0.12 + 0.10 + 0.08 = 0.30$
- What is the probability that there will be at least 2 calls for assistance in a day?  
 $P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1) = 1 - [P(1) + P(0)]$   
 $= 1 - (0.14 + 0.06) = 1 - 0.2 = 0.8$
- What is the probability that there will be *more than 2* calls for assistance in a day?  
 $P(X > 2) = 1 - P(X \leq 2) = 1 - [P(2) + P(1) + P(0)]$   
 $= 1 - (0.16 + 0.14 + 0.06) = 1 - 0.36 = 0.64$
- On average, how many calls would the fire station expect to get in a day? Calculate the variance and standard deviation as well.

$$EX = \sum xp(x), VX = \sum (x - EX)^2 p(x), SDX = \sqrt{VX}$$

$$EX = 0(0.06) + 1(0.14) + 2(0.16) + 3(0.14) + \dots + 10(0.03) = 3.97$$

You will see means that are not possible in the actual distribution. As in “the average number of kids per family is 1.97”, yet 1.97 children is not possible. It is read as most families **on average** have 2 kids, with some having less.

$$VX = \sum (x - EX)^2 p(x) =$$

$$(0 - 3.97)^2(0.06) + (1 - 3.97)^2(0.14) + (2 - 3.97)^2(0.16) + (3 - 3.97)^2(0.14)$$

$$+ (4 - 3.97)^2(0.12) + \dots + (10 - 3.97)^2(0.03) = 7.0891$$

$$SDX = \sqrt{VX} = \sqrt{7.0891} = 2.66$$

In an inspection of automobiles in Los Angeles, 60% of all automobiles had emissions that did not meet EPA expectations. For a random sample of 10 automobiles, find the following:

- What is this distribution? What are its parameter(s)?  
 $X \sim \text{bin}(n, p) \Rightarrow X \sim \text{bin}(10, 0.6)$   $P(X = x) = \binom{n}{x} p^x q^{n-x}$  with  $q = 1 - p$
- All 10 autos failed the inspection  
 $P(X = 10) = \binom{10}{10} (0.6)^{10} (0.4)^{10-10} = (1)(0.6)^{10}(1) = 0.006 = 0.6\%$
- Exactly 6 of the 10 failed the inspection  
 $P(X = 6) = \binom{10}{6} (0.6)^6 (0.4)^{10-6} = \binom{10}{6} (0.6)^6 (0.4)^4 = 210(0.6)^6 (0.4)^4 = 0.2508$   
 $= 25.08\%$
- Six or more failed the inspection  
 $P(X \geq 6) = P(6) + P(7) + \dots + P(10)$   
 $P(X = 7) = \binom{10}{7} (0.6)^7 (0.4)^{10-7} = \binom{10}{7} (0.6)^7 (0.4)^3 = 120(0.6)^7 (0.4)^3 = 0.2150$   
 $P(X = 8) = \binom{10}{8} (0.6)^8 (0.4)^{10-8} = \binom{10}{8} (0.6)^8 (0.4)^2 = 45(0.6)^8 (0.4)^2 = 0.1209$   
 $P(X = 9) = \binom{10}{9} (0.6)^9 (0.4)^{10-9} = \binom{10}{9} (0.6)^9 (0.4)^1 = 10(0.6)^9 (0.4)^1 = 0.0403$   
 $P(X \geq 6) = 0.2508 + 0.2150 + 0.1209 + 0.0403 + 0.006 = 0.633$
- All 10 passed the inspection  
 $P(X = 0) = \binom{10}{0} (0.6)^0 (0.4)^{10-0} = (1)(1)(0.4)^{10} = 0.0001 = 0.01\%$
- On average, how many autos would we expect to fail the emissions? Calculate the variance and standard deviation as well  
 $EX = np = (10)(0.6) = 6$   
 $VX = npq = 10(0.6)(0.4) = 2.4$   
 $SDX = \sqrt{npq} = \sqrt{2.4} = 1.55$
- On average, how many autos would we expect to pass the emissions? Calculate the variance and standard deviation as well  
 $EX = np = (10)(0.4) = 4$   
 $VX = npq = 10(0.4)(0.6) = 2.4$   
 $SDX = \sqrt{npq} = \sqrt{2.4} = 1.55$

A publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is 0.005 and errors are independent of each other. Find the following:

- What is this distribution? What are its parameter(s)? (no sample size = Poisson)

$$X \sim \text{pois}(\mu) \Rightarrow X \sim \text{pois}(0.005), P(X = x) = \frac{e^{-\mu} \mu^x}{x!}$$

- Exactly no errors are found

$$P(X = 0) = \frac{e^{-0.005} (0.005)^0}{0!} = e^{-0.005} \approx 0.995$$

- Exactly 1 error is found

$$P(X = 1) = \frac{e^{-0.005} (0.005)^1}{1!} = 0.004975 \approx 0.005$$

- At least 1 error is found

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - 0.995 = 0.005$$

- At least 2 errors are found

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1) = 1 - [P(1) + P(0)] \\ = 1 - (0.995 + 0.005) = 0 \text{?!? YES it is equal to 0 because the errors are VERY rare (0.005)}$$

- Exactly 4 errors

$$P(X = 4) = \frac{e^{-0.005} (0.005)^4}{4!} = 2.59 \times 10^{-11} = 0.0000000000259 \approx 0$$

- On average, how many typos would we expect to find? Calculate the variance and standard deviation as well

$$EX = \mu, VX = \mu, SDX = \sqrt{VX}$$

$$EX = 0.005, VX = 0.005, SDX = \sqrt{0.005} = 0.0707 \approx 0.071$$