

2-sample methods

Module 10 review

Statistics 251: Statistical Methods

Logistics

A (normal) CI is the same as a 2 tailed test. If your hypothesized value is in the CI, you cannot reject H_0 . If the hypothesized value is not in the CI, you can reject H_0 . This only applies to two-tailed tests and (normal) confidence intervals

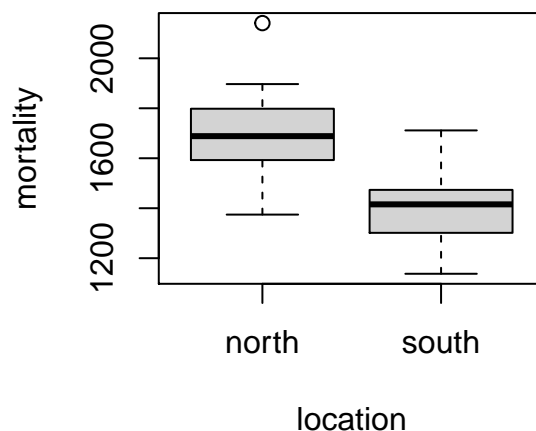
Checklist

- (1) State hypotheses, check assumptions if requested
- (2) State t statistic, df , and $pvalue$ from output
- (3) State test results
- (4) Make conclusion in context from results
- (5) State possible error that could have been made and discuss it within the context

Hard water

Hard water is an issue for many reasons, from a nuisance to clean to issues with mortality. The mortality rate for males in 61 randomly selected large towns in England and Wales was collected (per 100,000 people) and the water hardness was recorded as the calcium concentration (ppm) in the drinking water. The data notes for each town, whether it was north or south of Derby. Is there a significant difference in mortality rates in the two regions? Estimate the difference in means between the two areas with 95% confidence.

```
boxplot(mortality~location)
```



```
t.test(mortality~location)
```

Welch Two Sample t-test

```
data: mortality by location  
t = 7.4487, df = 58.466, p-value = 4.939e-10
```

alternative hypothesis: true difference in means is not equal to 0
 95 percent confidence interval:
 208.1627 361.1223
 sample estimates:
 mean in group north mean in group south
 1680.435 1395.792

$$H_0 : \mu_1 = \mu_2 \text{ vs. } H_a : \mu_1 \neq \mu_2$$

Assumptions:

- (1) Independence: random so yes
- (2) Randomization: yes
- (3) Normality: boxplots look mostly normal so yes

Organization of information:

$H_a : \neq$ (two tail test)

$\alpha = 0.05$ (assumed because not specifically stated otherwise)

$t = 7.4487$, $df = 58.466$, $pvalue = 4.939e-10 = 4.93 \times 10^{-10} \approx 0$

Results: $pvalue \approx 0 \leq \alpha(0.05) \therefore H_0$ is rejected

Conclusion: since the null is rejected, there is a significant difference in mortality rates in the two regions

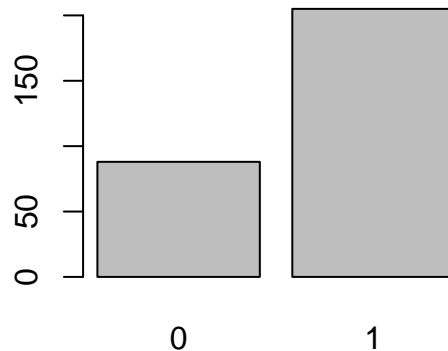
Error: since H_0 was rejected, a Type I error (reject null when null is true) could have been made; we think there is a significant difference in mortality rates in the two regions when there is not.

CI: (208.1627, 361.1223). With 95% confidence, the true difference in mean mortality rates between the regions is between 208.2 and 361.1. In other words, The North Region has between 208.2 and 361.1 more deaths per 100,000 people than the south region.

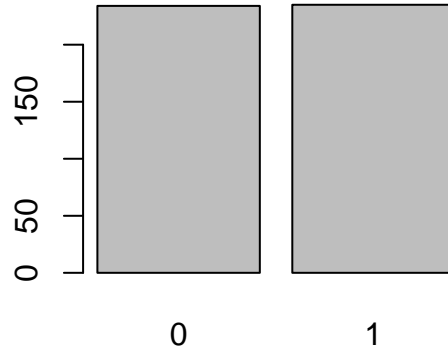
Tech use before bed

A 2011 poll randomly surveyed people asked about their sleep habits, and in particular, their use of technology around the time they try to go to bed. A poll found that 205 of 293 (70%) of Gen-Yers and 235 of 469 (50%) of Gen-Xers use technology within the hour before bed. Is there sufficient evidence that more Gen-Yers than Gen-Xers use technology within the hour before bed? Estimate the true difference in proportions with 99% confidence. Let $\alpha = 0.01$.

```
barplot(table(geny))
```



```
barplot(table(genx))
```



```
t.test(geny,genx,conf.level=.99,alternative='g')
```

Welch Two Sample t-test

```
data: geny and genx
t = 5.6085, df = 659.62, p-value = 1.502e-08
alternative hypothesis: true difference in means is greater than 0
99 percent confidence interval:
 0.1160172      Inf
sample estimates:
mean of x mean of y
0.6996587 0.5010661
```

```
t.test(geny,genx,conf.level=.99)
```

Welch Two Sample t-test

```
data: geny and genx
t = 5.6085, df = 659.62, p-value = 3.004e-08
alternative hypothesis: true difference in means is not equal to 0
99 percent confidence interval:
 0.1071192 0.2900660
sample estimates:
mean of x mean of y
0.6996587 0.5010661
```

$$H_0 : \pi_1 = \pi_2 \text{ vs. } H_a : \pi_1 > \pi_2$$

Assumptions:

- (1) Independence: random so yes
- (2) Randomization: yes
- (3) Normality: $n_1 = 293 \geq 60$ and $n_2 = 469 \geq 60$ so yes

Organization of information:

$H_a : >$ (upper tail test)
 $\alpha = 0.01$ (specifically stated)

$t = 5.6085$, $df = 659.62$, $pvalue = 1.502e-08 = 1.502 \times 10^{-8} \approx 0$

Results: $pvalue \approx 0 \leq \alpha(0.01) \therefore H_0$ is rejected

Conclusion: since the null is rejected, there is sufficient evidence that more Gen-Yers than Gen-Xers use technology within the hour before bed. There is a significant difference

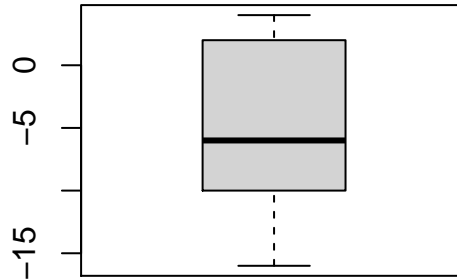
Error: since H_0 was rejected, a Type I error (reject null when null is true) could have been made; we think more Gen-Yers than Gen-Xers use technology within the hour before bed but they do not.

CI: $(0.1071192, 0.2900660) = (10.71\%, 29.01\%)$. With 99% confidence, the true difference in proportions of Gen-Yers vs. Gen-Xers that use technology within the hour before bed is between 10.71% and 29.01%. In other words, between 10.71% and 29.01% more Gen-Yers use technology before bed than Gen-Xers

Summer Course

Having done poorly on their math finals in June, six randomly selected students attend a summer course and retake the exams in August. Is there evidence that the course was effective? Let $\alpha = 0.1$. Estimate the true mean difference with 90% confidence.

```
boxplot(scorediff)
```



```
t.test(score~time,paired=T,conf.level=.9,alternative='g')
```

Paired t-test

```
data: score by time
t = 1.7541, df = 5, p-value = 0.06989
alternative hypothesis: true difference in means is greater than 0
90 percent confidence interval:
 0.8459554      Inf
sample estimates:
mean of the differences
          5.333333
```

```
t.test(score~time,paired=T,conf.level=.9)
```

Paired t-test

```
data: score by time
t = 1.7541, df = 5, p-value = 0.1398
alternative hypothesis: true difference in means is not equal to 0
90 percent confidence interval:
-0.7933564 11.4600230
sample estimates:
mean of the differences
          5.333333
```

$$H_0 : \mu_d = 0 \text{ vs. } H_a : \mu_d > 0$$

Assumptions:

- (1) Independence of subjects: random yes
- (2) Randomization: yes
- (3) Normality: boxplot of differences looks approximately normal yes
- (4) Two measurements on each subject yes

Organization of information:

$H_a : >$ (upper tail test)

$\alpha = 0.1$ (specifically stated)

$t = 1.7541$, $df = 5$, $pvalue = 0.06989$

Results: $pvalue = 0.06989 \leq \alpha(0.1) \therefore H_0$ is rejected

Conclusion: since the null is rejected, there is sufficient evidence to suggest that the course was effective. Results are significant.

Error: since H_0 was rejected, a Type I error (reject null when null is true) could have been made; we think the course was effective when it is not

CI: $(-0.7933564, 11.4600230)$. With 90% confidence, the true difference mean scores before and after the summer course is between -0.79 and 11.46; with 0 in the interval, there is no significant difference in the scores.